



Some Nonlinear Time-series Models and their Applications

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1. INTRODUCTION

At the very outset, I express my gratitude to the Indian Society of Agricultural Statistics for inviting me to be the Sessional President this year. It is my honour and proud privilege to deliver the Technical Address.

Well-known Box-Jenkins Autoregressive integrated moving average (ARIMA) methodology has virtually dominated analysis of time-series data since 1930s. However, it can be applied only when either the series under consideration is stationary or can be made so by differencing, detrending, or any other means. Another disadvantage is that this approach is ‘empirical’ in nature and does not provide any insight into the underlying mechanism. An alternative mechanistic approach, which is quite promising, is the Structural time-series modeling (STM). Here, the basic philosophy is that characteristics of the data dictate the particular type of model to be adopted from the family. Several families of models have been developed, which are applicable depending on whether the data exhibit prominent trend, or seasonal variations or cyclical fluctuations. For example, when the trend is dominant, three models, viz. Local level model, Local linear trend model, and Local linear trend model with intervention are available. A good description of these models is given in Harvey (2001). Ravichandran and Prajneshu (2002) carried out modelling and forecasting of India’s foodgrain production for the post-Green revolution era, viz. 1966-98 through two dynamical modelling techniques, viz. STM and Bayesian analysis of time-series.

However, both ARIMA models and STM are “Linear”. During the last three decades or so, the area of “Nonlinear time-series modelling” is rapidly growing (Fan and Yao 2003). Here, there are basically two approaches, viz. Parametric or Nonparametric. Evidently, if in a particular situation, we are quite sure about the functional form, we should use the former, otherwise the latter may be employed. Although several parametric families have been developed, the four most widely used are discussed in Section 2. Section 3 deals with two important nonparametric families. Finally, some future research problems are identified in Section 4.

2. PARAMETRIC NONLINEAR TIME-SERIES MODELS

(a) Bilinear Time-series Model

The most natural way to introduce nonlinearity into a linear ARIMA model is to add product terms. By restricting to products of time-series variable X_{t-j} and errors ε_{t-k} , the resultant bilinear model BL(p, q, r, s) is

$$X_t = \sum_{j=1}^p a_j X_{t-j} + \varepsilon_t + \sum_{k=1}^q b_k \varepsilon_{t-k} + \sum_{j=1}^r \sum_{k=1}^s c_{jk} X_{t-j} \varepsilon_{t-k} \quad (1)$$

where $\varepsilon_t \sim \text{IID}(0, \sigma^2)$, a_j , b_k and c_{jk} are parameters. Here p denotes lag in the linear part of autoregression in X_t , while r and s are lags in the nonlinear terms involving past observations and error series $\{\varepsilon_t\}$. The coefficients in past observations depend on past shocks thus enabling the model to capture data with high level crossings.

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Although bilinear models were proposed almost three decades ago and a number of theoretical contributions have been made in succeeding period, yet application of such models to real data is still a nontrivial task. Ghosh *et al.* (2006b) have developed computer programs in C-language for fitting of bilinear models. As mentioned earlier, bilinear models are of particular importance to describe those data sets that depict sudden bursts of large amplitude at irregular time epochs. As an illustration, India's marine products export data during the period 1961-62 to 1998-99 is considered. Based on normalized Akaike information criterion (NAIC), appropriate bilinear time-series model is fitted by applying Newton-Raphson iterative procedure.

(b) Autoregressive Conditional Heteroscedastic Time-series (ARCH) Models

In a path-breaking work, Engle (1982) proposed the ARCH model for which he was awarded the prestigious Nobel Prize in Economics in 2003. This entails a completely different class of models which is concerned with modelling volatility. The objective is not to give better point forecasts but rather to give better estimates of the variance which, in turn, allows more reliable forecast intervals leading to a better assessment of risk. The ARCH model allows the conditional variance to change over time as a function of squared past errors leaving the unconditional variance constant to model varying (conditional variance) or volatility of a time-series. It is often found that larger values of time-series also lead to larger instability (*i.e.*, larger variances), which is termed conditional heteroscedasticity. The process $\{\varepsilon_t\}$ is ARCH(q), if the conditional distribution of $\{\varepsilon_t\}$, given available information ψ_{t-1} , is

$$\varepsilon_t | \psi_{t-1} \sim N(0, h_t), h_t = a_0 + \sum_{i=1}^q a_i \varepsilon_{t-i}^2 \quad (2)$$

where $\varepsilon_t \sim \text{IID}(0, 1)$ and $a_i \geq 0$ are parameters.

SAS or EViews software packages may be employed for fitting of these models. Ghosh and Prajneshu (2003) applied these models to describe volatile monthly onion price data during April 1996 to October 2001. As the assumption of constant one-period ahead forecast variance did not hold, ARCH process was fitted and out-of-sample forecasts for four months were developed.

However, the conditional variance of ARCH(q) model, where q indicates the order of maximum lag, has the property that the unconditional autocorrelation function (acf) of squared residuals, if it exists, decays very rapidly compared to what is typically observed, unless the maximum lag q is long. To overcome this limitation of ARCH model, Bollerslev (1986) proposed the Generalized ARCH (GARCH) model, in which the unconditional autocorrelation function of squared residuals has slow decay rate. It also gives parsimonious models that are easy to estimate and, even in its simplest form, has proven surprisingly successful in predicting conditional variances. In the GARCH model, the conditional variance is assumed to be a linear function of its own lags and has the form

$$h_t = a_0 + \sum_{i=1}^q a_i \varepsilon_{t-i}^2 + \sum_{j=1}^p b_j h_{t-j} \quad (3)$$

This has the property that the unconditional autocorrelation function of ε_t^2 , if it exists, can decay slowly. As eq. (3) is a more parsimonious model of the conditional variance than a high-order ARCH model, it is preferred over the simpler ARCH alternative. The overwhelmingly most popular GARCH model in applications is the GARCH(1, 1) model.

For estimating parameters of GARCH model, most widely used method is the Gaussian maximum likelihood estimation (GMLE) method. The maximum likelihood estimators are derived by minimizing

$$L_T(\theta) = T^{-1} \sum_{t=v}^T \left[\ln \sqrt{\tilde{h}_t} - \ln f\left(\varepsilon_t / \sqrt{\tilde{h}_t}\right) \right] \quad (4)$$

where \tilde{h}_t is the truncated version of h_t (Paul *et al.* 2009). Besides normal distribution, the function $f(\cdot)$ can also follow t -distribution. The Akaike information criterion (AIC) and Bayesian information criterion (BIC) values for GARCH model with t -distributed errors are computed as

$$\text{AIC} = \sum_{t=v}^T \left[\ln \sqrt{\tilde{h}_t} - \ln f\left(\varepsilon_t / \sqrt{\tilde{h}_t + 2(p+q+1)}\right) \right] + 2(p+q+1) \quad (5)$$

and

$$\text{BIC} = \sum_{t=v}^T \left[\ln \sqrt{\hat{h}_t} - \ln f(\varepsilon_t / \sqrt{\hat{h}_t}) \right] + 2(p + q + 1) \ln(T - v + 1) \quad (6)$$

where \ln indicates natural logarithm.

Sometimes, volatility due to positive and negative shocks may be asymmetric. To this end, Nelson (1991) proposed the Exponential GARCH (EGARCH) model, represented by specifying the logarithm of conditional variance as

$$\ln(h_t) = a_0 + \beta \ln(h_{t-1}) + \alpha \left| \varepsilon_{t-1} / \sqrt{h_{t-1}} \right| + \gamma \varepsilon_{t-1} / \sqrt{h_{t-1}} \quad (7)$$

Ghosh *et al.* (2010b) considered all-India data of monthly export of fruits and vegetables seeds during the period April 2000 to January 2007 comprising 82 data points, obtained from Indiatat website (www.indiatat.com). The first 76 data points were employed for building the models while the data for the remaining 6 months were used for validation purpose. The data exhibit presence of volatility at several time-epochs. The EViews software package, Ver. 4 was used for data analysis. Further, AIC and BIC values for t -distributed errors in GARCH model were computed separately by writing computer programs in C. It is observed that the kurtosis is extremely high, which reflects a “leptokurtic” behaviour. Therefore, instead of Gaussian distribution, error distribution was assumed as Student’s t -distribution. The mean and conditional variance for fitted AR(1) - GARCH(1, 1) model were computed. A visual inspection shows that the fitted model is not able to capture properly the volatility present at various time-epochs in the data set, perhaps due to the fact that positive and negative shocks are not symmetric. To capture this asymmetric nature of volatility, EGARCH model was employed. The AIC and BIC values for fitted EGARCH model were respectively found to be much less than the corresponding values for the fitted GARCH model. This clearly shows the superiority of EGARCH model over GARCH model for the data under consideration. It is seen that the fitted model is able to capture quite well

the volatility present at various time-epochs. One-step ahead forecasts along with the corresponding forecast standard errors were computed, which could be of immense help to planners in formulating appropriate strategies.

(c) Mixture Nonlinear Time-series Models

These models may be employed to describe those data sets that depict sudden bursts, outliers and flat stretches at irregular time-epochs. Three models, viz. Gaussian mixture transition (GMTD), Mixed autoregressive (MAR) and MAR-ARCH were thoroughly studied by Ghosh *et al.* (2006a). Weekly wholesale onion price data during April 1998 to November 2001 were considered. After eliminating trend, seasonal fluctuations were studied by fitting Box-Jenkins airline model to residual series. The tests for presence of nonseasonal and seasonal stochastic trends and use of appropriate filters in airline models were also examined. Presence of ARCH was tested by Lagrange multiplier test. Estimation of parameters was done using Expectation-Maximization algorithm and the best model was selected on the basis of BIC. Out-of-sample forecasting was performed for one-step and two-step ahead prediction by naive approach. It is concluded that, for data under consideration, a three-component MAR and a two-component MAR-ARCH is the best in respective classes. Further, identified MAR-ARCH model is also shown to perform better than three-component MAR model identified earlier in terms of having fewer numbers of parameters and lower BIC value.

For seasonal data, Ghosh *et al.* (2010a) investigated Periodic autoregressive and Mixture-periodic ARCH models and applied these for modelling and forecasting of monthly rainfall data of Sub-Himalayan region of West Bengal, India during the period January 1990 to December 2006, obtained from the website (www.tropmet.res.in) of the Indian Institute of Tropical Meteorology, Pune, India. The data showed a periodic variation with marked volatility at some time-epochs. Salient feature of the work done is that the best predictor and prediction error variance for carrying out out-of-sample forecasting up to three-steps ahead were derived analytically by recursive use of conditional expectation and conditional variance.

(d) Threshold Autoregressive Time-series Models

Another important parametric nonlinear time-series family of models is that of Threshold autoregressive (TAR) models, initiated by Tong and Lim (1980). These assume different linear forms in different regions of the State space, which is usually dictated by one *threshold variable*, say X_{t-d} for some $d \geq 1$. The model is of the form

$$X_t = a_0^{(i)} + a_i^{(i)} X_{t-1} + \dots + a_p^{(i)} X_{t-p} + \varepsilon_t^{(i)}, \text{ if } X_{t-d} \in R_i \quad (8)$$

for $i = 1, \dots, k$, where R_i forms a partition of the real line and $\varepsilon_t^{(i)} \sim \text{IID}(0, \sigma_i^2)$. The simplest thresholding model is the Two-regime (*i.e.*, $k = 2$) TAR model with $R_1 = \{X_{t-d} \leq r\}$. The AR parameter depends on whether X_{t-d} exceeds the threshold value r ; hence the name TAR. Further, if $r = -\infty$, TAR model reduces to AR(1) model. A comprehensive discussion of TAR models is given in Tong (1995). STAR software package may be used for fitting of these models. A heartening feature of TAR models is that these are capable of describing cyclical fluctuations. Ghosh *et al.* (2006c) applied Self-exciting TAR (SETAR) model to describe India’s lac export annual data during the period 1901-2001. The data set exhibited prominent cycles of approximately 13 years. The best-identified model on the basis of minimum NAIC value was found to be SETAR two-regime model with delay parameter $d = 1$ and $p = 2$. Finally, out-of-sample forecasts for years 2002 and 2003 were obtained. For more efficient estimation of parameters of SETAR model, Iquebal *et al.* (2010) employed stochastic optimization technique of genetic algorithm, which is based on the principles of genetics and natural selection.

The SETAR moving average (SETARMA) is an important extension of SETAR model capable of describing cyclical data having sudden rise and fall, *i.e.*, the model is capable of exhibiting “limit cycle” behaviour. The SETARMA model of order $(2; p_1, p_2; q_1, q_2)$ can be expressed as

$$y_t = \begin{cases} \varphi_0^{(1)} + \sum_{j=1}^{p_1} \varphi_j^{(1)} y_{t-j} + \varepsilon_t^{(1)} + \sum_{w=1}^{q_1} \theta_w^{(1)} \varepsilon_{t-w}^{(K_{t-w-d})}, & \text{if } y_{t-d} \leq r \\ \varphi_0^{(2)} + \sum_{j=1}^{p_2} \varphi_j^{(2)} y_{t-j} + \varepsilon_t^{(2)} + \sum_{w=1}^{q_2} \theta_w^{(2)} \varepsilon_{t-w}^{(K_{t-w-d})}, & \text{if } y_{t-d} > r \end{cases} \quad (9)$$

where $\varepsilon_t^{(i)}$ is a white noise process with zero mean and finite variance $\sigma^{2(i)}$, $i = 1, 2$; p_i and q_i are nonnegative integers referred respectively as the autoregressive (AR) and moving average (MA) orders; $\varphi_j^{(i)}$ and $\theta_w^{(i)}$ are unknown AR and MA parameters, with $j = 1, 2, \dots, p_i$ and $w = 1, 2, \dots, q_i$; r is threshold value and d is delay parameter. Like an indicator variable, K_{t-w-d} takes value 1 or 2 according as y_{t-w-d} is less than or greater than r . It may be noted from eq. (9) that the SETARMA model is linear in each sub-space but it is nonlinear over the entire state space. Recently, Samanta *et al.* (2011) have developed the methodology for estimation of parameters of SETARMA two-regime model by using a very efficient stochastic optimization technique of Real-coded genetic algorithm. Subsequently, it was applied for modelling and forecasting of Indian mackerel time-series data.

Another parametric model for describing cyclicity is the Exponential autoregressive (EXPAR) family. An EXPAR (p) model may be written as

$$X_t = \left\{ \varphi_1 + \pi_1 \exp(-\gamma X_{t-1}^2) \right\} X_{t-1} + \dots + \left\{ \varphi_p + \pi_p \exp(-\gamma X_{t-1}^2) \right\} X_{t-p} + \varepsilon_t \quad (10)$$

where φ_i and π_i represent the autoregressive and exponential autoregressive parameters at lag i , $\gamma > 0$ is some scaling constant and $\{\varepsilon_i\}$ is white noise process with mean zero and variance σ_ε^2 . In practical situations, exact data generating process of time-series observations is not known. Therefore, fitted values from linear and nonlinear models may be used as explanatory variables to empirically describe the same. Recently, Ghosh *et al.* (2011a) combined the three models, viz. the ARIMA, EXPAR and SETAR models, which are capable of capturing the cyclical behaviour by using the Constant coefficient regression method as well as the Time-varying coefficient regression method through Kalman filtering technique. As an illustration, the models were then applied to describe annual Mackerel catch time-series data of Karnataka, India. It is found that the fitted model by using the latter approach performs the best.

3. NONPARAMETRIC NONLINEAR TIME-SERIES MODELS

Parametric families discussed above provide powerful tools for analyzing time-series data when models are correctly specified. However, any parametric model is at best only an approximation to the true stochastic dynamics that generates a given data set. A time-series may be of the type for which there is no suitable parametric model that gives a good fit to the data under consideration. In such a situation, “Nonparametric” approach may be employed.

(a) Functional-coefficient Autoregressive Model

A very versatile model of the above type is Functional-coefficient autoregressive (FCAR) model given by

$$X_t = a_1(X_{t-d}) X_{t-1} + \dots + a_p(X_{t-d}) X_{t-p} + \sigma(X_{t-d}) \varepsilon_t \quad (11)$$

where $\varepsilon_t \sim \text{IID}(0, 1)$ and is independent of X_{t-1}, X_{t-2}, \dots . The FCAR model (Fan and Yao 2003) depends critically on choice of model-dependent variable X_{t-d} , which is one of the lagged variables. Ghosh *et al.* (2010c) applied FCAR model for forecasting of India’s annual export lac data during the period 1900 to 2000. Comparison of the performance of FCAR model vis-à-vis the SETAR and ARIMA models was also made from the viewpoint of dynamic one-step and two-step ahead forecasts along with Mean square prediction error, Mean absolute prediction error and Relative mean absolute prediction error. The SAS, Ver. 9.1 and SPSS software packages were used for data analysis. Superiority of FCAR model over SETAR and ARIMA models was demonstrated for the data under consideration.

(b) Wavelet Analysis

Currently, there is a lot of interest in employing “Wavelet analysis” for nonparametric nonlinear time-series modelling. Novel idea of wavelets is that these are localized in both time and space whereas traditional Fourier bases are localized only in frequency but not in time. The theory of wavelets permits decomposition of functions into localized oscillating components and so is an ideal tool for modelling and forecasting purposes. Wavelet analysis can be carried out either in Time domain or in Frequency domain. An excellent description of various aspects of Wavelet methodology

is given in Percival and Walden (2000) and Abramovich *et al.* (2000). A further improvement in this methodology is incorporation of the concept of “thresholding”. Various types of thresholding are discussed in Jansen (2001). For Wavelet thresholding approach in time domain, Sunilkumar and Prajneshu (2004) carried out modelling and forecasting of India’s meteorological subdivisions rainfall data. This methodology was extended by Sunilkumar and Prajneshu (2008) to the situation when the errors are autocorrelated and applied it to describe India’s marine fish production. Wavelet thresholding methodology was also employed by Sunilkumar and Prajneshu (2009) for detection of jump points in time-series data. In respect of India’s oilseed yield data during 1939-2002, existence of one jump point in 1988 was established, thereby reflecting the success of “Technology Mission on Oilseeds” set up by the Government of India.

Ghosh *et al.* (2011b) investigated the Wavelet approach in frequency domain for analyzing time-series data. As an illustration, Indian monsoon rainfall time-series data from 1879-2006 was considered. It is found that the size of the test for testing significance of trend in respect of Daubechies wavelet is more than that for Haar wavelet. Further, Haar wavelet generally performed better than Daubechies wavelet in terms of power of the test. An important conclusion emerging out of this study is the presence of a declining trend in the data, which may have serious implications from ‘Global Warming’ viewpoint.

4. SOME FUTURE RESEARCH PROBLEMS

Current status for estimation of parameters of SETAR models is that Real-coded genetic algorithm is employed. As future work, some other stochastic optimization techniques could also be tried. Particle swarm optimization (PSO) is a population based stochastic optimization technique, inspired by social behaviour of bird flocking and fish schooling (Gazi and Passino 2011). Similar to Genetic algorithms, PSO performs searches using a population (called swarm) of individuals (called particles) that are updated from iteration to iteration. Another potential optimization technique is the Ant colony optimization (ACO) technique, which borrows its features from the ability

of some ant species to find, collectively, the shortest path between two points (Solnon 2010). There is a need to employ PSO and ACO techniques for estimation of parameters of SETAR models.

In this article, several parametric families of nonlinear time-series models have been discussed. However, there are some other families for which research effort is required to be directed before applying these to data from Indian agriculture. The SETAR model assumes that regime-switching occurs at some particular value of the transition variable. In reality, policy decisions are taken on a regular basis and accordingly, the Smooth transition autoregressive (STAR) model has been proposed. Medeiros and Veiga (2005) extended this model and introduced the Neuro-coefficient smooth transition autoregressive model. As a future research problem, efficient estimation procedure for application of this model may be developed.

Stochastic volatility (SV) parametric nonlinear time-series model to describe time-varying volatility have some extra flexibility for modelling kurtosis compared to the GARCH family, but at the cost of introducing an additional stochastic term. The parameters of this model can be estimated by employing the Extended Kalman filtering (EKF) technique, which is a modification of usual KF. The EKF approach is to apply the standard KF (for *linear* systems) to *nonlinear* systems with additive white noise by continually updating a *linearization* around the previous state estimate, starting with an initial guess. In other words, we only consider a linear Taylor approximation of the system function at the previous state estimate and that of the observation function at the corresponding predicted position. This approach gives a simple and efficient algorithm to handle a nonlinear model. Further, Ristic *et al.* (2004) have given a detailed description of a newly developing powerful technique of Particle filters (PF), which are applicable for nonlinear and non-Gaussian filtering. Polson *et al.* (2008) have developed a simulation-based approach to sequential parameter learning and filtering in general state-space models. As a future research problem, estimation procedure may be developed for SV models using EKF/PF techniques.

The FCAR nonparametric model depends critically on the choice of one of the lagged variables, which

limits the scope of its applications. A generalization of this family of models is to allow a linear combination of past values, called indices, as a model-dependent variable. This leads to the family of Adaptive FCAR (AFCAR) models (Fan and Yao 2003). The estimation procedure and efficient algorithms for fitting of FCAR model need to be developed. Further, in this article, the work done dealing with Wavelet analysis in time domain and frequency domain has been described. There is a need to carry out research work in order to extend this type of work for bivariate/multivariate data.

Artificial neural network (ANN) methodology suffers from the drawbacks of overfitting and convergence at local minima. Recently, a very promising technique of Nonlinear Support vector machine (NSVM), based on the principles of Statistical learning theory, has been proposed. It implements the Structural risk minimization principle, which has been shown to be superior to traditional Empirical risk minimization principle implemented in ANN models. An excellent description of NSVM is available in Ivanciuc (2007). As a future research problem, Nonlinear Support vector machine may be applied for prediction of time-series data.

All the models discussed in the article are applicable to those data sets in which the data points are expressible in terms of single point values. Thus, it is assumed that there is no internal variation in an observation and so the analysis deals with variation between observations only. In contrast, Symbolic data analysis (Diday and Noirhomme-Fraiture 2008) is concerned with the internal variation of each observation plus the variation between observations. Maia *et al.* (2008) proposed a forecasting model for interval-valued time-series data. It is a challenging task to extend this type of work for Nonlinear time-series models when the data are interval-valued.

Another promising area, related to Symbolic data analysis, is "Fuzzy time-series modelling", where the response variable lies in an interval. However, this interval is not 'crisp' or 'precise', but is rather imprecise, vague or fuzzy. Singh (2009) developed a computational method of forecasting based on higher-order time-series. Considerable amount of research work needs to be carried out to develop 'fuzzy' versions of various families of Nonlinear time-series models.

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