Available online at www.isas.org.in/jisas
JOURNAL OF THE INDIAN SOCIETY OF
AGRICULTURAL STATISTICS 65(3) 2011 331-338

A Systematic Approach for Unequal Allocations under Ranked Set Sampling with Skew Distributions

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Received 21 June 2010; Revised 11 July 2011; Accepted 19 July 2011

SUMMARY

Ranked Set Sampling (RSS) is a useful technique for improving the estimates of mean and variance when the sampling units in a study can be more easily ranked than actually measured. Under equal allocation, RSS is found to be more precise than simple random sampling (SRS). Further gain in precision of the estimate may be obtained with appropriate use of unequal allocation. For skewed distributions, the optimum gain in precision is obtained through unequal allocation based on Neyman's approach, in which the sample size corresponding to each rank order is proportional to its standard deviation. However, the unavailability of the standard deviations of the rank orders makes the Neyman's approach impractical. The two models, viz., 't-model' and '(s, t)-model' suggested by Kaur *et al*. (1997) are also impractical due to their dependence on population parameters of rank orders and complexities in finding the optimum values of 't' and '(s, t)'. In this article, we propose a simple and systematic approach for unequal allocation for RSS with skew distributions. The proposed approach performs better than SRS and RSS with equal allocation. It also appears to perform better than the RSS with unequal allocation using 't-model' and quite close to the '(s, t)-model' in most of the situations we have considered. The performance of the proposed procedure relative to existing models has been numerically evaluated for some skewed distributions.

Keywords : Ranked set sampling, Relative precision, Neyman's allocation, Positively skewed distributions, Order statistics.

1. INTRODUCTION

Ranked Set Sampling (RSS), introduced by McIntyre (1952), is a cost effective sampling scheme that can be utilized to potentially increase precision. It is highly useful when actual measurement of the variable of interest is costly or time-consuming but the ranking of the set of items according to the variable can be done without actual measurements. Such situations normally arise in environmental monitoring and assessment that require observational data. Since the inception of RSS by McIntyre (1952) and development of its mathematical foundation by Takahasi and Wakimoto (1968), various researchers have investigated the utility of RSS and the conditions under which it may be useful and cost-effective. RSS has been satisfactorily used to estimate pasture yield by McIntyre (1952, 1978), forage yields by Halls and Dell (1966), mass herbage in a paddock by Cobby *et al*. (1985), shrub phytomass by Martin *et al*. (1980) and Muttlak and McDonald (1992), tree volume in a forest by Stokes and Sager (1988), root weight of Arabidopsis thaliana by Barnett and Moore (1997) and bone mineral density in a human population by Nahhas *et al*. (2002). A few other situations where RSS may be applied have been discussed by Patil *et al*. (1994). A complete review of the applications and theoretical work on RSS can be found in Patil *et al*. (1994), Kaur *et al*. (1995) and Chen *et al*. (2004).

For the present discussion, we restrict ourselves to the problem of unequal allocation in RSS for skewed distributions. For the case of equal allocation or balanced RSS designs, it has been shown by Takahasi

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and Wakimoto (1968) that the relative precision (RP) of RSS with respect to simple random sampling (SRS) lies between 1 and $(k+1)/2$, where k is the set size [The relative precision (RP) may be defined as the ratio of the error variances of the two designs that are different but are based upon the same sampling unit and sample size]. However, with the use of unequal allocation, Takahasi and Wakimoto (1968) showed that the RP of RSS relative to SRS lies between 0 and k. This shows that an appropriate use of unequal allocation can increase the performance of RSS beyond that achievable with equal allocation. However, if it is not applied properly, the performance of RSS with unequal allocation can be even worse than that of SRS. For RSS with unequal allocation, McIntyre (1952) suggested the use of optimum allocation based on Neyman's approach in which the sample units are allocated into ranks in proportion to the standard deviations of each rank. However, Neyman's optimal approach is impractical due to the fact that the standard deviations of the ranks are rarely available beforehand. In many environmental situations, the data obtained is skewed towards the right tail of the distribution. For instance, verification data obtained after the remediation of a site is generally skewed with heavy right tail. McIntyre (1952) observed that skewness has an adverse effect on the performance of RSS and suggested more measurements in the "longest tail of the distribution". For skewed distributions, Takahasi (1970), Yanagawa and Shirahata (1976), and Yanagawa and Chen (1980) suggested the use of random allocations in RSS. It has been usually observed that for a positively skew distribution, the variance of the order statistic increases with the rank order. Utilizing this, Kaur *et al*. (1994) and Kaur *et al*. (1997) suggested two 'near' optimal approaches for positively skewed distributions and called them as the 't-model' and '(s, t)-model'. They found that the performance of both models was better than the equal allocation model. They also studied the role played by skewness, kurtosis and coefficient of variation for obtaining the allocation factor(s) and in devising the rules-of-thumb. For the right choice of allocation factors, the '(s, t)-model' performs better than the 't-model', although the performance of Neyman's method is the best. Although Kaur *et al*. (1994, 1997) observed that the 't' and '(s, t)' models were near optimum, their use in real situations may also be restricted due to their dependence on the population parameters of rank orders and complexities involved in determining the allocation factors 't' and 's'. Moreover, if the allocation factor(s) are fractional, a number of adjustments are required to make them integers. The rules-of-thumb suggested by them for this purpose also provide only a rough idea of the allocation factor(s).

In this article we suggest a simple and systematic approach for unequal allocation in RSS for skewed distributions. The proposed procedure performs better than the equal allocation model. It also appears to perform better than the 't-model' and quite close to the '(s, t)- model' for most of the situations. In Section 2, we discuss the basic framework of RSS with equal and unequal allocation. Section 3 describes the proposed approach for unequal allocation with skewed distributions. In Section 4 we discuss some examples using positively skewed distributions to demonstrate the utility of the proposed procedure. The results of the study have been concluded in Section 5.

2. RANKED SET SAMPLING WITH EQUAL AND UNEQUAL ALLOCATION

In balanced ranked set sampling or RSS with equal allocation, a simple random sample of size k is taken from the population and is ranked on the basis of personal judgment or a concomitant variable. The smallest observation from this set is chosen for measurement. A second sample of size k is taken from the population and the observation judged to be the second smallest is selected for measurement. The process is continued until the item with the largest rank, i.e., the k^{th} item, in the k^{th} sample of size k is selected for measurement. This completes a RSS cycle. This cycle is then repeated m times to get $n = km$ observations for measurement out of $N = k^2m$ observations selected from the population.

2.1 RSS with Equal Allocation

Let $Y_{(i:k)j}$, $i = 1, 2, ..., k$; $j = 1, 2, ..., m$ denote the measured unit for the i^{th} rank order in the j^{th} cycle. For fixed i, the $Y_{(i:k)j}$, $j = 1, 2,...,m$ are i.i.d. with mean $\mu_{(i:k)}$ and variance $\sigma^2_{(i:k)}$. Let μ and σ^2 be the mean and variance of the population. Then, an unbiased estimate of population mean is the ranked set sample mean given as

$$\overline{Y}_{(k)eql} = \frac{1}{km}\sum_{i=1}^{k}\sum_{j=1}^{m} Y_{(i:k)j} \tag{1}$$

where the subscript 'eql' denotes equal allocation. The variance of $\bar{Y}_{(k)eql}$ is

$$Var\bar{Y}_{(k)eql} = \frac{1}{k^2 m}\sum_{i=1}^{n}\sigma_{(i:k)}^2 . \quad (2)$$

For SRS with $n = km$ measurements, the variance of the mean is

$$Var\left(\bar{Y}_{SRS}\right) = \frac{\sigma^2}{km}.$$

Therefore the relative precision (RP) of RSS with respect to SRS is

$$RP_{SRS:eql} = \frac{Var(\bar{Y}_{SRS})}{Var(\bar{Y}_{(k)eql})} = \frac{\sigma^2}{\frac{1}{k}\sum_{i=1}^{k}\sigma_{(i:k)}^2} \quad (3)$$

$$= \frac{\sigma^2}{\overline{\sigma^2}}$$

where is the average of the within-rank variances.

2.2 RSS with Unequal Allocation

For unequal allocation, suppose $m_i (> 0)$ units are measured corresponding to the i^{th} rank, $i = 1, 2, ..., k$. This results in $n = m_1 + m_2 + ... + m_k$ measurements for the sample. Denoting the measured units by $Y_{(i:k)}$, $i = 1, 2, ..., k$; $j = 1, 2, ..., m_i$, an unbiased estimator of the population mean is given by

$$\bar{Y}_{(k)uneql} = \frac{1}{k}\sum_{i=1}^{k}\frac{T_i}{m_i}, \quad (4)$$

where the subscript 'uneql' denotes unequal allocation and $T_i = \sum_{j=1}^{m_i} Y_{(i:k)j}$.

The variance of $\bar{Y}_{(k)uneql}$ is

$$Var(\bar{Y}_{(k)uneql}) = \frac{1}{k^2}\sum_{i=1}^{k}\frac{\sigma_{(i:k)}^2}{m_i}. \quad (5)$$

For Neyman's optimum allocation, we have

$$m_i = \frac{n\sigma_{(i:k)}}{\sum_{i=1}^{k}\sigma_{(i:k)}} \quad (6)$$

and

$$Var\left(\bar{Y}_{(k)opt}\right) = \frac{1}{nk^2}\left(\sum_{i=1}^{k}\sigma_{(i:k)}\right)^2 = \frac{\bar{\sigma}^2}{n} \quad (7)$$

where $\bar{\sigma} = \frac{1}{k}\sum_{i=1}^{k}\sigma_{(i:k)}$ is the average of the within-rank standard deviations. The relative precision of Neyman's optimum allocation relative to SRS is given by

$$RP_{SRS:opt} = \frac{Var\left(\bar{Y}_{SRS}\right)}{Var\left(\bar{Y}_{(k)opt}\right)} = \frac{\sigma^2}{\bar{\sigma}^2} \quad (8)$$

and the relative precision of Neyman's optimum allocation relative to RSS with equal allocation is

$$RP_{eql:opt} = \frac{Var\left(\bar{Y}_{(k)eql}\right)}{Var\left(\bar{Y}_{(k)opt}\right)} = \frac{\overline{\sigma^2}}{\bar{\sigma}^2} . \quad (9)$$

2.2.1 The t-model

For the 't-model', the highest order statistic is quantified t times more frequently than the remaining order statistics. For this model, the allocation of units is

$$m' \equiv m_1 = m_2 = ... = m_{k-1} = \frac{m_k}{t} \ (t \geq)1 \quad (10)$$

and the sample size $n = (k - 1 + t)m'$. For this model

$$Var\left(\bar{Y}_{(k)t}\right) = \frac{1}{k^2 m'} (a' + b'/t) \quad (11)$$

where $a' = \sum_{i-1}^{k-1}\sigma_{(i:k)}^2$ and $b' = \sigma_{(k:k)}^2$. The relative precision $\bar{Y}_{(k)t}$ of relative to SRS is

$$RP_{SRS:t} = \frac{k^2\sigma^2}{(k-1+t)\,(a'+b'/t)} \quad (12)$$

and the relative precision of $\bar{Y}_{(k)t}$ with respect to RSS with equal allocation is

$$RP_{eql:t} = \frac{k^2\overline{\sigma^2}}{(k-1+t)(a'+b'/t)} \quad (13)$$

On optimizing (13), the optimal value of t is obtained as

$$t_{opt} = \sqrt{\frac{b'(k-1)}{a'}} \quad (14)$$

2.2.2 The (s, t)-model

In (s, t)-model, the two largest observations are assigned more weight than others. For this model

$$m'' \equiv m_1 = m_2 = ... = \frac{m_{k-1}}{s} = \frac{m_k}{t} \ (1 \leq s \leq t) \quad (15)$$

and the sample size $n = (k - 2 + s + t)m''$. The variance of the estimate for this model is

$$Var\left(\bar{Y}_{(k)st}\right) = \frac{1}{k^2 m''}\,(a + b/s + c/t) \quad (16)$$

where $a = \sum_{i=1}^{k-2} \sigma^2_{(i:k)}$, $b = \sigma^2_{(k-1:k)}$, and $c = \sigma^2_{(k:k)}$.

The relative precision of the '(s, t)-model' with respect to SRS and RSS with equal allocation is given as

$$RP_{SRS:st} = \frac{k^2\sigma^2}{(k-2+s+t)\,(a+b/s+c/t)} \quad (17)$$

and

$$RP_{eql:st} = \frac{k^2\overline{\sigma^2}}{(k-2+s+t)\,(a+b/s+c/t)} \quad (18)$$

The optimum values of (s, t) are obtained by optimizing (18) as

$$s^* = \sqrt{\frac{(k-2)b}{a}} \quad \text{and } t^* = \sqrt{\frac{(k-2)c}{a}} \quad (19)$$

As discussed in Section 1, the 't' and '(s, t)' models have difficulties in implementation and in obtaining the allocation factor(s). The following section deals with a simple and systematic approach for unequal allocation in RSS with skewed distributions.

3. THE PROPOSED SYSTEMATIC APPROACH FOR UNEQUAL ALLOCATION

In Section 2, we have seen that the Neyman's allocation proposed by McIntyre (1952) for unequal allocation provides the precise estimates for skewed distributions. However this approach is impractical. Two near optimum models, viz. the 't-model' and the '(s, t)-model' were suggested by Kaur *et al.* (1994,1997). However, as discussed in Section 1, it is difficult to implement those models in practical situations where the population parameters are unknown. Utilizing the facts that the variances of the order statistics for positively skewed distributions increase with their rank orders and $m_i \propto \sigma_{(i:k)}$, we propose a very simple and systematic approach for RSS with unequal allocation for positively skewed distributions. The proposed approach performs better than SRS and RSS with equal allocation for positively skewed distributions. The performance of the proposed approach also turns out to be better than or quite close to that of 't' and '(s, t)'models.

We know that for positively skewed distribution, $\sigma^2_{(1:k)} \le \sigma^2_{(2:k)} \le ... \le \sigma^2_{(k:k)}$ and for optimum allocation $m_i \propto \sigma_{(i:k)}$. Combining these two results, it may be inferred that for positively skewed distributions, the higher order statistics should be measured more often than the lower order statistics. For this reason, we propose that the r^{th} order statistic should be measured r-times in the ranked set sample. That is, the first order statistic should be measured only once, the second order statistic twice and similarly the k^{th} order statistic in the ranked set sample of size k should be measured k times. The proposed procedure for RSS with unequal allocation for positively skewed distributions is as follows:

Randomly draw k units from the population and rank them on the basis of inspection or a concomitant variable. Select the smallest ranked unit for measurement. Again draw k units randomly from the population and after ranking them select the second smallest unit for measurement. Repeat this process twice so as to select two second smallest units. Now again randomly draw k units from the population and select the third smallest unit for measurement. Repeat this process three times so as to select three third smallest units for measurement. Continue this process until k units of the k^{th} rank (largest rank) are selected for measurement. Thus, for the proposed model,

$$m_i = i, \quad \text{for } i = 1, 2, \ldots, k. \quad (20)$$

For this model, the number of units to be measured will be $n = 1 + 2 + ... + k = \frac{k(k+1)}{2}$, and the total units randomly drawn from the population will be $N = \frac{k^2(k+1)}{2}$. Denoting the quantified units $Y_{(i:k)j}$, $i = 1, 2, ..., k$; $j = 1, 2, ..., i$ by an unbiased estimate of the population mean (μ) is given by

$$\bar{Y}_{(k)sys} = \frac{1}{k}\sum_{i=1}^{k}\frac{S_i}{i}, \quad (21)$$

where $S_i = \sum_{j=1}^{i} Y_{(i:k)j}$. The variance of $\bar{Y}_{(k)sys}$ is given as

$$Var(\bar{Y}_{(k)sys}) = \frac{1}{k^2}\sum_{i=1}^{k}\frac{\sigma^2_{(i:k)}}{i} = \frac{1}{k^2}\sum_{i=1}^{k} a_i\,, \quad (22)$$

where $a_i = \dfrac{\sigma^2_{(i:k)}}{i}$. The relative precision of $\bar{Y}_{(k)sys}$ compared to SRS is

$$RP_{SRS:sys} = \frac{\sigma^2/n}{Var\left(\bar{Y}_{(k)sys}\right)} = \frac{2k\sigma^2}{(k+1)\sum_{i=1}^{k} a_i}. \quad (23)$$

Comparing with the RSS with equal allocation, the relative precision of the proposed estimator is

$$RP_{eql:sys} = \frac{Var\left(\bar{Y}_{(k)eql}\right)}{Var\left(\bar{Y}_{(k)sys}\right)} = \frac{2\sum_{i=1}^{k}\sigma^2_{(i:k)}}{(k+1)\sum_{i=1}^{k} a_i}. \quad (24)$$

It can be easily shown that $RP_{eql:sys} > 1$ if $\sigma^2_{(1:k)} \le \sigma^2_{(2:k)} \le \ldots \le \sigma^2_{(k:k)}$.

Since RSS with equal allocation is always more efficient than SRS, the proposed scheme is always more efficient than SRS and RSS with equal allocation. The relative precision of the proposed estimator with respect to the 't-model' is given by

$$RP_{t:sys} = \frac{Var\left(\bar{Y}_{(k)t}\right)}{Var\left(\bar{Y}_{(k)sys}\right)} = \frac{2(k-1+t)\,(a'+b'/t)}{k(k+1)\sum_{i=1}^{k} a_i}. \quad (25)$$

The relative precision of the proposed estimator compared to the '(s, t)-model' is

$$RP_{st:sys} = \frac{Var\left(\bar{Y}_{(k)st}\right)}{Var\left(\bar{Y}_{(k)sys}\right)}.$$

$$= \frac{2(k-2+s+t)\,(a+b/s+c/t)}{k(k+1)\sum_{i=1}^{k} a_i} \quad (26)$$

It is difficult to compare theoretically the proposed approach with the 't-model' and the '(s, t)-model'. However, in the next section we illustrate with the help of some examples from positively skewed distributions that the proposed approach performs better than the 't-model' and quite close to the '(s, t)-model' in most of the situations.

4. EXAMPLES

In this section we numerically evaluate the performance of existing methods and the proposed method for standard Lognormal [LN(0,1)] and standard Gamma [G(r), for r = 1, 2 and 3] distributions.

The performance of different unequal allocation models relative to SRS for standard Lognormal and Gamma distributions for set sizes 3, 4 and 5 is shown in Table 1. It is observed from Table 1 that for all cases the performance of the proposed method is better than

Table 1. Relative precision (RP) compared with SRS of equal allocation, t-model (optimum t), (s, t)-model (optimum (s, t)), proposed allocation method and Neyman allocation for standard Lognormal [LN(0,1)] and standard Gamma G(*r*), *r* = 1, 2 and 3, distributions; for *k* = 3, 4 and 5.

Distributions	k	Relative precision (RP)				
		Eq. alloc.	*t*-model	(s,t)-model	Proposed Method	Neyman's allocation
LN(0,1)	3	1.339	2.036	2.120	1.859	2.120
	4	1.471	2.433	2.605	2.174	2.640
	5	1.589	2.783	3.049	2.449	3.139
G(1)	3	1.636	1.968	2.039	2.012	2.039
	4	1.920	2.367	2.497	2.475	2.538
	5	2.190	2.740	2.924	2.920	3.029
G(2)	3	1.753	1.953	1.987	1.962	1.990
	4	2.096	2.378	2.443	2.403	2.460
	5	2.424	2.785	2.881	2.828	2.930
G(3)	3	1.801	1.946	1.966	1.913	1.966
	4	2.169	2.380	2.421	2.326	2.430
	5	2.524	2.799	2.863	2.720	2.887

that of the equal allocation method. For the LN(0,1) distribution, the performance of the proposed method is marginally lower than that of the 't' and the '(s, t)'models when the allocation factors 't' and '(s, t)' take the optimal values. For the standard Gamma distribution with parameter $r = 1$, the performance of the proposed method is found to be better than that of the 't-model' and even better than that of the '(s, t)-model' for $k = 5$. For the standard Gamma distribution with parameter $r = 2$, the performance of the proposed model is better than that of the 't-model' and quite close to that of the '(s, t)-model'. For the standard Gamma distribution with parameter $r = 3$, the performance of the proposed model is quite close to that of the 't' and the '(s, t)' models.

To make a comparison amongst the various methods for unequal allocation, the multiple plots of $RP_{SRS:eql}$, $RP_{SRS:t}$, $RP_{SRS:st}$, $RP_{SRS:opt}$ and $RP_{SRS:sys}$ versus 't' for the standard Lognormal and the Gamma distributions were obtained for k = 3, 4 and 5. The relative precision remains an increasing function of set size (k) in all the cases. For all the cases the proposed method performs better than the equal allocation model. With the increase in t ($t > 4$), the RP of 't' and (s,t)-models starts decreasing for G(1), G(2) and G(3) models, whereas for the proposed model it remains constant and hence the proposed model performs better than 't' and (s,t)-models for these distributions when $t > 4$.

The plots for RP for (i) Equal allocation; (ii) t-model; (iii) (s, t)-model; (iv) Proposed method; (v) Neyman allocation for standard lognormal [LN(0, 1)] and Gamma [G(r), for r=1, 2 and 3] distributions for k=3 are given in Fig. 1. From Fig. 1, we find that for all the cases, the proposed method performs better than equal allocation model. It also performs better than t-model and quite close to (s,t) and Neyman allocation models for G(1) and G(2) distributions.

The plots for RP for these distributions for k=4 are given in Fig. 2. From Fig. 2, we find that for all the cases, the proposed method performs better than equal allocation model. It also performs better than t-model and quite close to (s,t) and Neyman allocation models for G(2) distributions. For G(1) distribution, the proposed model performs even better than (s,t)-model.

The plots for RP for k=5 for the five models under comparison are given in Fig. 3. From Fig. 3, we find

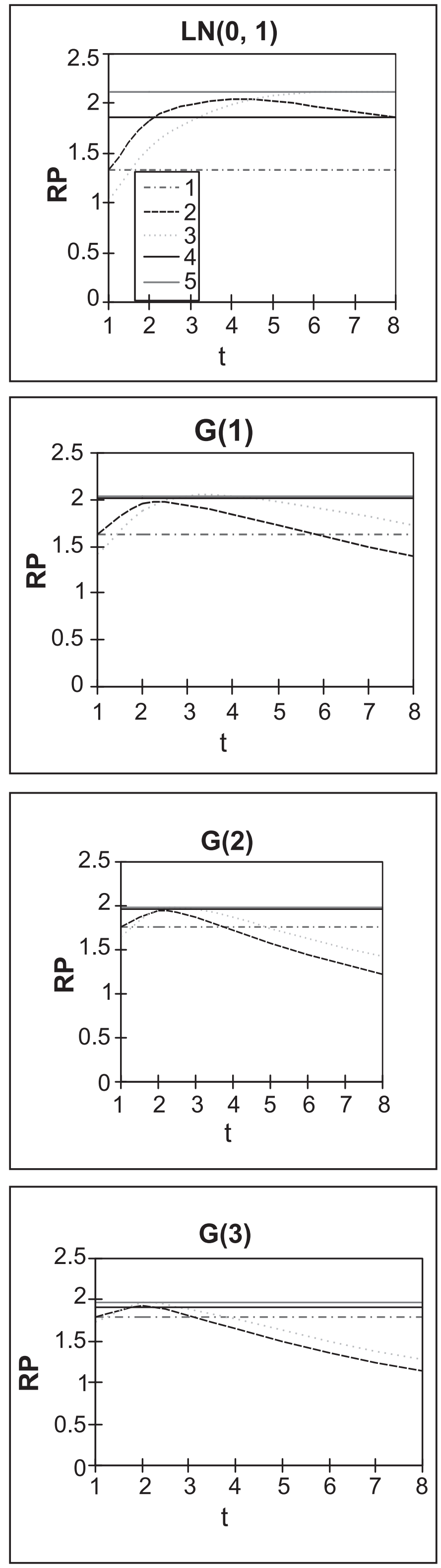

Fig. 1. Relative Precision for (1) Equal allocation; (2) t-model; (3) (s, t)-model; (4) Proposed method; (5) Neyman allocation for standard lognormal [LN(0, 1)] and Gamma [G(r), for r=1, 2 and 3] distributions for k=3.

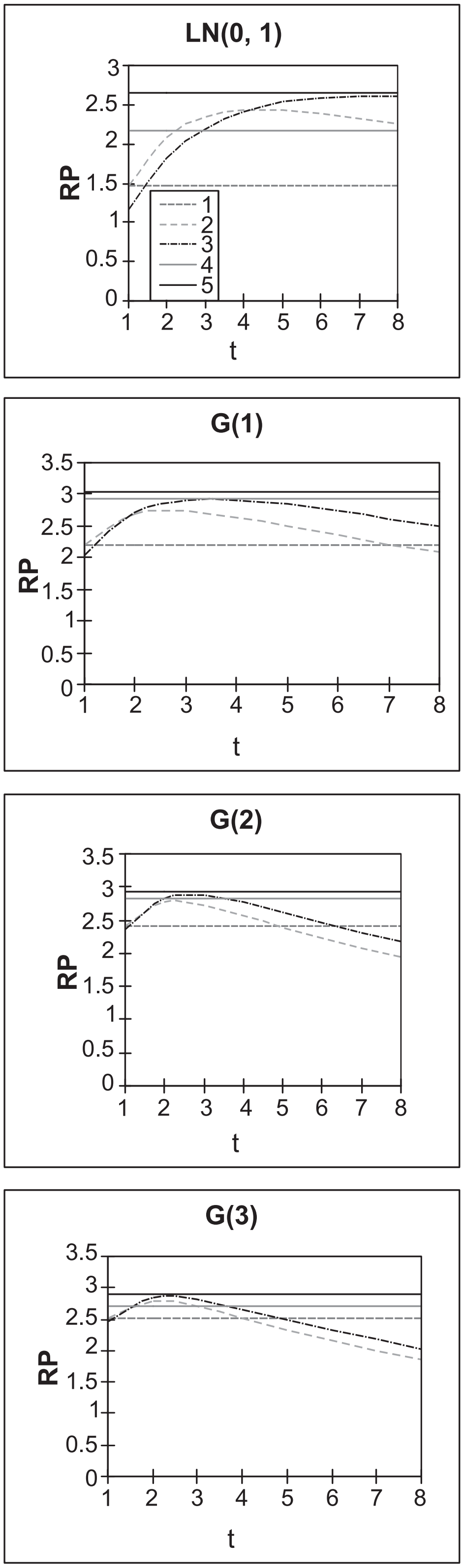

Fig. 2. Relative Precision for (1) Equal allocation; (2) t-model; (3) (s, t)-model; (4) Proposed method; (5) Neyman allocation for standard lognormal [LN(0, 1)] and Gamma [G(*r*), for *r*=1, 2 and 3] distributions for *k*=4.

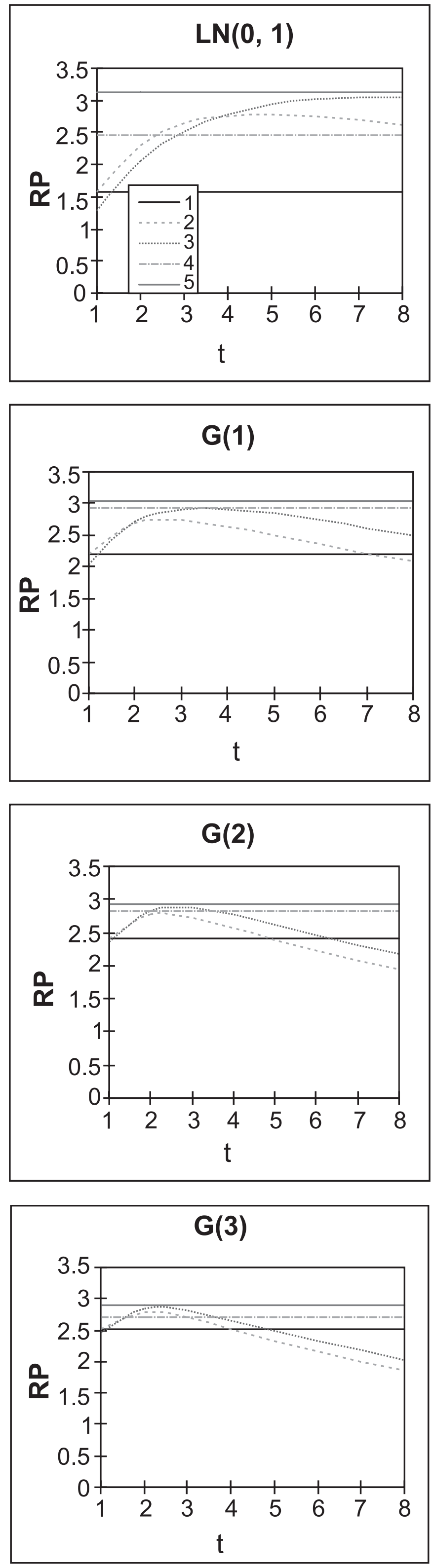

Fig. 3. Relative Precision for (1) Equal allocation; (2) t-model; (3) (s, t)-model; (4) Proposed method; (5) Neyman allocation for standard lognormal [LN(0, 1)] and Gamma [G(*r*), for *r*=1, 2 and 3] distributions for *k*=5.

that for k=5, the results are quite similar to the results obtained for k=4. For all the cases, the proposed method performs better than equal allocation model. It also performs better than t-model and quite close to (s,t) and Neyman allocation models for G(2) distributions. For G(1) distribution, the proposed model performs even better than (s,t)-model.

5. CONCLUSION

In this article, we have proposed a simple and systematic approach for unequal allocation for RSS with skew distributions. The proposed approach appears to perform better than SRS and RSS with equal allocation. This approach also performs better than RSS with unequal allocation using t-model, and quite close to the (s-t)-model suggested by Kaur *et al*. (1977) and the Neyman allocation model in most of the situations. The proposed approach overcomes the drawbacks of 't' and (s,t)-models and is more practical. On the basis of these results, we may conclude that the proposed method is a good alternative to the 't' and '(s, t)' models for positively skewed distributions.

ACKNOWLEDGEMENT

The authors are grateful to the Editor and an anonymous referee for their constructive comments and suggestions, which led to considerable improvement in presentation of this manuscript.

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