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A πps Sampling Scheme

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SUMMARY

In this paper a πps Sampling Scheme has been suggested which is a judicious combination of PPS Systematic Sampling and SRSWOR. The efficiency of the proposed πps Sampling Scheme has been compared with PPSWR and SRSWOR. Importantly, the limitation of the proposed scheme along with the choice of sample size by PPS Systematic Sampling and SRSWOR are the highlights of the Paper.

Keywords: Inclusion probability, Horvitz Thompson estimator, PPS systematic sampling.

1. INTRODUCTION

A Sampling Scheme in which inclusion probabilities are proportional to size is termed as πps Sampling Scheme. For this Sampling Scheme the Horvitz Thompson (1952) Estimator for population total $Y = \sum_{i=1}^{N} y_i$ is given by

$$\hat{Y}_{HT} = \sum_{i \varepsilon s} \frac{y_i}{\pi_i} \tag{1}$$

where y_i is the value of the study variable y for the i^{th} unit of the population and π_i be the inclusion probability for the i^{th} unit which is assumed to be positive for $i = 1, \ldots N$.

The variance of $\hat{Y}_{HT}$ is given by

$$V(\hat{Y}_{HT}) = \sum_{i=1}^{N} \sum_{j>i}^{N} (\pi_i \pi_j - \pi_{ij}) \left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2 \tag{2}$$

where π_{ij} is the Inclusion Probabilities for a pair of units (i, j).

Yates Grundy (1953) form of Variance Estimator for above variance is given by:

$$\hat{V}(\hat{Y}_{HT}) = \sum_{i=1}^{n} \sum_{j>i}^{n} \left(\frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}}\right) \left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2 \tag{3}$$

The desirable properties of a πps Sampling in addition to $\pi_i \; \alpha \; X_i$ are:

- π_{ij} (the inclusion probabilities for a pair of units i and j) > 0, for all i and j. This is necessary condition for estimation of variance.
- $(\pi_i \pi_j - \pi_{ij}) > 0$, for all i and j. This is desirable condition for non negativity of variance estimator.
- Narain (1951) condition: $\pi_{ij} \leq \frac{2(n-1)}{n} \pi_i \pi_j$. This is necessary condition for πps to be more efficient than PPSWR.
- DesRaj (1966) condition: $\pi_{ij} > \frac{(n-1)}{n} \pi_i \pi_j$. This is sufficient condition for πps to be more efficient than PPSWR.

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There have been numerous attempts to evolve πps Sampling Schemes for sample size 2. Important among these are Brewer (1963), Hanurav (1967), Durbin (1967), Padam Singh (1978) etc. These schemes generally satisfy all desirable properties. For general case of sample size $n > 2$, there have also been several attempts. But none of the known Sampling Schemes for $n > 2$ satisfy all the desirable properties. The hunt is on in developing new πps Sampling Schemes satisfying most of these conditions.

The PPS systematic Sampling is one of the πps Sampling Schemes which is commonly used in practice because of its ease of selection and operational convenience. However, π_{ij} is zero for many pairs of i & j and hence puts limitation on variance estimation. There have been attempts to modify PPS Systematic Sampling or use it in conjunction with other Schemes so as to evolve a πps Scheme. Mention may be made of Hartley and Rao (1962), Das and Mohanthy (1970), Agrawal *et al*. (1986) etc. Present research is also an attempt in this direction by judiciously combining the PPS systematic sampling with SRSWOR.

2. PROPOSED SAMPLING SCHEME

Suppose that the population under study consists of N distinct and identifiable units and a sample of size n is desired to be drawn from it. Let Y_i and X_i denote the values of the characteristic under study and auxillary character respectively for the i^{th} unit of the population. Conventionally, small letters y_i and x_i will correspond to the i^{th} unit in the sample. Further it is assumed that $P_i = \frac{X_i}{X}$ where X = sum of X_i.

The Proposed scheme consists of the following steps:

a. Select n_1 units from N units of the population by usual PPS systematic sampling scheme with probability for i^{th} unit proportional to P_i'.

b. Select $(n - n_1)$ units from the remaining $(N - n_1)$ units of the population by Simple Random Sampling without replacement (SRSWOR).

For this sampling scheme the inclusion probability for i^{th} unit is given by

π_i = Probability that i^{th} unit is selected at step (a) + Probability that i^{th} unit is not selected at step (a) × Probability that i^{th} unit is selected at step (b)

$$\pi_i = n_1 P_i' + (1 - n_1 P_i')\frac{(n - n_1)}{(N - n_1)} \tag{4}$$

For πps Sampling, $\pi_i = nP_i$, we should have

$$nP_i = n_1 P_i' + (1 - n_1 P_i')\frac{(n - n_1)}{(N - n_1)} \tag{5}$$

$$P_i' = \frac{1}{n}\frac{(N - n_1)}{(N - n)}\left(nP_i - \frac{(n - n_1)}{(N - n_1)}\right) \tag{6}$$

Since P_i' has to be positive, to ensure this, the condition to be satisfied is:

$$\text{Min}\left(\frac{X_i}{X}\right) > \frac{1}{n}\frac{(n - n_1)}{(N - n_1)} \tag{7}$$

Since nPi <1, this mean the following condition is also to be satisfied.

$$\text{Max}\left(\frac{X_i}{X}\right) < \frac{1}{n}\frac{(n - n_1)}{(N - n_1)} + \frac{n_1(N - n)}{(N - n_1)} \tag{8}$$

3. INCLUSION PROBABILITY FOR A PAIR OF UNITS (*i*, *j*)

The computation of π_{ij} is explained as under

π_{ij} = Probability that

(i) Both i and j^{th} units are selected at step (a) +

(ii) Probability that only i^{th} unit is selected at step (a) and j^{th} unit is selected at step (b) +

(iii) Probability that the j^{th} unit is selected at step (a) and i^{th} unit is selected at step (b)+

(iv) None of i and j^{th} unit is selected at step (a) × both i and j^{th} units are selected at step (b)

The Probability of selecting both i^{th} and j^{th} units at step (a) can be computed using the results given by Agrawal *et al*. (1986) say $\pi_{ij}(a)$.

The Probability of selecting only unit i at step (a) is $n_1 P_i' - \pi_{ij}(a)$ and probability of selecting unit j at step

(b) is $\dfrac{n_2}{(N-n_1)}$. Thus Probability of selecting unit i at step (a) and unit j at step (b) is given by

$$(n_1P'_i - \pi_{ij}(a))\ \frac{n_2}{(N-n_1)} \tag{9}$$

Similarly, the Probability of selecting unit j at step (a) and i^{th} unit i at step (b) is given by

$$(n_1P'_i - \pi_{ij}(a))\ \frac{n_2}{(N-n_1)} \tag{10}$$

Further, the probability of not selecting either of i and j at step (a) together with probability of selecting both i^{th} and j^{th} units at step (b) is given by

$$((1 - \pi_i(a) - \pi_j(a) + \pi_{ij}(a))\ \frac{n_2(n_2-1)}{(N-n_1)(N-n_1-1)} \tag{11}$$

Thus adding all these, the π_{ij} is given by

$$\pi_{ij} = \pi_{ij}(a) + (n_1P_i^{'} - \pi_{ij}(a))\frac{n_2}{(N-n_1)}$$
$$+ (n_1P_i^{'} - \pi_{ij}(a))\frac{n_2}{(N-n_1)}$$
$$+ \frac{n_2(n_2-1)}{(N-n_1)(N-n_1-1)}(1 - n_1P'_i - n_1P'_j + \pi_{ij}(a)) \tag{12}$$

4. ILLUSTRATIONS

To illustrate the selection of the sample and computation of inclusion probabilities following examples are considered

Example 1

Let us consider the following population of size $N = 5$, with X values as under. Let a sample of size 3 be drawn with $n_1 = 2$ and $n_2 = 1$. The probabilities of selection to be used for PPS Systematic Sampling along with corresponding X'_i are also indicated in the Table 1.

Table 1. Population 1

S.No.	X_i	P'_i	X'_i
1	14	0.10	2
2	16	0.15	3
3	18	0.20	4
4	20	0.25	5
5	22	0.30	6

All Possible samples of size 2 out of 5 using PPS systematic sampling with X_i are

(1, 4), (1, 4), (2, 4), (2, 4), (2, 5), (3, 5), (3, 5), (3, 5), (3, 5), (4, 5)

The values of π_{ij} (a) are given by

$$\pi_{12}(a) = 0,\ \pi_{13}(a) = 0,\ \pi_{14}(a) = 2/10,$$
$$\pi_{15}(a) = 0,\ \pi_{23}(a) = 0,\ \pi_{24}(a) = 2/10,$$
$$\pi_{25}(a) = 1/10,\ \pi_{34}(a) = 0,\ \pi_{35}(a) = 4/10$$
$$\pi_{45}(a) = 1/10$$

Let one more unit be drawn by SRS. Then the possible samples of size 3 using PPS Systematic sample for $n_1 = 2$ and SRSWOR for $n_2 = 1$ is given by

(1, 4, 2) (1, 4, 3) (1, 4, 5)
(1, 4, 2) (1, 4, 3) (1, 4, 5)
(2, 4, 1) (2, 4, 3) (2, 4, 5)
(2, 4, 1) (2, 4, 3) (2, 4, 5)
(2, 5, 1) (2, 5, 3) (2, 5, 4)
(3, 5, 1) (3, 5, 2) (3, 5, 4)
(3, 5, 1) (3, 5, 2) (3, 5, 4)
(3, 5, 1) (3, 5, 2) (3, 5, 4)
(3, 5, 1) (3, 5, 2) (3, 5, 4)
(4, 5, 1) (4, 5, 2) (4, 5, 3)

The Inclusion Probability for individual units is given by

$$\pi_1 = 14/30,\ \pi_2 = 16/30,\ \pi_3 = 18/30,$$
$$\pi_4 = 20/30,\ \pi_5 = 22/30$$

The $\pi_{ij}s$ for pairs of units are given in table

Table 2. Inclusion Probabilities for pairs of units: Population 1

i, *j*	2	3	4	5
1	4/30	6/30	9/30	8/30
2		7/30	7/30	6/30
3			9/30	14/30
4				12/30

Importantly, $\Sigma\pi_i = n = (90/30) = 3$,
$\Sigma\Sigma\pi_{ij} = n(n-1) = 6$

Example 2

Consider another example as under with $N = 8$, $n = 4$, $n_1 = 2$ and $n_2 = 2$. The values of X, P'_i and X'_i are given in Table 3

Table 3. Population 2

S. No.	X_i	P_i	P'_i	X'_i
1	130	0.09848485	0.04545	2
2	140	0.10606061	0.06818	3
3	150	0.11363636	0.09091	4
4	160	0.12121212	0.11364	5
5	170	0.12878788	0.13636	6
6	180	0.13636364	0.15909	7
7	190	0.14393939	0.18182	8
8	200	0.15151515	0.20455	9

All possible samples of size 4 using PPS systematic sampling for $n_1 = 2$ and SRSWOR for $n_2 = 2$ are:

(1, 6, 2, 3) (1, 6, 2, 4) (1, 6, 2, 5) (1, 6, 2, 7) (1, 6, 2, 8) (1, 6, 3, 4) (1, 6, 3, 5) (1, 6, 3, 7)(1, 6, 3, 8) (1, 6, 4, 5) (1, 6, 4, 7) (1, 6, 4, 8) (1, 6, 5, 7) (1, 6, 5, 8) (1, 6, 7, 8)

(1, 6, 2, 3) (1, 6, 2, 4) (1, 6, 2, 5) (1, 6, 2, 7) (1, 6, 2, 8), (1, 6, 3, 4)(1, 6, 3, 5) (1, 6, 3, 7) (1, 6, 3, 8) (1, 6, 4, 5) (1, 6, 4, 7) (1, 6, 4, 8) (1, 6, 5, 7) (1, 6, 5, 8) (1, 6, 7, 8)

(2, 6, 1, 3) (2, 6, 1, 4) (2, 6, 1, 5) (2, 6, 1, 7) (2, 6, 1, 8) (2, 6, 3, 4)(2, 6, 3, 5) (2, 6, 3, 7) (2, 6, 3, 8) (2, 6, 4, 5) (2, 6, 4, 7) (2, 6, 4, 8) (2, 6, 5, 7) (2, 6, 5, 8) (2, 6, 7, 8)

(2, 6, 1, 3) (2, 6, 1, 4) (2, 6, 1, 5) (2, 6, 1, 7) (2, 6, 1, 8) (2, 6, 3, 4)(2, 6, 3, 5) (2, 6, 3, 7)(2, 6, 3, 8) (2, 6, 4, 5) (2, 6, 4, 7) (2, 6, 4, 8) (2, 6, 5, 7) (2, 6, 5, 8) (2, 6, 7, 8)

(2, 6, 1, 3) (2, 6, 1, 4) (2, 6, 1, 5) (2, 6, 1, 7) (2, 6, 1, 8) (2, 6, 3, 4) (2, 6, 3, 5) (2, 6, 3, 7) (2, 6, 3, 8) (2, 6, 4, 5) (2, 6, 4, 7) (2, 6, 4, 8) (2, 6, 5, 7) (2, 6, 5, 8) (2, 6, 7, 8)

(3, 7, 1, 2) (3, 7, 1, 4) (3, 7, 1, 5) (3, 7, 1, 6) (3, 7, 1, 8) (3, 7, 2, 4) (3, 7, 2, 5) (3, 7, 2, 6) (3, 7, 2, 8) (3, 7, 4, 5) (3, 7, 4, 6) (3, 7, 4, 8) (3, 7, 5, 6) (3, 7, 5, 8) (3, 7, 6, 8)

(3, 7, 1, 2) (3, 7, 1, 4) (3, 7, 1, 5) (3, 7, 1, 6) (3, 7, 1, 8) (3, 7, 2, 4)(3, 7, 2, 5) (3, 7, 2, 6)(3, 7, 2, 8) (3, 7, 4, 5) (3, 7, 4, 6) (3, 7, 4, 8) (3, 7, 5, 6) (3, 7, 5, 8) (3, 7, 6, 8)

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(3, 7, 1, 2) (3, 7, 1, 4) (3, 7, 1, 5) (3, 7, 1, 6) (3, 7, 1, 8) (3, 7, 2, 4) (3, 7, 2, 5)(3, 7, 2, 6)(3, 7, 2, 8) (3, 7, 4, 5) (3, 7, 4, 6) (3, 7, 4, 8) (3, 7, 5, 6) (3, 7, 5, 8) (3, 7, 6, 8)

(4, 7, 1, 2) (4, 7, 1, 3) (4, 7, 1, 5) (4, 7, 1, 6) (4, 7, 1, 8) (4, 7, 2, 3) (4, 7, 2, 5) (4, 7, 2, 6) (4, 7, 2, 8) (4, 7, 3, 5) (4, 7, 3, 6) (4, 7, 3, 8) (4, 7, 5, 6) (4, 7, 5, 8) (4, 7, 6, 8)

(4, 7, 1, 3) (4, 7, 1, 5) (4, 7, 1, 6) (4, 7, 1, 8) (4, 7, 2, 3) (4, 7, 2, 5) (4, 7, 2, 6) (4, 7, 2, 8) (4, 7, 3, 5) (4, 7, 3, 6) (4, 7, 3, 8) (4, 7, 5, 6) (4, 7, 5, 8) (4, 7, 6, 8) (4, 7, 1, 2)

(4, 7, 1, 3) (4, 7, 1, 5) (4, 7, 1, 6) (4, 7, 1, 8) (4, 7, 2, 3) (4, 7, 2, 5) (4, 7, 2, 6) (4, 7, 2, 8) (4, 7, 3, 5) (4, 7, 3, 6) (4, 7, 3, 8) (4, 7, 5, 6) (4, 7, 5, 8) (4, 7, 6, 8) (4, 7, 1, 2)

(4, 7, 1, 3) (4, 7, 1, 5) (4, 7, 1, 6) (4, 7, 1, 8) (4, 7, 2, 3) (4, 7, 2, 5) (4, 7, 2, 6) (4, 7, 2, 8) (4, 7, 3, 5) (4, 7, 3, 6) (4, 7, 3, 8) (4, 7, 5, 6) (4, 7, 5, 8) (4, 7, 6, 8) (4, 7, 1, 2)

(4, 8, 1, 2) (4, 8, 1, 3) (4, 8, 1, 5) (4, 8, 1, 6) (4, 8, 1, 7) (4, 8, 2, 3) (4, 8, 2, 5) (4, 8, 2, 6) (4, 8, 2, 7) (4, 8, 3, 5) (4, 8, 3, 6) (4, 8, 3, 7) (4, 8, 5, 6) (4, 8, 5, 7) (4, 8, 6, 7)

(5, 8, 1, 2) (5, 8, 1, 3) (5, 8, 1, 4) (5, 8, 1, 7) (5, 8, 2, 3) (5, 8, 2, 4) (5, 8, 2, 6) (5, 8, 2, 7) (5, 8, 3, 4) (5, 8, 6, 7) (5, 8, 3, 6) (5, 8, 3, 7) (5, 8, 4, 6) (5, 8, 4, 7) (5, 8, 1, 6)

(5, 8, 1, 2) (5, 8, 1, 3) (5, 8, 1, 4) (5, 8, 1, 7) (5, 8, 2, 3) (5, 8, 2, 4) (5, 8, 2, 6) (5, 8, 2, 7) (5, 8, 3, 4) (5, 8, 6, 7) (5, 8, 3, 6) (5, 8, 3, 7) (5, 8, 4, 6) (5, 8, 4, 7) (5, 8, 1, 6)

(5, 8, 1, 2) (5, 8, 1, 3) (5, 8, 1, 4) (5, 8, 1, 7) (5, 8, 2, 3) (5, 8, 2, 4) (5, 8, 2, 6) (5, 8, 2, 7) (5, 8, 3, 4) (5, 8, 6, 7) (5, 8, 3, 6) (5, 8, 3, 7) (5, 8, 4, 6) (5, 8, 4, 7) (5, 8, 1, 6)

(5, 8, 1, 2) (5, 8, 1, 3) (5, 8, 1, 4) (5, 8, 1, 7) (5, 8, 2, 3) (5, 8, 2, 4) (5, 8, 2, 6) (5, 8, 2, 7) (5, 8, 3, 4) (5, 8, 6, 7) (5, 8, 3, 6) (5, 8, 3, 7) (5, 8, 4, 6) (5, 8, 4, 7) (5, 8, 1, 6)

(5, 8, 1, 2) (5, 8, 1, 3) (5, 8, 1, 4) (5, 8, 1, 7) (5, 8, 2, 3) (5, 8, 2, 4) (5, 8, 2, 6) (5, 8, 2, 7) (5, 8, 3, 4) (5, 8, 3, 5) (5, 8, 3, 6) (5, 8, 3, 7) (5, 8, 4, 6) (5, 8, 4, 7) (5, 8, 1, 6)

(5, 8, 1, 2) (5, 8, 1, 3) (5, 8, 1, 4) (5, 8, 1, 7) (5, 8, 2, 3) (5, 8, 2, 4) (5, 8, 2, 6) (5, 8, 2, 7) (5, 8, 3, 4) (5, 8, 3, 5) (5, 8, 3, 6) (5, 8, 3, 7) (5, 8, 4, 6) (5, 8, 4, 7) (5, 8, 1, 6)

(6, 8, 1, 2) (6, 8, 1, 3) (6, 8, 1, 4) (6, 8, 1, 5) (6, 8, 1, 7) (6, 8, 2, 3) (6, 8, 2, 4) (6, 8, 2, 5) (6, 8, 2, 7) (6, 8, 3, 4) (6, 8, 3, 5) (6, 8, 3, 7) (6, 8, 4, 5) (6, 8, 4, 7) (6, 8, 5, 7)

(6, 8, 1, 2) (6, 8, 1, 3) (6, 8, 1, 4) (6, 8, 1, 5) (6, 8, 1, 7)(6, 8, 2, 3) (6, 8, 2, 4) (6, 8, 2, 5) (6, 8, 2, 7) (6, 8, 3, 4) (6, 8, 3, 5) (6, 8, 3, 7) (6, 8, 4, 5) (6, 8, 4, 7) (6, 8, 5, 7)

For this the Values of π_i is given by

$$\pi_1 = 130/330,\ \pi_2 = 140/330,\ \pi_3 = 150/330,$$
$$\pi_4 = 160/330,\ \pi_5 = 170/330,\ \pi_6 = 180/330,$$
$$\pi_7 = 190/330,\ \pi_8 = 200/330$$

$$\Sigma\pi_i = (1320/330) = 4 = n$$

The 330*π_{ij} values for all pairs are given in Table 4.

Table 4. 330* Inclusion Probability for a pair of units : Population 2

(*i*, *j*)	1	2	3	4	5	6	7	8
1	–	42	46	50	54	70	61	66
2	42	–	50	53	58	80	66	70
3	46	50	–	58	62	66	94	74
4	50	58	58	–	66	70	98	84
5	54	58	62	66	–	74	78	118
6	70	80	66	70	74	–	82	98
7	61	74	94	98	78	82	–	90
8	66	70	74	84	118	98	90	–

$\Sigma\pi_{ij} = n(n-1) = 90$

5. CHOICE OF SAMPLE TO BE SELECTED BY PPS SYSTEMATIC SAMPLING AND SRSWOR

One of the important issue relates to choice of n_1 out of n. It is observed that the condition $(\pi_i\pi_j - \pi_{ij}) > 0$ is more likely to be met when larger sample is drawn by SRSWOR. Thus, if situation permits it is desirable to select relatively large sample by SRSWOR. The choice of n_1 is seen to be a function of the following (i) Population size (ii) Sample Size (iii) Minimum and Maximum values of X_i s. Table below provides the choice of n_1 for different populations according to the coefficient of Variation and Sample Size.

Table 5. Choice of n_1 out of n

N	C.V(%)	n	n_1	n_2
41	28	10	6	4
		15	7	8
		20	8	12
		25	9	16
		30	8	22
41	43	10	7	3
		15	8	7
		20	12	8
		25	17	8
		30	22	8
41	52	10	6	4
		15	7	8
		20	8	12
		25	9	16
		30	8	22
41	66	10	3	7
		15	4	11
		20	4	16
		25	3	22
		30	3	27

As expected, it is observed that for populations with higher C.V. i.e. larger variations, the sample to be selected by SRSWOR (n_i) could be considerably higher.

6. COMPARISON OF EFFICIENCIES

Though it can not be theoretically established , it is generally expected that πps Sampling Scheme would have smaller variance than PPSWR. Therefore, it is important to compare how the proposed πps Sampling Scheme performs in comparison to PPSWR and SRSWOR. For this population of size 5 and 8 given in the section on illustrations have been considered**.**

Following table provides the details of the populations along with computations of variances.

Table 7. Values of Y and X for Population 1

S.No.	X_i	Y_i
1	14	211
2	16	242
3	18	273
4	20	302
5	22	331

Table 8. Computation of variance : Population 1

Pairs	π_{ij}	π_i	π_j	Y_i	Y_j	$(\pi_i\pi_j - \pi_{ij})\left(\frac{Y_i}{\pi_i} - \frac{Y_j}{\pi_j}\right)^2$
(1, 2)	0.133333	0.466667	0.533333	211	242	0.298493
(1, 3)	0.200000	0.466667	0.600000	211	273	0.653078
(1, 4)	0.300000	0.466667	0.666667	211	302	0.008164
(1, 5)	0.266667	0.466667	0.733333	211	331	0.045870
(2, 3)	0.233333	0.533333	0.600000	242	273	0.135410
(2, 4)	0.233333	0.533333	0.666667	242	302	0.068760
(2, 5)	0.200000	0.533333	0.733333	242	331	1.088333
(3, 4)	0.300000	0.600000	0.666667	273	302	0.400010
(3, 5)	0.466667	0.600000	0.733333	273	331	–0.352620
(4, 5)	0.400000	0.666667	0.733333	302	331	0.238003

Variance of Suggested πps Scheme = 2.58

Variance of Equivalent PPSWR = 5.00

Variance of Equivalent SRSWOR = 300.09

Table 9. Values of Y and X for Population 2

S.No.	X_i	Y_i
1	130	196
2	140	212
3	150	228
4	160	242
5	170	257
6	180	273
7	190	286
8	200	302

Table 10. Computation of variance : Population 2

Pairs	π_{ij}	π_i	π_j	Y_i	Y_j	$(\pi_i\pi_j - \pi_{ij})\left(\frac{Y_i}{\pi_i} - \frac{Y_j}{\pi_j}\right)^2$
(1, 2)	0.127273	0.393939	0.424242	196	212	0.188671661
(1, 3)	0.139394	0.393939	0.454545	196	228	0.654385934
(1, 4)	0.151515	0.393939	0.484848	196	242	0.099389030
(1, 5)	0.163636	0.393939	0.515152	196	257	0.070879205
(1, 6)	0.212121	0.393939	0.545455	196	273	0.024146500
(1, 7)	0.184848	0.393939	0.575758	196	286	0.027024606
(1, 8)	0.200000	0.393939	0.606060	196	302	0.022424869
(2, 3)	0.151515	0.424242	0.454545	212	228	0.146937698
(2, 4)	0.160606	0.424242	0.484848	212	242	0.015656776
(2, 5)	0.175758	0.424242	0.515152	212	257	0.029685594
(2, 6)	0.242424	0.424242	0.545455	212	273	–0.006786879
(2, 7)	0.200000	0.424242	0.575758	212	286	0.392607087
(2, 8)	0.212121	0.424242	0.606060	212	302	0.090104673
(3, 4)	0.175758	0.454545	0.484848	228	242	0.273372847
(3, 5)	0.187879	0.454545	0.515152	228	257	0.342057381
(3, 6)	0.200000	0.454545	0.545455	228	273	0.058096870
(3, 7)	0.284848	0.454545	0.575758	228	286	–0.547476701
(3, 8)	0.224242	0.454545	0.606060	228	302	0.558278152
(4, 5)	0.200000	0.484848	0.515152	242	257	0.002953806
(4, 6)	0.212121	0.484848	0.545455	242	273	0.098826416
(4, 7)	0.296970	0.484848	0.575758	242	286	–0.101675500
(4, 8)	0.254545	0.484848	0.606060	242	302	0.026803328
(5, 6)	0.224242	0.515152	0.545455	257	273	0.148511512
(5, 7)	0.236364	0.515152	0.575758	257	286	0.277267561
(5, 8)	0.357576	0.515152	0.606060	257	302	–0.015376221
(6, 7)	0.248485	0.545455	0.575758	273	286	0.928467472
(6, 8)	0.296970	0.545455	0.606060	273	302	0.162655260
(7, 8)	0.272727	0.575758	0.606060	286	302	0.186243906

Variance of Suggested PPS Scheme = 4.15

Variance of Equivalent PPSWR = 8.61

Variance of Equivalent SRSWOR = 169.35

Following table summarizes the gain in efficiency of suggested πps scheme over PPSWR and SRSWOR.

Table 11. Gain in Efficiency (%)

Sampling Scheme	Population1	Population 2
Suggested πps Sampling over PPSWR	93.70	107.45
Suggested πps Sampling over SRSWOR	11514.02	3976.84
PPSWR over SRSWOR	5894.15	1865.02

As expected, the proposed πps schemes performs better than PPSWR with gain in efficiency around 100%. As compared to SRSWOR the gain in efficiency is substantial. The populations considered represent the situations where sampling with varying probabilities are used. Importantly, for both the populations the study variable is assumed as directly related to the auxillary character i.e. the size. In such situation the proposed πps sampling scheme could be used with advantage.

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