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Dialectical Estimation

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SUMMARY

A novel approach of *dialectical estimation* is discussed in which averaging two somewhat contradictory estimates from the same individual reduces error which is larger in magnitude than the reduction in sampling error expected when two estimates from the same individual are obtained on the basis of an internal probability distribution. Such procedures are useful in judgment sampling where a single judge is available for evaluation and his subjective assessment is affected by systematic error in addition to random error. Hegelian dialectics is the basis of such a strategy and can lead to a series of dialectical averages that tend to the true value of the characteristic under investigation.

Keywords: Hegelian dialectics, Dialectical estimation, Judgment sampling, Internal probability distribution, Date-estimation experiment, Mental tool.

1. INTRODUCTION

My first encounter with Prof. GR Seth was a strange one, we being fellow travelers in a phat-phat (auto-rickshaw with a motor-cycle) on the roads of Delhi from Union Public Service Commission (UPSC), Shahjahan Road to his residence on Mall Road in the year 1955. Later on, I was fortunate enough to work under his able guidance when I joined IASRI, then known as Institute of Agricultural Research Statistics (IARS), in 1961 as Assistant Professor in the Training Unit of the Institute. He was then Professor/Deputy Statistical Advisor and took fancy of me in view of my throughout first class academic career. The Institute had embarked on collaboration with IARI on teaching of Agricultural Statistics for the award of M.Sc. and Ph.D. degrees and was in the process of finalizing the syllabus of the courses. He called me one of those days and directed me to prepare the syllabus single-handedly and over-night on the basis of those then existing for the Professional Statistician's Certificate (PSC) course of the Institute and M.Sc. course of the University. It was a hard task for me, I having no training at the IARS and coming from the Indian Veterinary Research Institute (IVRI), Izatnagar, three years after obtaining a M.Sc. degree in Mathematical Statistics from Lucknow University. I, however, did manage to draw up a syllabus of M.Sc. and Ph.D. courses that included core as well as advanced courses as per requirement of the Post-graduate School of IARI which had then recently acquired the deemed University Status from the University Grants Commission. The syllabus was duly approved and continued to form the foundation basis for all syllabi that came into being subsequently.

Dr. Seth made me to teach most of the courses like those on probability, statistical methods, experimental designs, sample surveys, statistical genetics etc. including experimental statistics courses given as 'minor' to IARI students. But I was hesitant to teach the course on 'statistical inference' though I had learnt it in my M.Sc. course at Lucknow University. I therefore attended the course given by him to the students that was based on his own learning of the subject at Stanford University in USA and reflected his thorough grasp of the subject. It made a lasting

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impression on me and helped me in my own teaching of the subject when called upon to do so once or twice when there was no one to teach it. He also recommended me to go for a one year training to Canada in computer science under Colombo Plan to help in building up the then existing IBM system at the Institute. However, due to the wishes of Dr. VG Panse, the Statistical Advisor and Head of the Institute my training was shifted to UK in statistical genetics. Subsequently, as the first Director of the Institute, he arranged to create positions of Sr. Statistician and Sr. Professor, both in statistical genetics in the IV Five Year Plan, so as to help me, after my return from training, in building up a school of statistical genetics at the Institute in accordance with the wishes of Dr. Panse. In retrospect I realized that behind his this generous gesture was his desire to compensate for the sacrifice that I made to opt for the Colombo Plan training in preference to joining a post of Sr. Scientist in Defense Science Laboratory for which I had been selected by the UPSC and the post was a Sr. Class I as against a Junior Class I post of Assistant Professor that I was holding at that time. In fact, when I went to him for advice in this regard, he said that it has to be entirely my choice. Needless to add that my Colombo Plan training at the Institute of Animal Genetics, Edinburgh, UK and subsequent developments led to the emergence of IASRI as world center in statistical genetics that is now being further strengthened into Bioinformatics field due to the efforts of the present Director, Dr. VK Bhatia, an outstanding statistical geneticist who worked with me from mid-seventies after doing his Ph.D. with me.

Prof. Gobind Ram Seth was a renowned statistician, well known for his fine contributions in sampling theory and practice, theory of parametric and non-parametric inference and sequential analysis in addition to his significant work in developing the Institute in more than one way in collaboration with Dr. V.G. Panse. I pay my humble tribute to his memory by discussing a recently emerging topic on *dialectical estimation* useful in judgment sampling.

2. JUDGMENT SAMPLING

Modern investigations in almost all activities involve quantitative data generation followed by sound statistical analysis to draw valid inferences there from. Since in most cases census of all the units in the population is costly and time consuming, drawing simple random samples of a pre-determined size is usually the norm in such investigations. Such a procedure provides with an estimate of the average characteristic of the population along with an estimate of the sampling error of the estimate that decreases with the increase in sample size. For each unit in the sample, the characteristic is either measured objectively with an instrument or judged subjectively. In the former case, the observation taken is sum of the true value of the characteristic in the population and the random error due to measurement that can be reduced by taking repeated measurements. The random error follows a normal distribution and if the instrument is accurate and unbiased, has a very small standard deviation so that taking only a single measurement suffices. But if it is not so, repeated measurements of the unit are to be taken. For instance, in chemical experiments usually duplicate samples are taken for measurements and in recently emerging microarray experiments triplicate or more number of samples are taken. On the other hand, when the characteristic is judged by an expert, it is sum of the true value, the random error and a systematic bias of the expert – the tendency to over- or under- estimate the true value – which gets introduced and which cannot be reduced by repetition of judging the observation. It is this judgment sampling that I wish to discuss in this paper.

There are several situations, particularly in psychology, where judges are used to subjectively evaluate the characteristics under study. In the field of agriculture too, several times we have to resort to some type of judgment in our evaluations. For instance, the Ministry of Agriculture under Govt. of India issues several crop forecasts in advance of the actual harvesting of the crop each year for estimation of total crop production. There are four forecasts, first in middle of September, the second in January, the third in March end and the final fourth one in the month of June. The assessments are made by the State Govt. based on the reports from field offices of the State Dept. of Agriculture, mainly guided by the visual observations and therefore involve judgment.

In judgment sampling, if we have several judges evaluating the same unit, the average of their evaluations provides with a more accurate estimate than the estimate given by a randomly chosen judge. When the correlation between the errors in judgment by different judges is low, the bias component, like the error component, is expected to average out, leading to a more accurate estimate provided of course the number of judges is very large. This is because there are likely to be as many judges who underestimate the true value as those who overestimate the true value. However, if *only* one judge is available for evaluation, how one should go about improving the estimates. A useful strategy is to ask the same judge to re-evaluate the unit such that it is at odds with the first one. This second estimate can be termed as a *dialectical* estimate. The averaging of the first and the second dialectical estimate can lead to an estimate with reduced estimation error that can be larger in magnitude than that expected by solely reducing the random error. This is because the dialectical estimate is based on different knowledge and therefore has a different error. The mind of the same judge averages its own conflicting opinions. This dialectical approach has its origin in Hegelian philosophy which we discuss in the next section.

3. CONCEPT OF DIALECTICS

Friedrich Hegel, a German philosopher, believed that the *thought* process moves in a three-beat rhythm that he called 'dialectic'. It begins with an idea- a thesis- then proceeds to develop into its opposite, the antithesis; after that the mind sees the relatedness of thesis and antithesis and weaves them together into a synthesis. This synthesis, in turn, becomes another thesis, and the dialectic continues. This Hegel's *dialectical idealism* was questioned by Karl Marx who interpreted the three-beat rhythm as just opposite of that given by Hegel and created by the class struggle in a society determined by man's economic needs. He claimed it to be more real and laid the foundations of *dialectical materialism* that dominated half of the world by way of communism. Dialectics, however, fell into disrepute due to political reasons mainly because of its advocacy by communist Russia. It is, on the other hand, a way of explaining the world faced with a set of opposing, dichotomic and contradictory processes. In fact dialectics is only another way of saying 'thinking correctly'.

I have discussed elsewhere a dialectical perspective to agricultural research for sustainable food production (Narain 1992, 1997, 1998a, 1998b, 2002, 2006), my recent article on this subject having appeared in INSA proceedings (Narain 2008). Here I wish to adopt it in connection with judgment sampling.

4. DIALECTICAL ESTIMATION

The three-beat rhythm (dialectic) of the thought process referred to in the previous section can be utilized to develop dialectical estimates. The thesis in the beginning corresponds to the first estimate (y_1) by a judge. Its opposite- the antithesis –corresponds to the dialectical and the second estimate (y_2) by the judge and averaging the two estimates (synthesis) is the *dialectical average* (y_d). Herzog and Hertwig (2009) calls it *dialectical bootstrapping*. This y_d then becomes the basis for generating the second round of dialectical average. And the process is continued till we get the true value.

Suppose the true value being judged is μ. A randomly chosen judge estimates it, for the first time, as y_1 which is at a distance of x from μ, so that $y_1 = \mu + x$, the estimate thereby having an error of x units. The *same* judge is then asked to give a dialectical estimate which is at odds with the first one. For this purpose, he is supposed to think that y_1 is well off the true value. He needs to consider reasons for having such thinking in terms of considerations that might have gone wrong and what would these imply. Based on such a perspective, he makes a second (y_2), the so called dialectical estimate that is at odds with y_1. The possible range in which the second estimate is supposed to fall is from the upper value at $y_1 = \mu + x$ to the lower value at (μ-3x), a distance of 3x from the true value on the *opposite* side of the first estimate. When the second estimate is at the lower value, the dialectical average is (μ-x), committing an absolute error of x units, the same as that with the first estimate. But when the dialectical estimate is *above* the lower value, say

(μ-2x), the dialectical average becomes $y_{d(1)}$= (μ-x/2), giving a *smaller* absolute error of half of that with the first estimate. We need to take the second estimate as much opposite to the first estimate as possible, the maximum being at a distance of 3x (three times the error in the first estimate) away from the true value on the opposite side. Since, however, dialectical average in this case happens to give the same absolute error as the first estimate, we consider a distance of 2x units away from the true value on the opposite side of the true value. This gives the dialectical average $y_{d(1)}$. Now considering $y_{d(1)}$ units as his first estimate, the judge goes about, with the same thinking process as earlier, developing an estimate which is at odds with $y_{d(1)}$. The possible range for this dialectical estimate now becomes from the lower value of $y_{d(1)}$ to the upper value of (μ + 3x/2). Again, if the dialectical estimate coincides with the upper value, the dialectical average becomes (μ + x/2), committing an absolute error of x/2 units, the same as that with $y_{d(1)}$. But if the dialectical estimate is *below* the upper value, say at a distance of 2x/2 on the opposite side of $y_{d(1)}$ which means (μ + x), the second dialectical average becomes $y_{d(2)}$= (μ + x/4), giving a *still smaller* absolute error of one-fourth of that with the first estimate (y_1). This dialectics process can be continued to yield dialectical averages with further smaller absolute errors. For instance, the n-th dialectical average can be expressed as

$$y_{d(n)} = [\mu + (-1)^n x/2^n].$$

This shows that the absolute error is ($1/2^n$) times of that with the first estimate. Obviously with infinitely large n, the error tends to be zero and the dialectical average tends to be the true value μ. It is also to be noted that when the error in the first estimate (x) is small, the dialectical averages would also have smaller errors.

5. ILLUSTRATION

We illustrate the process of dialectical estimation by describing the empirical results given in Herzog and Hertwig (2009) on date-estimation task. It was based on a survey of N = 101 participants who were students at the University of Basel, Switzerland. For participating in the study, they received a flat fee of 10 Swiss francs and the chance to win one of two iPods in a lottery. The date-estimation task was created, using the *Wikipedia* encyclopedia available on-line, by selecting 40 historical events such as the discovery of electricity etc., 10 each from the 16th, 17th, 18th, and 19th centuries. Each participant was randomly assigned to one of the two conditions: (a) dialectical bootstrapping condition (N_1= 50) and (b) reliability condition (N_2=51). The latter condition was introduced for comparative purpose to see whether the dialectical approach performs better or not. In each condition, the participants were asked to generate their estimates of the historical events *without* knowing that they would be asked later to generate a second estimate. In (a), after they gave their estimates, they were asked to give dialectical estimates (while their first estimates were displayed in front of them) that are at odds with the first one. They were told beforehand that the more accurate of the two estimates for each question would be selected and that the chances of winning an iPod would increase as the absolute errors of these 'better' estimates decreased. This incentive was given to elicit *bold* second estimates. In (b), participants simply made a second estimate without knowing their first estimates. They were told beforehand that one of the two estimates for each question would be randomly selected and that the absolute errors of these selected estimates would determine the chance of winning an iPod. This incentive was to elicit a participant's 'best' estimate on both the occasions.

The results were expressed in terms of the median absolute deviation between his or her estimates and the actual dates. This was averaged across participants to give the accuracy measure of each estimate. On the other hand, the accuracy gain (%) for a participant was defined as the median decrease in error of the average of the two estimates relative to the first estimate, across items. In condition (a), the accuracy gain averaged 4.1 percentage points with a 95% confidence interval of 2.0% - 6.4% compared to condition (b) where it was only 0.3% with confidence interval of 0.0% - 0.8%. The distribution of the dialectical gains across participants indicated that in three-fourths of the participants the accuracy increased, the maximum increase being about 30%. In the rest, it decreased, the maximum decrease being about 15%.

6. DISCUSSION

Dualism in the world is a basic phenomenon. We have up and down, good and bad, rich and poor, beautiful and ugly, intelligent and dull, love and hate and so on. Hegel described the synthesis of such phenomena in terms of dialectics and gave a powerful thought process as a three-beat rhythm as discussed earlier. It is interesting that this concept of dialectics can be used to generate a sort of *dialectical estimate* when an individual judge or evaluator exploits it to improve upon his conflicting opinions through a thinking process in his mind. Herzog and Hertwig (2009) used it in a novel way to build a second dialectical estimate in an opinion survey based on *consider-the-opposite* strategy that led to the conclusion 'the wisdom of many in one mind'. But he did not go to the next step of building a third dialectical estimate. As we have seen here the Hegelian dialectics is a repeated process; thesis-antithesis-synthesis, synthesis = thesis-antithesis-synthesis and so on. That means the second dialectical estimate can lead to a third one and probably a fourth one before coming to a stop and giving a dialectical average. This further reduces the individual judge's error in making the prediction as shown here theoretically. When I raised this point with Dr. Herzog, he (Herzog 2009), remarked and I quote

"We truncated the dialectical process after the first step and one could indeed continue the process to create a second, third and so on dialectical estimate, possibly/probably further increasing accuracy. However, as research from the forecast combination literature suggest, there will be a decreasing marginal increase in accuracy for each further step (as the degrees of freedom to come up with another dialectical, yet realistic/plausible estimate decrease) and most of the gain will probably be accomplished with, say, two to 5 estimates. We actually thought about investigating these questions empirically."

The dialectical estimate is *not* obtained in a random manner but is generated by looking at the first estimate such that it is at odds with the first estimate which is, however, based on a random process. It is interesting to find that repeating the dialectics process *ad infinitum* gives the dialectical average which tends to converge to the true value. A single mind is thus capable of simulating the dialectics process so as to lead to the true value. In actual practice, however, we may not need to go to a series of dialectics and terminate the process at the first dialectical average as shown empirically by Herzog and Hertwig (2009) in a date-estimation experiment.

The whole purpose of taking a dialectical average of the first random and the second dialectical estimates is to come up with an estimate with lower error and therefore higher accuracy than the first random estimate. The dialectical average has been shown, both theoretically as well as empirically, to have smaller error in magnitude than the estimate based on a draw from the internal probability distribution of the individual.

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DISCUSSION OF DIALECTICAL ESTIMATION

Raj S. Chhikara

Professor Narain is to be commended for his thought provoking ideas in his contribution to the special issue of JISAS published in honor of Professor G.R. Seth. First, it highlights the significance of Professor Seth's contribution to the field of statistics and to the growth of IASRI, a premiere global institute in agricultural statistics. Next, it puts forward a novel idea of dialectical approach to parametric estimation involving "judgment" sampling.

I take here some liberty in making the following comments more so from the point of clarifying my understanding of this "novel approach" as put forward in the article that I was not familiar with.

Professor Narain views dialectics in terms of "thinking correctly" and this has the implication of one being correct, at least thinking in the direction of being right, which may not necessarily be the case when a parameter is estimated. The factual knowledge would come into play in the process of achieving correctness. To what extent one has the facts and how those facts are evaluated or weighed in judging or arriving at a conclusion by an individual will reflect upon the correctness of his thought and hence the outcome. The estimation approach as described in the paper amounts to weighing pros and cons to determine a sequence of estimates on opposite sides of some parametric value an individual has to consider. This process, called dialectical, involves critical thinking which may not necessarily be the same as "thinking correctly". So it seems one is to view dialectics in terms of "thinking critically" and dialectical estimation as proposed to be a method of accounting pros and cons in the process of developing a sequence of estimates and then weighing them to arrive at an estimate for the parameter.

Next, the underlying assumptions to the proposed dialectical estimation is the notion that an individual can be represented by a probability distribution and that a systemic approach of repeatedly estimating its mean and averaging the outcomes would lead closer to the true mean value of the distribution. The dialectical approach as described is shown to provide a consistent estimator. This, however, is predicate upon knowing x, the error in the first estimate, which is not the case unless the mean μ is known so that the first estimate $(\mu + x)$ and the subsequent estimates $\mu - 3x$ or $\mu - 2x$, etc. can be specified. So its implementation may be problematic in case of judgment sampling involving a lone individual. However, as described in the application, one is able to achieve a dialectical estimate based on multiple participants/individuals using multiple items for responses.

In the application discussed in the paper the error is computed for each participant by the mean absolute deviation based upon multiple responses. The average taken further over all participants provides an estimate of mean which is no more from an individual's probability distribution but for the one combined across all the participants. In this setting it is a viable approach and implementable.

The above dialectical estimation is contrasted with that of a "randomly best" estimation construct. It is shown that based on the accuracy gain and confidence intervals, the latter method performs better.

The thesis Professor Narain has advocated for using dialectics in estimation of parameter is visionary. It bears not only upon the use of facts, but also upon how to use systematically the thought process to estimate a quantity which could have its own a-priori distribution. So this approach may be seen inherently as Bayesian and thus it has potential for rigorous treatment and investigation going forward.

REJOINDER

Prem Narain

I am indeed grateful to Prof. Raj Chhikara for taking interest in my work and discussing certain issues on the novel approach of dialectical estimation. I offer the following comments to clarify the issues involved.

1. The dialectical approach that I have discussed here in connection with estimation in judgment sampling has a much wider context in such diverse areas as agriculture, genetics, psychology, anthropology, ecology, evolution, neurobiology, developmental biology etc. where *dialectics* is only another way of saying 'thinking correctly'. It, however, need not be so in statistical estimation where we are faced with uncertainty. I therefore

welcome Prof. Chhikara's suggestion of interpreting it as 'thinking critically' in the statistical context.

2. The process of 'dialectical averaging' can be repeated as many times as one is able to do so and therefore the exercise was done to demonstrate theoretically how the accuracy would behave if this is done. In actual practice, however, one's forecast or prediction of the forthcoming event could end up with one round of dialectical process, as was provided in the application discussed. The key idea is that, compared to the first estimate of the event, the second dialectical estimate when averaged with the first estimate is expected to lead to a smaller error in prediction and hence a gain in accuracy, the error being available after the event has taken place. Similarly, taking the dialectical average as the new first estimate, the third dialectical estimate when averaged with this estimate is expected to give a still smaller error in prediction and consequently a higher gain in accuracy. This reasoning, in theory, could go on as long as possible. While this is true for a given individual, when several individuals are taken in the sample, we have a distribution of errors that indicates in what percentage of cases the dialectical process provides with a gain in accuracy. This is likely to be high with data generated empirically as found in the application discussed.

3. The dialectical way of improving upon the accuracy in prediction or judgment is a mental process where each individual in the sample can construct conflicting realities of the event under consideration and can possibly benefit from dialectical averaging.