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A Note on Consul and Geeta Distributions

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SUMMARY

It has been observed that the two basic Lagrangian distributions, the Consul distribution of Consul (1983) and the Geeta distribution of Consul (1990) are nothing but the particular cases of shifted generalized negative binomial distribution (GNBD) of Jain and Consul (1971). The two distributions do not have identities separate from that of the GNBD and so the various properties of these two distributions can easily be deduced from those of the GNBD. A number of other such particular distributions of the GNBD may also be obtained.

Keywords: Generalized geometric distribution, Generalized negative binomial distribution, Particular cases, Shifted distribution.

1. INTRODUCTION

A discrete random variable X is said to have Consul distribution if its probability mass function (pmf) is given by [see Consul and Shenton (1975), Consul (1983)]

$$P_1(x) = \frac{1}{x}\binom{\beta x}{x-1}\alpha^{x-1}(1-\alpha)^{\beta x - x+1};\ 0 < \alpha < 1,$$
$$1 \le \beta \le \alpha^{-1},\ x = 1, 2, \ldots \quad (1.1)$$

As at $\beta = 1$, this distribution reduces to the geometric distribution, Famayo (1997) named it generalized geometric distribution and discussed some of its properties and its applications.

The mean and variance of this distribution are obtained as

$$\mu = 1/(1-\alpha\beta),\quad \sigma^2 = \alpha\beta(1-\alpha)/(1-\alpha\beta)^3 \quad (1.2)$$

Another distribution named as Geeta distribution has been defined and its various properties have been studied by Consul (1990). The pmf of this distribution is given by

$$P_2(x) = \frac{1}{\beta x - 1}\binom{\beta x - 1}{x}\alpha^{x-1}(1-\alpha)^{\beta x - x};$$
$$0 < \alpha < 1,\ 1 \le \beta \le \alpha^{-1},\ x = 1, 2, \ldots \quad (1.3)$$

The mean and variance of this distribution have been obtained as

$$\mu = (1-\alpha)/(1-\alpha\beta);\quad \sigma^2 = (\beta-1)\,\alpha\,(1-\alpha)/(1-\alpha\beta)^3 \quad (1.4)$$

Both the distributions (1.1) and (1.3) belong to the class of basic Lagrangian distributions having pmf as

$$P_3(x) = \frac{1}{\Gamma(x+1)}\left[\frac{d^{x-1}}{dz^{x-1}}(g(z))^x\right]_{z=0};\ x \in N \quad (1.5)$$

where $g(z)$ is a successively differentiable function such that $g(1) = 1$ and $g(0) \ne 0$. Selecting $g(z) = (1 - \alpha + \alpha z)^{\beta}$ and $g(z) = [(1-\alpha)/(1-z\alpha)]^{\beta-1}$ in (1.5), the Consul distribution (1.1) and the Geeta distribution (1.3) can be obtained respectively.

Jain and Consul (1971) defined a three-parameter generalized negative binomial distribution (GNBD) by its pmf

$$P_4(x; m, \beta, \alpha) = \frac{m}{m+\beta x}\binom{m+\beta x}{x}\alpha^x(1-\alpha)^{m+\beta x - x};$$
$$0 < \alpha < 1,\ 1 < \beta < \alpha^{-1};\ x = 0, 1, 2, \ldots \quad (1.6)$$

At $\beta = 1$, this distribution reduces to the negative binomial distribution and at $\beta = 0$ to the binomial distribution. Its mean and variance have been obtained as

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$\mu = m\alpha/(1-\alpha\beta)$ and $\sigma^2 = m\alpha(1-\alpha)/(1-\alpha\beta)^3$ (1.7)

The GNBD (1.6) belongs to the class of generalized Lagrangian distributions which has the pmf

$$P_5(x) = f(0);\ x = 0$$

$$= \frac{1}{\Gamma(x+1)} D^{x-1}\left[g(z)^x f'(z)\right]_{z=0};\ x \in N \quad (1.8)$$

Selecting $g(z) = (1-\alpha+\alpha z)^\beta$ and $f(z) = (1-\alpha+\alpha z)^m$, the GNBD (1.6) can be obtained. The GNBD with support 1, 2, 3, ... is obviously of the form

$$P_6(x) = \frac{m}{m+\beta+\beta x}\binom{m+\beta+\beta x}{x-1} \alpha^{x-1}(1-\alpha)^{m-\beta+\beta x-x+1}; \quad (1.9)$$

In this paper it has been observed that the Consul distribution (CD) and the Geeta distribution (GD) both of which seem to have their own separate identities are nothing but the particular cases of the shifted GNBD (1.9) and so all the properties possessed by the GNBD would also be possessed by the Consul and Geeta distributions.

2. CD AND GD AS PARTICULAR CASES OF GNBD

The CD (1.1) for the support 0, 1, 2, … can be written as

$$P_7(x) = \frac{1}{x+1}\binom{\beta+\beta x}{x}\alpha^x(1-\alpha)^{\beta+\beta x-x}$$

$$= \frac{\beta}{\beta+\beta x}\binom{\beta+\beta x}{x}\alpha^x(1-\alpha)^{\beta+\beta x-x} \quad (2.1)$$

which is a GNBD $P_4(x;\beta,\beta,\alpha)$. It can be verified that taking $m = \beta$ in (1.7), the mean, of course after adding one, and the variance of the CD given in (1.2) are obtained. The higher moments of the CD can accordingly be obtained from the corresponding moments of the GNBD.

Moreover, using lattice path analysis, Mishra (1982) obtained a generalized geometric distribution (GGSD) having its pmf

$$P_8(x) = \frac{1}{1+\beta x}\binom{1+\beta x}{x}\alpha^x(1-\alpha)^{1+\beta x-x}$$

$$x = 0, 1, 2, \ldots \quad (2.2)$$

which is a particular case of the GNBD (1.6) with $m = 1$. The shifted GGSD (2.2) with support 1, 2, 3, ... is nothing but the CD (1.1).

The GD (1.3) for the support 0, 1, 2, ... can be written as

$$P_9(x) = \frac{1}{\beta-1+\beta x}\binom{\beta-1+\beta x}{x+1}\alpha^x(1-\alpha)^{\beta-1+\beta x-x} \quad (2.3)$$

$$= \frac{\beta-1}{\beta-1+\beta x}\binom{\beta-1+\beta x}{x}\alpha^x(1-\alpha)^{\beta-1+\beta x-x} \quad (2.4)$$

which is a GNBD, $P_4(x;\beta-1,\beta,\alpha)$. Taking $m = \beta-1$ in (1.7) it can be easily seen that the same mean, of course after adding one, and the same variance as given in (1.4) are obtained.

Thus it is clear that the CD (2.1) and the GD (2.3) are the GNBD's with $m = \beta$ and $m = \beta - 1$ respectively. It can also be seen that selecting $g(z) = f(z) = (1-\alpha+\alpha z)^\beta$, the CD can be obtained and selecting $g(z) = (1-\alpha+\alpha z)^\beta$, $f(z) = (1-\alpha+\alpha z)^{\beta-1}$ the GD can be obtained. So the basic nature of these distributions is the same as that of the GNBD. All the properties that are possessed by the GNBD can be shown to be possessed by these two distributions also by taking accordingly the values of m as β and $\beta - 1$.

3. OTHER SIMILAR DISTRIBUTIONS

We define a GNBD for integer k as

$$P_4(x;\beta+k,\beta,\alpha) = \frac{\beta+k}{\beta+k+\beta x}\binom{\beta+k+\beta x}{x}\alpha^x(1-\alpha)^{\beta+k+\beta x-x};$$

$$0 < \alpha < 1,\ \alpha\beta < 1,\ x = 0, 1, 2, \ldots \quad (3.1)$$

Writing $x - 1$ in place of x we get the shifted GNBD with support 1, 2, 3, … as

$$P_6(x;\beta+k,\beta,\alpha) = \frac{\beta+k}{k+\beta x}\binom{k+\beta x}{x-1}\alpha^{x-1}(1-\alpha)^{k+\beta x-x+1} \quad (3.2)$$

Obviously the CD and the GD can be obtained from (3.2) taking $k = 0$ and $k = 1$ respectively. Many such similar distributions can be obtained by selecting the values of k. Some of them are given below.

$$P_6\,(x\,;\beta+1,\beta,\alpha) = \frac{\beta+1}{\beta x - x + 2}\binom{\beta x}{x-1}\alpha^{x-1}(1-\alpha)^{\beta x - x + 2} \qquad (3.3)$$

$$P_6\,(x\,;\beta+2,\beta,\alpha) = \frac{\beta+2}{\beta x - x + 3}\binom{1+\beta x}{x-1}\alpha^{x-1}(1-\alpha)^{\beta x - x + 3} \qquad (3.4)$$

$$P_6\,(x\,;\beta-2,\beta,\alpha) = \frac{\beta-2}{\beta x - x - 1}\binom{\beta x-3}{x-1}\alpha^{x-1}(1-\alpha)^{\beta x - x - 1} \qquad (3.5)$$

The distribution (3.2) satisfies the properties of both over dispersion, *i.e.*, the variance is greater than the mean and under dispersion *i.e.*, the variance is less than the mean. We have for this distribution

$$\mu = (1+\alpha k)/(1-\alpha\beta) \quad \text{and}$$

$$\sigma^2 = (\beta + k)\,\alpha\,(1-\alpha)/(1-\alpha\beta)^3 \qquad (3.6)$$

and so $$\sigma^2/\mu = (\beta+k)\,\alpha\,(1-\alpha)/(1+\alpha k)\,(1-\alpha\beta)^2 \qquad (3.7)$$

which gives variance greater than, equal to and less than the mean as k is greater than, equal to and less than $[1-\alpha\beta\,(3-\alpha\beta-\alpha)]/(2\beta-\alpha\beta^2-1)$ respectively.

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REFERENCES

Consul, P.C. (1983). Lagrange and related probability distributions. *Encyclopedia Statist. Scis.*, **4**, S. Kotz, N.L. Johnson and C.B. Read (Editors), 448-454, New York, Wiley.

Consul, P.C. (1990a). Geeta distribution and its properties. *Comm. Statist. -Theory Methods*, **19**, 3051-3068.

Consul, P.C. (1990b). Two stochastic models for Geeta distribution. *Comm. Statist. -Theory Methods*, **19**, 3699-3706.

Consul, P.C. and Shenton, L.R. (1975). On the probabilistic structure and properties of discrete Lagrangian distributions. In : *Statistical Distributions in Scientific Work*, G.P. Patil, S. Kotz and J.K. Ord (Editors), 41-57, D. Reidal Publishing Company, Boston, M.A.

Famoye, F. (1997). Generalised geometric distribution and some of its applications. *J. Math. Sci.*, **8**, 219-237.

Jain, G.C. and Consul, P.C. (1971). A generalized negative binomial distribution. *SIAM J. Appl. Math.*, **21**, 501-513.

Mishra, A. (1982). A generalization of geometric distribution. *J. Bihar Math. Soc.*, **6**, 14-17.