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Ambiguities in the Basics of Probability Theory and Implications

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SUMMARY

The purpose of this very short paper is to discuss a fundamental ambiguity in probability theory, and an implication of the same (in the Foundations of Physics). Details of this and other implications will be discussed elsewhere. The paper does not present any new mathematical result; rather, it emphasizes a certain feature in the basics of probability theory.

Keywords: Probability theory, Wave function.

INTRODUCTION

Probability theory is a very important discipline with far reaching applications. It has features which, mathematically, can be quite advanced. Much of this theory involves working with relatively complex and complicated events, which are themselves built up of simple events. Once we agree on these simple events and their probabilities, then the theory of more complex events is built upon this ground. However, at the very basic levels, there is an ambiguity which we now discuss.

THE AMBIGUITY

To focus our attention on the main point in this paper without unnecessary confusion, we shall begin with an example.

Suppose we have a bunch of pencils. Each pencil is of one of 7 colors. Each pencil has exactly one of the 26 Roman alphabets (*a*,......, *z*) engraved on it. Each pencil has one of five different kinds of bases: circle, square, triangle, pentagon, and hexagon. Each pencil has either one, or two, or three dots on it. Finally, each pencil is either thin or thick pointed.

Thus, in this example, we have an object called 'pencil'. This object has 5 attributes, namely, type of color (7 kinds), one of the letters from the alphabet engraved on it (26 symbols), type of base (5 types), number of dots (3 levels), and types of tip (2).

Now, suppose we have n such objects (pencils) randomly thrown on the floor of a room, where, for simplicity we shall assume that no two objects are identical. A child walks in, and randomly picks up a pencil; this pencil is seen to be of green color with a triangle base. The nature of its tip, whether it is thick-point or thin-point is not visible.

People sitting in the room ask the question: 'What is the probability (say, p) that the pencil is thin-tipped?' We shall assume that the value of n is unknown. We shall also assume that people do not try to examine any further the pencils on the floor.

A Bayesian in the room says that, in the absence of any information, he would say that the probability is 2/3, because he normally finds that in the stores, about one-third of the pencils on sale are thick tipped. Meanwhile, a man walks in and says that he knows about the pencils on the floor (before the child picked

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up one pencil), and that there were 17 green pencils out of which 3 were thin-tipped. Since the choice by the child is randomly made, he says that, *obviously*, p = 3/17. Now, another man comes; he says that there were 10 triangle-based pencils out of which 5 were thin-tipped. He says that, *obviously,* p = 5/10. This gives rise to a new question: "Which value of p should be accepted: one of the 3 values just suggested (namely, 2/3, 3/17, and 5/10), or something else?"

Consider the value 3/17 for *p;* that should be right since there are 17 green pencils and 3 of them have thin tip, and the drawing is random. But, so is the value 5/10, based on the pencils with triangular base. It is clear that the fact that the two values are different is a result of conditioning. Both of these values are conditional probabilities, one conditioned on green color and the other on triangular base. Still, the question is: "which of these two conditional probability values should we accept?" Also, why should we restrict to only these two conditionings? Why not employ others, like 'the alphabet symbol' or the 'number of dots' on the pencils?

Under what criterion can we chose one attribute for conditioning over another attribute, for example, 'color' over 'shape of base'? Shouldn't the concept 'probability' be free of conditioning attributes? Incidentally, what is the problem here, and what constitutes its solution?

Does p have a definite fixed value which we can approximate in some sense?

Suppose the value of n, the total number of objects (pencils) is 120, out of which 50 are thin-tipped. Then, for the population as a whole, the probability is 50/120 or 5/12. Is this a better estimate in some sense?

Consider the situation where the child comes back to the room, and is asked about the pencil. He says he does not recall if it had a thin or thick tip, but it does have the alphabet symbol 'K' on it. Suppose now a person in the room says that he does have information on the alphabet attribute K; he knows that there were only 3 pencils with 'K' written on them, and all of these pencils have a thick tip. Thus, in the light of conditioning on the attribute 'alphabet' symbol, the problem gets resolved in a sense, and we know that the pencil in question has a thick tip, and the value of p (conditioned on this attribute) is 0.

So, we have a multitude of values for p : 2/3, 3/17, 5/10, 50/120, and 0/3. The last one brings into our realm of consciousness the real fact about the pencil, namely, that it is thick-tipped. It thus resolves the problem by *removing one aspect of the question*, in the sense that so far as our interest in the tip of that specific pencil (whether it is thick or thin point) is concerned, we know the fact (namely, that it is thick-tipped). However, there is *another aspect of the question, namely what can we say about p if a pencil is drawn randomly from the pool of pencils on the floor* (assuming that the pencil the child took is placed back on the floor). From the viewpoint of this aspect, all the above values of p (namely, 2/3, 3/17, 5/10, 60/120, 0/3, and other such values obtained by conditioning in various ways) still have interest and validity associated with them. Indeed, they all are conditional probabilities, associated with different types of conditioning. Whether or not we know that the tip of the pencil that the child took is thin or thick, these probabilities will still continue to be relevant, depending on the question asked. For example, if we ask about the probability (say, q) that if two pencils are drawn with replacement (one out of the pencils with triangle base, and the other out of the green pencils) , then at least one of them will be thick-tipped, we shall compute q to be: [1-(3/17)(5/10)], which is a definite number (31/34). Are we out of ambiguity? Note that, here, we are assuming no further knowledge beyond the fact that we know the proportion of thin-tipped pencils among the green color pencils, and also among the pencils with triangle base; however, whatever knowledge we do use here, that knowledge itself constitutes the conditioning.

Consider probability theory in general; if E is an event, let p(E) denote the probability that E occurs. Consider the question Q: "What is p(E)?" Then, Q has two aspects: (i) interest in E itself, and (ii) interest in the process by which E is answered. Part (i) of Q is answered once we know whether E occurred or not. Part (ii) deals with the process by which (or circumstances under which) the question concerning E is being answered. Part (i) deals with bringing a fact (concerning whether E occurred or not) into our realm of consciousness. Part (ii) involves the 'conditioning that we use' (i.e., the 'perspective that we have') to answer the question on E.

Thus, the question Q does not have a fixed answer in an absolute sense. The answer depends on the perspective we use, the conditioning we are under. This is the basic ambiguity in probability theory. But, this ambiguity is no disaster. We simply work under the conditioning we find ourselves in.

However, problem arises when we forget that we are under a conditioning, that whatever we are doing is under some particular perspective, and that the picture we are projecting may change as the perspective changes.

FOUNDATIONS OF PHYSICS AND PROBABILITY THEORY

Readers familiar with the Foundations of Physics will recall that the Schroedinger's Wave Equation involves a so-called 'wave function', that is usually denoted by ψ. The wave function ψ satisfies the said wave equation. We would further recall that Max Born gave a probabilistic interpretation to ψ, which is basic to Quantum Mechanics. It is that the probability that a particle would be found in a region (say, R) equals

$$\psi^* \psi \mathrm{d}x$$

the integral being taken over all x (the space variable) in the region R. (Here, we must recall that ψ is defined over the complex field, and ψ^* denotes the complex conjugate of ψ.)

In the light of the discussion in the preceding section, it should be remembered that the probability that is being talked about must be only under some perspective. As the perspective changes, the probability could change. Each probability determination would be right under its own perspective. But, the point is that there may be other perspectives!

Since there may be other perspectives, Quantum Mechanics is incomplete! That is what Einstein maintained, though this is denied currently by many physicists who have laid the anchor of Physics in the high seas of probability theory.

DEDICATION

This paper is particularly dedicated to the memory of Dr. G.R. Seth, who was Deputy Director of ICAR Statistical Wing where I worked as a 'Statistical Investigator' during January 1958 to August 1959. At that time, I was also enrolled for Ph.D. in Stochastic Processes under Dr. Adhikari in Lucknow University. But, in September 1958, Dr. Seth asked me to solve a problem mathematically in the field of 'Statistical Experimental Design Theory'. I happened to succeed in that. At the urging of Dr. P.V. Sukhatme (who was in Rome those days at FAO), the Director arranged a seminar on March 15, 1959, in which there were two speakers, one of whom was myself. The seminar was also attended by some top Indian statisticians, like Dr. K.R. Nair, who had come to Delhi for a meeting. After the seminar, I was urged by Nair, Seth, and others to work in Design under R.C. Bose who was then at the University of North Carolina, Chapel Hill. That is how I came to USA in 1959.

Dr. Seth continually gave encouragement and advice, which became instrumental in my being able to come abroad for further study.

I would like to take this opportunity to also respectfully thank K.S. Krishnan and T.P. Abraham under whom I worked then. Dr. Daroga Singh and V.N. Amble (the two other Assistant Directors besides Abraham) also helped me immensely. I also got encouragement from B.V. Sukhatme and A.R. Roy.

Although all these men are gone, their nobility lives on in memory.