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Analysis of Balanced Incomplete Block Design with Mixed-up Plots

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SUMMARY

In this paper, we consider the problem of analysis of Balanced Incomplete Block Design (BIBD) in which yields of two plots get mixed-up. Estimates of mixed-up values, explicit expression for bias and test statistic for testing the hypothesis on homogeneity of treatment effects are developed. Also an illustrative example is given.

Keywords: Mixed-up plots, Balanced Incomplete Block Design.

1. INTRODUCTION

In certain experiments, especially in agriculture, due to either accidental error or negligence of the experimental staff or loss of proper labels of the observation kits of some plots, it becomes almost impossible to identify the actual yields of those plots, but however the sum total of the yields of all those plots are available. In this case we say that the yields of such plots are mixed-up. Bose and Mahalanobis (1938) have narrated their experience with such problems in some important experiments.

Bose and Mahalanobis (1938) have obtained estimates of the mixed-up values involved in some basic designs when there are only two mixed-up yields. Bose (1938) has given a general method of estimating mixed-up values when several observations get mixed-up together. For some other references on mixed-up analysis see Chakrabarthi (1962) and Deo and Kharshikar (1988). Techniques of estimation of mixed-up values have been attempted in other ways as well. To estimate mixed-up observations, iterative and non-iterative computer procedures are given by Preece and Gower (1974) and John and Lewis (1975) respectively. Another approach of estimation of mixed-up values is by using the covariance method and is given in Nair (1940) and Patricia (1981).

In certain experiments, because of shortage of experimental apparatus or facilities or physical size of the block, we may be constrained in choosing enough numbers of plots in a block to apply treatments. In such experiments Balanced Incomplete Block Design (BIBD) is one of the most commonly used experimental design. In BIBD with mixed-up observations, due to the non-availability of explicit form for test statistic and other expressions required in the analysis, researchers in applied sciences resort to other crude methods. So in this paper we consider a BIBD with two mixed-up plots and derive reasonable estimates of mixed-up yields, obtain explicit expression for bias involved in the treatment sum of squares and develop expression for the test statistic to test the hypothesis on homogeneity of treatment effects, for each of the different cases.

2. ANALYSIS OF BIBD WITH TWO MIXED-UP PLOTS

Consider a BIBD with v treatments, b blocks each containing k plots, each treatment occurring r times and

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any two treatments occurring together in λ blocks. If Y_{ij} is the yield of the i^{th} treatment in the j^{th} block, the mathematical model assumed will be,

$Y_{ij} = n_{ij}(\alpha_i + \beta_j + e_{ij})$ $i = 1, 2,..., v; j = 1, 2,..., b$
where

$$n_{ij} = \begin{cases} 1 & \text{if } i^{th} \text{ treatment occurs in } j^{th} \text{ block} \\ 0 & \text{otherwise} \end{cases}$$

α_i and β_j are the i^{th} treatment and j^{th} block effects respectively and e_{ij}, $i = 1, 2,..., v$, j = 1, 2,..., b are independent and identically distributed N(0, σ^2) variables. We denote i^{th} treatment total by T_i and j^{th} block total by B_j provided there are no mixed-up values in the i^{th} treatment and j^{th} block. In this case if we also define $Q_i = T_i - \sum_j \frac{n_{ij}B_j}{k}$, then the error sum of squares associated with the above model is,

$$R_0^2 = \sum_{ij} n_{ij}Y_{ij}^2 - \sum_j \frac{B_j^2}{k} - \frac{k}{\lambda v}\sum_i Q_i^2 \sim \sigma^2\chi^2_{(kb-b-v+1)}. \tag{2.1}$$

The conditional error sum of squares subject to $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_v$ is

$$R_1^2 = \sum_{ij} n_{ij}Y_{ij}^2 - \sum_j \frac{B_j^2}{k} \sim \sigma^2\chi^2_{(kb-b)} \text{ (Under } H_0) \tag{2.2}$$

$$= R_0^2 + T$$

where $T = \frac{k}{\lambda v}\sum_i Q_i^2 \sim \sigma^2\chi^2_{(v-1)}$ (Under H_0) (2.3)

If the observation corresponding to Y_{ij} get mixed-up with another observation, then we write B'_j and T'_i to denote the sum of all non mixed-up values in the j^{th} block and sum of all non mixed-up values for the i^{th} treatment respectively. Suppose the two yields Y_{lm} and Y_{pq} are mixed-up and 'a' be the total yield of these two mixed-up plots. Let $Y_{lm} = X$ then $Y_{pq} = a - X$. To estimate X express R_0^2 given in (2.1) as a function of X, say $R_0^2(X)$, then the value $\hat{X}$ of X which minimizes $R_0^2(X)$ is taken as an estimate of X. If we define $T(\hat{X})$ as the treatment sum of squares T obtained from (2.3) after replacing X by $\hat{X}$, then it need not be the correct treatment sum of squares for testing H_0. Hence to obtain the bias in $T(\hat{X})$ first we express R_1^2 given in (2.2) as a function of X (that is $R_1^2(X) = R_0^2(X) + T(X)$) and determine the value X^* of X which minimizes $R_1^2(X)$. Then the bias involved in $T(\hat{X})$ is

$$B = R_1^2(\hat{X}) - R_1^2(X^*) \tag{2.4}$$

If explicit expression for bias is obtained, then one can write the adjusted treatment sum of squares as $T' = T(\hat{X}) - B$ and has $\sigma^2\chi^2_{(v-1)}$ under $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_v$. Then to test H_0, we use the F statistic

$$F = \frac{T'/(v-1)}{[R_0^2(\hat{X})]/(kb-b-v)} \sim F(v-1, kb-b-v)$$

In the next three sections we obtain the explicit expressions for $\hat{X}$, X^* and B so as to complete the analysis of BIBD with two mixed-up values.

3. ANALYSIS WHEN THE MIXED-UP PLOTS BELONG TO DIFFERENT BLOCKS AND DIFFERENT TREATMENTS

Let $Y_{lm} = X$ and $Y_{pq} = a - X$ be the mixed-up values in which 'a' is known. Let the i^{th} treatment be denoted by t_i, $i = 1, 2,..., v$ and S be the set of all these v treatments, which is partitioned as $S = S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6$ where, $S_1 = \{t_i : n_{im} = 0, n_{iq} = 0\}$ represents the set of all treatments which do not occur in the m^{th} and q^{th} blocks. $S_2 = \{t_i : n_{im} = 1, n_{iq} = 1 \text{ and } i \neq l, p\}$ represents the set of all treatments other than t_l and t_p but occur in both m^{th} and q^{th} blocks. $S_3 = \{t_i : n_{im} = 1, n_{iq} = 0 \text{ and } i \neq l, p\}$ represents the set of all treatments other than t_l and t_p which occur in the m^{th} block but not in the q^{th} block. $S_4 = \{t_i : n_{im} = 0, n_{iq} = 1 \text{ and } i \neq l, p\}$, represents the set of all treatments other than t_l and t_p which occur in the q^{th} block but not in the m^{th} block, $S_5 = \{t_l\}$ and $S_6 = \{t_p\}$.

When the mixed-up plots belong to different blocks and different treatments, there may arise the following four different cases.

3.1 Case 1

Consider the case when t_l occurs in the m^{th} block but not in the q^{th} block, and t_p occurs in the q^{th} block but not in the m^{th} block. That is $n_{lm} = 1$, $n_{lq} = 0$, $n_{pm} = 0$, $n_{pq} = 1$. In this case the cardinal number of the set S_3 and S_4 are equal. Let it be c_1.

The error sum of squares is

$$R_0^2(X) = X^2 + (a-X)^2 - \frac{(B'_m + X)^2}{k} - \frac{(B'_q + a - X)^2}{k} - \frac{k}{\lambda v}\left\{\sum_{i\in S_3} Q_i^2 + \sum_{i\in S_4} Q_i^2 + Q_l^2 + Q_p^2\right\}$$
$$+ \text{ Terms independent of } X \qquad (3.1.1)$$

where

$$\sum_{i\in S_3} Q_i^2 = \sum_{i\in S_3}\left(Q_i^{(m)} - \frac{X}{k}\right)^2$$

$$\text{where} \quad Q_i^{(m)} = T_i - \frac{(\sum'_j n_{ij}B_j + B'_m)}{k} \qquad (3.1.2)$$

$$\sum_{i\in S_4} Q_i^2 = \sum_{i\in S_4}\left(Q_i^{(q)} - \frac{a}{k} + \frac{X}{k}\right)^2$$

$$\text{where} \quad Q_i^{(q)} = T_i - \frac{(\sum'_j n_{ij}B_j + B'_q)}{k} \qquad (3.1.3)$$

$$Q_l^2 = \left[Q_l^{(1)} + \frac{X(k-1)}{k}\right]^2$$

$$\text{where} \quad Q_l^{(1)} = T'_l - \frac{(\sum'_j n_{lj}B_j + B'_m)}{k} \qquad (3.1.4)$$

$$Q_p^2 = \left[Q_p^{(1)} + \frac{a(k-1)}{k} - \frac{X(k-1)}{k}\right]^2$$

$$\text{where} \quad Q_p^{(1)} = T'_p - \frac{(\sum'_j n_{pj}B_j + B'_q)}{k} \qquad (3.1.5)$$

Use of equations (3.1.2) to (3.1.5) in (3.1.1) gives

$$R_0^2(X) = X^2 + (a-X)^2 - \frac{(B'_m + X)^2}{k} - \frac{(B'_q + a - X)^2}{k} - \frac{k}{\lambda v}\left\{\sum_{i\in S_3}\left(Q_i^{(m)} - \frac{X}{k}\right)^2 + \sum_{i\in S_4}\left(Q_i^{(q)} - \frac{a}{k} + \frac{X}{k}\right)^2 + \left[Q_l^{(1)} + \frac{X(k-1)}{k}\right]^2 + \left[Q_p^{(1)} + \frac{a(k-1)}{k} - \frac{X(k-1)}{k}\right]^2\right\}$$

+ Terms independent of X

Equating $\frac{\partial R_0^2(X)}{\partial X} = 0$ we get,

$$\hat{X} = \frac{a}{2} + \frac{\lambda v(B'_m - B'_q) + kQ_{(1)}}{2d_1}$$

where

$$Q_{(1)} = \sum_{i\in S_4} Q_i^{(q)} - \sum_{i\in S_3} Q_i^{(m)} + (k-1)(Q_l^{(1)} - Q_p^{(1)})$$

and, $d_1 = \lambda v(k-1) - c_1 - (k-1)^2$

In this case the value X^* which minimizes the conditional error sum of squares $R_1^2(X)$ is

$$X^* = \frac{a}{2} + \frac{(B'_m - B'_q)}{2(k-1)}$$

Using the relation (2.4), the non-negative bias in $T(\hat{X})$ is given by

$$B = \frac{\left[\left\{c_1 + (k-1)^2\right\}(B'_m - B'_q) + k(k-1)Q_{(1)}\right]^2}{2k(k-1)d_1^2}$$

Now to test $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_v$, the test statistic to be used is

$$\frac{T'/(v-1)}{[R_0^2(\hat{X})]/(kb-b-v)} \sim F(v-1,\ kb-b-v)$$

3.2 Case 2

Consider the case when t_l occurs both in the m^{th} and q^{th} blocks and t_p occurs in the q^{th} block but not in the m^{th} block. That is $n_{lm} = 1$, $n_{lq} = 1$, $n_{pq} = 1$ and $n_{pm} = 0$. In this case if the cardinal number of the set S_4 is c_2 then it is easy to see that the cardinal number of S_3 is $c_2 + 1$. In this case we can write Q_l^2 as

$$Q_l^2 = \left[T_l' - \frac{(\sum_j' n_{lj} B_j + B_{(m,q)})}{k} + X - \frac{a}{k} \right]^2$$

where $B_{(m,q)} = B_m' + B_q'$

$$= \left[Q_l^{(2)} + X - \frac{a}{k} \right]^2$$

where $Q_l^{(2)} = T_l' - \frac{(\sum_j' n_{lj} B_j + B_{(m,q)})}{k}$

All other terms involved in R_0^2 are same as that of Case 1. Then the error sum of squares can be written as

$$R_0^2(X) = X^2 + (a-X)^2 - \frac{(B_m' + X)^2}{k}$$

$$- \frac{(B_q' + a - X)^2}{k} - \frac{k}{\lambda v} \left\{ \sum_{i \in S_3} \left(Q_i^{(m)} - \frac{X}{k} \right)^2 \right.$$

$$+ \sum_{i \in S_4} \left(Q_i^{(q)} - \frac{a}{k} + \frac{X}{k} \right)^2 + \left(Q_l^{(2)} + X - \frac{a}{k} \right)^2$$

$$\left. + \left(Q_p^{(1)} + \frac{a(k-1)}{k} - \frac{X(k-1)}{k} \right)^2 \right\}$$

+ Terms independent of X

By equating $\frac{\partial R_0^2(X)}{\partial X}$ to zero we get,

$$\hat{X} = \frac{a}{2} + \frac{\lambda v (B_m' - B_q') + k Q_{(2)}}{2 d_2}$$

where

$$Q_{(2)} = \sum_{i \in S_4} Q_i^{(q)} - \sum_{i \in S_3} Q_i^{(m)} + k Q_l^{(2)} - (k-1) Q_p^{(1)}$$

and $d_2 = \lambda v(k-1) - c_2 - (k-1)^2 - k$

As in case 1, here also we get X^* as

$$X^* = \frac{a}{2} + \frac{(B_m' - B_q')}{2(k-1)}$$

Using relation (2.4), the non-negative bias in $T(\hat{X})$ is given by

$$B = \frac{\left[\left\{ c_2 + k + (k-1)^2 \right\} (B_m' - B_q') + k(k-1) Q_{(2)} \right]^2}{2k(k-1) d_2^2}$$

Now to test $H_0 : \alpha_1 = \alpha_2 = \ldots = \alpha_v$, the test statistic to be used is

$$\frac{T'/(v-1)}{[R_0^2(\hat{X})]/(kb-b-v)} \sim F(v-1, kb-b-v)$$

3.3 Case 3

Consider the case when t_p occurs both in the m^{th} and q^{th} blocks and t_l occurs in the m^{th} block but not in the q^{th} block. That is $n_{pm} = 1$, $n_{pq} = 1$, $n_{lm} = 1$ and $n_{lq} = 0$. In this case, if the cardinal number of the set S_3 is c_3 then as in Case 2 the cardinal number of S_4 is $c_3 + 1$. In this case Q_p^2 can be written as

$$Q_p^2 = \left[Q_p^{(2)} + \frac{a(k-1)}{k} - X \right]^2$$

where $Q_p^{(2)} = T_p' - \frac{(\sum_j' n_{pj} B_j + B_{(m,q)})}{k}$

All other terms involved in R_0^2 are same as that of Case 1. Then the error sum of squares can be written as

$$R_0^2(X) = X^2 + (a-X)^2 - \frac{(B_m' + X)^2}{k}$$

$$- \frac{(B_q' + a - X)^2}{k} - \frac{k}{\lambda v} \left\{ \sum_{i \in S_3} \left(Q_i^{(m)} - \frac{X}{k} \right)^2 \right.$$

$$+ \sum_{i\in S_4}\left(Q_i^{(q)} - \frac{a}{k} + \frac{X}{k}\right)^2 + \left(Q_l^{(1)} + \frac{X(k-1)}{k}\right)^2$$

$$+ \left.\left(Q_p^{(2)} + \frac{a(k-1)}{k} - X\right)^2\right\}$$

+ Terms independent of X

By equating $\frac{\partial R_0^2(X)}{\partial X}$ to zero we get,

$$\hat{X} = \frac{a}{2} + \frac{\lambda v(B'_m - B'_q) + kQ_{(3)}}{2d_3}$$

where

$$Q_{(3)} = \sum_{i\in S_4} Q_i^{(q)} - \sum_{i\in S_3} Q_i^{(m)} - k\,Q_p^{(2)} + (k-1)Q_l^{(1)}$$

and $\quad d_3 = \lambda v(k-1) - c_3 - (k-1)^2 - k$

Here also we get X^* as, $X^* = \frac{a}{2} + \frac{(B'_m - B'_q)}{2(k-1)}$

Using relation (2.4), the non-negative bias in $T(\hat{X})$ is given by

$$B = \frac{\left[\{c_3 + k + (k-1)^2\}(B'_m - B'_q) + k(k-1)Q_{(3)}\right]^2}{2k(k-1)d_3^2}$$

Now to test $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_v$, the test statistic to be used is

$$\frac{T'/(v-1)}{[R_0^2(\hat{X})]/(kb-b-v)} \sim F(v-1, kb-b-v)$$

3.4 Case 4

Consider the case when t_l and t_p occurs both in the m^{th} and q^{th} blocks. That is $n_{lm} = 1$, $n_{lq} = 1$, $n_{pm} = 1$ and $n_{pq} = 1$. In this case clearly the cardinal numbers of both S_3 and S_4 are equal and let it be denoted by c_4. In this case we can write Q_l^2 and Q_p^2 as follows

$$Q_l^2 = \left[Q_l^{(2)} + X - \frac{a}{k}\right]^2$$

$$Q_p^2 = \left[Q_p^{(2)} + \frac{a(k-1)}{k} - X\right]^2$$

All other terms involved in R_0^2 are same as that of Case 1**.** Then the error sum of squares can be written as

$$R_0^2(X) = X^2 + (a-X)^2 - \frac{(B'_m + X)^2}{k} - \frac{(B'_q + a - X)^2}{k}$$

$$- \frac{k}{\lambda v}\left\{\sum_{i\in S_3}\left(Q_i^{(m)} - \frac{X}{k}\right)^2\right.$$

$$+ \sum_{j\in S_4}\left(Q_j^{(p)} - \frac{a}{k} + \frac{X}{k}\right)^2 + \left(Q_m^{(2)} + X - \frac{a}{k}\right)^2$$

$$+ \left.\left(Q_q^{(2)} + \frac{a(k-1)}{k} - X\right)^2\right\}$$

+ Terms independent of X

By equating $\frac{\partial R_0^2(X)}{\partial X}$ to zero we get

$$\hat{X} = \frac{a}{2} + \frac{\lambda v(B'_m - B'_q) + kQ_{(4)}}{2d_4}$$

where $Q_{(4)} = \sum_{ji\in S_4} Q_i^{(q)} - \sum_{i\in S_3} Q_i^{(m)} + k\left(Q_l^{(2)} - Q_p^{(2)}\right)$

and $\quad d_4 = \lambda v(k-1) - c_4 - k^2$

As in the previous cases, the value X^* which minimizes the conditional error sum of squares $R_1^2(X)$ is

$$X^* = \frac{a}{2} + \frac{(B'_m - B'_q)}{2(k-1)}$$

Using relation (2.4), the non-negative bias in $T(\hat{X})$ is given by

$$B = \frac{\left[(c_4 + k^2)(B'_m - B'_q) + k(k-1)Q_{(4)}\right]^2}{2k(k-1)d_4^2}$$

Now to test $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_v$, the test statistic to be used is

$$\frac{T'/(v-1)}{[R_0^2(\hat{X})]/(kb-b-v)} \sim F(v-1,\ kb-b-v)$$

4. MIXED-UP PLOTS BELONG TO SAME TREATMENT

Let $Y_{lm} = X$ and $Y_{lq} = a - X$ be the mixed-up values in which 'a' is known. Let S be partitioned as $S = S_1 \cup S_2 \cup S_3 \cup S_4$. Where $S_1 = \{t_i : n_{im} = 0,\ n_{iq} = 0,\ i = 1, 2,..., v\}$ represents the set of all treatments which do not occur in the m^{th} and q^{th} blocks. $S_2 = \{t_i : n_{im} = 1,\ n_{iq} = 1,\ i = 1, 2,...,v\}$ represents the set of all treatments which occur in both m^{th} and q^{th} blocks. $S_3 = \{t_i : n_{im} = 1,\ n_{iq} = 0,\ j = 1, 2,..., v\}$ represents the set of all treatments occur in the m^{th} block but not in the q^{th} block. $S_4 = \{t_i : n_{im} = 0,\ n_{iq} = 1,\ i = 1, 2, ..., v\}$ represents the set of all treatments which occur in the q^{th} block but not in the m^{th} block. In this case the cardinal number of S_3 and S_4 are equal. Let it be c_5. Also $\sum_{i \in S_4} Q_i^2$ and $\sum_{i \in S_3} Q_i^2$ are all same as that of Case 1 in Section 3. Then the error sum of squares is

$$R_0^2(X) = X^2 + (a-X)^2 - \frac{(B'_m + X)^2}{k} - \frac{(B'_q + a - X)^2}{k} - \frac{k}{\lambda v}\left\{\sum_{i \in S_3}\left(Q_i^{(m)} - \frac{X}{k}\right)^2 + \sum_{i \in S_4}\left(Q_i^{(q)} - \frac{a}{k} + \frac{X}{k}\right)^2\right\}$$
$$+ \text{ Terms independent of } X$$

By equating $\frac{\partial R_0^2(X)}{\partial X}$ to zero we get

$$\hat{X} = \frac{a}{2} + \frac{\lambda v(B'_m - B'_q) + kQ_{(5)}}{2d_5}$$

where $Q_{(5)} = \sum_{i \in S_4} Q_i^{(q)} - \sum_{i \in S_3} Q_i^{(m)}$

and $d_5 = \lambda v(k-1) - c_5$

In this case the value X^* which minimizes the conditional error sum of squares $R_1^2(X)$ is

$$X^* = \frac{a}{2} + \frac{(B'_m - B'_q)}{2(k-1)}$$

Using relation (2.4), the non-negative bias in $T(\hat{X})$ is given by

$$B = \frac{\left[c_5(B'_m - B'_q) + k(k-1)Q_{(5)}\right]^2}{2k(k-1)d_5^2}$$

Now to test $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_v$, the test statistic to be used is

$$\frac{T'/(v-1)}{[R_0^2(\hat{X})]/(kb-b-v)} \sim F(v-1,\ kb-b-v)$$

5. MIXED-UP PLOTS BELONG TO SAME BLOCK

Let $Y_{lm} = X$ and $Y_{pm} = a - X$ be the mixed-up values in which 'a' is known. The set of all treatments S be partitioned as, $S = S_1 \cup S_2 \cup S_3$ where $S_1 = \{t_i : i \neq l, p\}$ represents the set of all treatments other than t_l and t_p, $S_2 = \{t_l\}$ and $S_3 = \{t_p\}$. We have

$$Q_l^2 = \left[Q_l^{(3)} + X - \frac{a}{k}\right]^2$$

where $$Q_l^{(3)} = T_l' - \frac{(\sum_j' n_{lj}B_j + B'_m)}{k}$$

$$Q_p^2 = \left[Q_p^{(3)} + \frac{a(k-1)}{k} - X\right]^2$$

where $$Q_p^{(3)} = T'_p - \frac{(\sum_j' n_{pj}B_j + B'_m)}{k}$$

The error sum of squares is

$$R_0^2(X) = X^2 + (a-X)^2 - \frac{k}{\lambda v}\left\{\sum_{i \in S_3}\left(Q_i^{(m)} - \frac{X}{k}\right)^2 + \sum_{i \in S_4}\left(Q_i^{(q)} - \frac{a}{k} + \frac{X}{k}\right)^2\right\}$$
$$+ \text{ Terms independent of } X$$

By equating $\dfrac{\partial R_0^2(X)}{\partial X}$ to zero we get

$$\hat{X} = \frac{a}{2} + \frac{k\,Q_{(6)}}{2\,(\lambda v - k)}$$

where, $Q_{(6)} = Q_l^{(3)} - Q_p^{(3)}$

In this case the value X^* which minimizes the conditional error sum of squares $R_1^2(X)$ is

$$X^* = \frac{a}{2}$$

Using relation (2.4), the non-negative bias in $T(\hat{X})$ is given by

$$B = \frac{\left[kQ_{(6)}\right]^2}{2(\lambda v - k)^2}$$

Now to test $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_v$, the test statistic to be used is

$$\frac{T'/(v-1)}{[R_0^2(\hat{X})]/(kb-b-v)} \sim F(v-1,\, kb-b-v)$$

6. APPLICATION

The following example from Das and Giri (1986) is used to illustrate the applications of the results derived in this paper. Table 6.1 gives the results of an experiment in BIBD with $v = 13$, $b = 13$, $k = 4$, $r = 4$ and $\lambda = 1$. Block 1, for example contained treatments t_3, t_6, t_9 and t_{11}, and the figures in the table in the columns 1 to 13 are the yields obtained.

Though no mixed-up values occur in this experiment, for illustration purpose, two plot yields are chosen as mixed-up for each of the possible cases. Using the theory developed in this paper we have obtained the estimators of the mixed-up plots, bias and test statistic for all possible cases.

- Suppose $Y_{4,\,6}$ and $Y_{12,\,7}$ are mixed-up, then we have situation in Section 3.1 with a =121. The estimated values are $Y_{4,\,6}$ = 61.2 and $Y_{12,\,7}$ = 59.8 (the actual yields in these plots were 60 and 61 respectively). Also, B = 0.449, 0.449, $R_0^2(\hat{X})$ = 2229.4, $T(\hat{X})$ = 1389.36 and F = 1.3498.
- If $Y_{4,10}$ and $Y_{11,11}$ are mixed-up, then we have situation in Section 3.2 with a =116. The estimate

Table 6.1

Treatment	Block												
	1	2	3	4	5	6	7	8	9	10	11	12	13
t_1							69		77		72	63	
t_2				54					65	57			61
t_3	50	45						68	75				
t_4		38				60				60	62		
t_5				54		65	62	65					
t_6	39					54						67	63
t_7					47			66			52		79
t_8		65		70	60							54	
t_9	58				61		52			53			
t_{10}			31		64	65			63				
t_{11}	49		37	34							56		
t_{12}		44	63				61						53
t_{13}			53					75		70		84	

values are, $Y_{4,10}$ = 61.17 and $Y_{11,\ 11}$ = 54.83 (the actual yields in these plots were 60 and 56 respectively). Also, B = 51.04, $R_0^2(\hat{X})$ = 2229.83, $T(\hat{X})$ = 1407.5014 and F = 1.318.

- If the mixed-up values are $Y_{1,\ 9}$ and $Y_{2,\ 10}$, then we have situation in Section 3.3 with a =134. In this case the estimated values are, $Y_{1,\ 9}$ = 77.33 and $Y_{2,\ 10}$ = 56.67 (the actual yields in these plots were 77 and 57 respectively). Also, B = 73.5, $R_0^2(\hat{X})$= 2224.54, $T(\hat{X})$ = 1401.22 and F = 1.29.
- Suppose mixed-up are $Y_{3,\ 8}$ and $Y_{3,\ 9}$, then we have situation in Section 4 with a =143. The estimated values are, $Y_{3,\ 8}$ = 69.44 and $Y_{3,\ 9}$ = 73.55 (the actual yields in these plots were 68 and 75 respectively). Also, B = 7.8776, $R_0^2(\hat{X})$ = 2209.588, $T(\hat{X})$ = 1396.6 and F = 1.36.
- When $Y_{1,\ 11}$ and $Y_{4,\ 11}$, are mixed-up then we have situation in Section 5 with a =134. The estimated values are, $Y_{1,11}$ = 71.88 and $Y_{4,\ 11}$ = 62.11 (the actual yields in these plots were 72 and 62 respectively). Also, B = 47.8, $R_0^2(\hat{X})$ = 2221.473, $T(\hat{X})$ = 1395.1735 and F = 1.321.

The appropriate tabular value of F at 5% level of significance is 2.96, which is greater than the calculated F value in all the above cases. So the test indicates that in all cases the treatment effects are homogeneous. It may be noted that if the original data is used as such (without any mixed-up case), the F statistic for testing the homogeneity of treatment is equal to 1.4 which is also not significant.

Note : If the number of mixed-up values is more than two, then getting closed form of expression for the error sum of squares and corrected treatment sum of squares become tedious. However the results of this paper helps any practicing statistician to choose appropriate exact estimates and test statistics for a BIBD with only two mixed-up values.

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