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Length-weight Relationship and Growth Pattern of *Tor putitora* (Hamilton) under Monoculture and Polyculture Systems: A Case Study

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SUMMARY

The population of the endangered coldwater fish species, Tor putitora has been sharply declined in the recent past and is threatened with multifaceted dangers. In the present investigation, an attempt has been made to develop the length-weight relationship of *Tor putitora* under monoculture and polyculture systems for direct use in fishery assessment and also to describe growth pattern in terms of weight of this fish species in the above two different culture systems. The von-Bertalanffy model was found to be the best suitable model to describe the growth pattern of *Tor putitora*.

Key words: Endangered, Fish stock, Growth pattern, Monoculture, Polyculture.

1. INTRODUCTION

In the recent past, tremendous downfall in the catches of *Tor putitora* (golden mahseer) has been experienced not only in the Himalayan region but also in other parts of India. The reasons behind this unabated declining trend are many, mostly related to anthropogenic activities and the environmental degradation, which in the pace of development has severely damaged the eco-systems holding mahseer and allied fisheries. Besides environmental stresses, indiscriminate killing of adults and juveniles has been recognized as a major factor responsible for declining of mahseer in the Himalayan region. *Tor putitora* is an endangered coldwater fish species that is a popular fish as food and as a source of recreation for anglers. As mahseer stock is threatened with multifaceted dangers, which are partly due to overexploitation and consequently reduced yield from many fish stocks. The size of fish plays an important role in fish stock assessment. In the present scenario, it may be worthy to know the growth pattern of this fish species under monoculture and polyculture systems, so that we could be able to provide proper management of mahseer stocks more precisely. Since, the size of fish is a primary driver for many key processes in fisheries systems and the majority of the data underlying stock assessments are size structured, the fitting of non-linear models will be of immense help to provide formal advice on stock management. Again, length-weight relationship of fish is important in fisheries biology because they allow the estimation of the average weight of the fish of a given length group by establishing a mathematical relationship between the two (Beyer 1987). They are also useful for assessing the relative well being of the fish population (Bolger and Conolly 1989). As length and weight of fish are among the important morphometric characters, they can be used for the purpose of taxonomy and ultimately in fish stock assessment. Once if we establish the mathematical relationship between length and weight of fish, we will be able to see the changes during various developmental events of life such as metamorphosis, the onset of maturity, etc. Thus, the present study aims primarily to establish the length-weight relationship more precisely of *Tor putitora* under monoculture and polyculture systems, which can be very useful in fish

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stock assessment, secondly, to identify most suitable non-linear model for describing the growth pattern of *Tor putitora* over different period of time under monoculture and polyculture systems.

2. MATERIALS AND METHODS

A non-linear statistical model is one in which at least one derivative with respect to at least one parameter being a function of the parameter(s). Details of non-linear models have been given by Ratkowsky (1990). The length-weight relationship was calculated using the formula: $W = aL^b$ (Pauly 1984), where '*W*' is the weight of the fish in gm and '*L*' is the length of the fish measured in mm; '*a*' and '*b*' are parameters, the later being called the rate of allometric. Also, by taking logarithmic transformation on both sides of the above equation, we get the linearize model. $\log W = \log a + b \log L$.

The following non-linear models (Seber and Wild 1989) have been tried to explain the growth pattern in the dataset considered.

(i) Von-Bertalanffy model

$$W_t = W_\infty - \left(W_\infty - K\right)\exp\left(-bt\right) + e$$

(ii) Logistic model

$$W_t = \frac{W_\infty}{\left[1 + b\exp\left(-Kt\right)\right]} + e\ ;\ b = \frac{W_\infty}{W(0)} - 1$$

(iii) Gompertz model

$$W_t = W_\infty \exp\left[-b\exp\left(-Kt\right)\right] + e$$

$$b = \ln\left[W_\infty / W(0)\right]$$

(iv) Richards model

$$W_t = W_\infty\left[1 + b\exp\left(-Kt\right)\right]^{(-1/d)} + e$$

$$b = \left[\mathrm{W}_\infty^d / \mathrm{W}^d(0)\right] - 1$$

where, W_t is the observed fish weight during time *t*; *K*, *b*, W_∞, *d* are the parameters, and *e* is the error term. The parameter *K* is the intrinsic growth rate and the parameter W_∞ represents asymptotic size (in weights) of the fish for each model. Symbol b represents different functions of the initial value *W*(0) and d is the added parameter in Richards model.

Model Fitting

Being a part of the project programme conducted at NRCCWF, Bhimtal under the NATP, Joshi *et al*. (2004) monitored growth performance of Tor putitora in different culture systems. The mahseer stock were rearing at Pantnagar, Uttarakhand and raising under monoculture and polyculture systems with the stocking densities of 4000 and 1600 per hectare respectively in earthen ponds of 0.1 hectare each. During the rearing period of about two years, the average length and weight of mahseer was obtained at different unequal time intervals for monoculture and polyculture systems separately. In monoculture system, the specimens of this fish species ranged 234-431 mm in length and 320-950 gm in weight while in polyculture system, specimens varied between 244-445 mm in length and 325-980 gm in weight. Further, the initial and average weight gain in *Tor putitora* at an unequal interval during the rearing period of about two years viz. 0, 2, 5, 8, 11, 14, 17 and 22 months are respectively 320, 400, 475, 550, 650, 725, 825 and 950 gm in monoculture system whereas 325, 430, 500, 605, 710, 795, 850 and 980 gm in polyculture system. These data were further utilized for the present investigation. For model fitting, age of *Tor putitora* given in months was converted to corresponding approximate age in years. However, ordinary least squares method is used for fitting of linearized model.

There are four main methods available in literature (Seber and Wild 1989) to obtain estimate of the unknown parameters of a non-linear regression model, namely (a) Gauss-Newton Method, (b) Steepest-Descent Method, (c) Levenberg-Marquardt Technique and (d) Do Not Use Derivative (DUD) Method. Levenberg-Marquardt method is the most widely used and reliable procedure for computing non-linear least square estimates and has been applied under the present study.

To examine model performance, a measure of how the predicted and observed variables covary in time is needed. Thus, the coefficient of determination, R^2 is generally used. However, Kvalseth (1985) has emphasized that, although R^2 given by

$$R^2 = 1 - \frac{\sum\left(W_t - \hat{W}_t\right)^2}{\sum\left(W_t - \bar{W}\right)^2}$$

is quite appropriate even for non-linear models, uncritical use of and sole reliance on R^2 statistics may fail to reveal important data characteristics and model inadequacies. Hence, summary statistics like Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Square Error (MSE) are also used.

$$\text{RMSE} = \left[\sum_{t=1}^{n} \left(W_t - \hat{W}_t \right)^2 \Big/ n \right]^{1/2}$$

$$\text{MAE} = \sum_{t=1}^{n} \left| \left(W_t - \hat{W}_t \right) \right| / n$$

$$\text{MSE} = \left[\sum_{t=1}^{n} \left(W_t - \hat{W}_t \right)^2 \Big/ (n - p) \right]$$

where

$\hat{W}_t$ Predicted fish weight of tth observation

$\bar{W}$ Average fish weight

n Number of observations, $t = 1, 2, ..., n$

p Number of parameters involved in the model

The better model will have the least values of these statistics. It is, further, recommended for residual analysis to check the assumptions made for the model to be developed. Thus, independence or the randomness assumption of the residuals needs to be tested before taking any final decision about the adequacy of the model developed. To test the independence assumption of residuals run test procedure is available in the literature (Ratkowsky 1990). However, the normality assumption is not so stringent for selecting non-linear models because their residuals may not follow normal distribution while it must be strictly follow for linear or linearize models.

The Analysis of Covariance (ANCOVA) method was applied to check if the regression lines of length-weight relationship for monoculture and polyculture are parallel using SPSS Syntax commands (Singh and Nayak 2007). The above non-linear models were fitted using the Non-linear Regression option on SPSS 12.0 version. Different sets of initial parameter values were tried to meet the global convergence criterion for best fitting of the non-linear models.

3. RESULTS AND DISCUSSION

The slopes of the two regression lines due to various culture systems are not significant different since (F = 0.506) and (p = 0.491) for the corresponding ANCOVA test as shown in Table 1. Thus, we have to fit a model of combined data for monoculture and polyculture systems regarding length-weight relationship of Tor putitora. The two different forms of allometric model (non-linear and linearized forms) are fitted to the combined data for monoculture and polyculture systems. The estimates of parameter, goodness of fit statistics and residual analysis results of the fitted models is presented in Table 2. In this case, R^2 values are approximately

Table 1. User-specified contrasts test results on comparing length-weight relationship in monoculture and polyculture systems of *Tor putitora* by ANCOVA method

Comparison of culture systems	Monoculture v/s polyculture
Mean Square	1.429×10^{-4}
F-value	0.506
p-value	0.491
Comment	Not significant

Table 2. Summary statistics of the models fitted to length-weight dataset of *Tor putitora*

	Non-linear model $W = aL^b$	Linearize model $\log W = \log a + b \log L$
Parameter Estimates		
a or Log *a*	0.016	–1.680 (0.021)*
b	1.810	1.761
Goodness of Fit Statistics		
R^2	0.994	0.993 (0.993)
RMSE	16.001	1.118×10^{-2} (18.314)
MAE	12.113	1.000×10^{-2} (14.022)
MSE	292.616	1.429×10^{-4} (293.472)
Residual Analysis		
Run Test (|Z|)	1.908	1.109
Shapiro-Wilk Test p-value	0.832	0.123

* The bracketed values are related to conversion of the linearize models to original (non-linear) models by taking antilogarithm.

same in all irrespective of the nature of models. The best-fitted model is decided based on the values of RMSE, MAE and MSE. Further, we have examined whether the assumptions about residuals are satisfied for the models or not. The run test |Z| values to check independence assumption of the residuals are below the critical value (1.96) of normal distribution at 5% level of significance, shows the suitability of the fitted models. Moreover, Shapiro-Wilk test p-values for the residuals clearly indicate that residuals are normally distributed in Table 2. Thus, the independence as well as normality assumption about residuals is satisfied for fitting of the models. On comparing the performance, the non-linear model gives better result to its corresponding linearize model. Hence, we conclude that non-linear model appears to describe more precisely the length-weight relationship of *Tor putitora* than its corresponding linearize model.

The estimates of parameters, R^2, RMSE, MAE, MSE, run test statistic (|Z|) value, Shapiro-Wilk test p-values for the above growth models are again given in Table 3. The same criteria are used to identify the best model, since R^2 values being almost equal in all the models and von-Bertalanffy model performs well for dataset of monoculture system. Further, we have examined whether the assumptions about residuals are satisfied for this model or not. The run test |Z| value to check independence assumption of the residuals is 1.146, which is below the critical value 1.96. Shapiro-Wilk test p-values given in Table 3 also clearly show that the normality assumption is not violated. Hence, von-Bertalanffy model seems to describe more precisely the growth pattern of *Tor putitora* under the monoculture system. The corresponding asymptotic size (in weight) of *Tor putitora* is 8913 gm approximately in the monoculture system. Furthermore, by working out the ratio of the weight during the last interval considered in the dataset to the asymptotic size, it is seen that only 11% of the maximum size is already achieved so far

Table 3. Summary statistics of the models fitted to the growth dataset of *Tor putitora*

	Monoculture System				Polyculture System			
	Von-Bertalanffy	Logistic	Gompertz	Richards	Von-Bertalanffy	Logistic	Gompertz	Richards
Parameter Estimates								
K	327.840	0.096	0.050	0.050	333.938	0.120	0.073	0.073
b	0.003	2.765	1.584	0.001	0.026	2.251	1.316	0.004
W_∞	8912.834	1266.355	1618.974	1618.577	1804.003	1126.581	1269.385	1269.320
d	-	-	-	0.001	-	-	-	0.000
Goodness of Fit Statistics								
R^2- value	0.998	0.998	0.998	0.998	0.997	0.995	0.996	0.996
RMSE	8.010	9.345	8.430	8.431	11.675	14.209	12.520	12.520
MAE	7.851	7.326	6.941	7.326	11.225	10.801	10.463	10.053
MSE	102.663	139.718	113.706	142.148	218.089	323.053	250.780	313.495
Residual Analysis								
Run test |Z| value	1.146	1.146	1.146	1.146	0.382	0.382	0.382	0.382
Shapiro Wilk Test p-value	0.720	0.913	0.738	0.738	0.531	0.637	0.550	0.531
Asymptotic size (in weight) achieved at present	11%	75%	58%	59%	54%	87%	77%	77%

and hence, there is immense scope for further increase in weight of this fish species under the monoculture system. Similarly, von-Bertalanffy model is again found appropriate for the dataset under polyculture system. The required assumptions about residuals of this model are also satisfied. Further, it is observed that 54% of the maximum size has already been achieved and scope for further increase in weight of this fish species is more limited under polyculture system as compare to monoculture system. Also, von-Bertalanffy model fitted to the data of monoculture system shows better fit than the polyculture system.

REFERENCES

Beyer, J.E. (1987). On length-weight relationship. *Fishbyte*, **5,** 11-13.

Bolger, T. and Conolly, P.L. (1989). The selection of suitable indices for measurement and analysis of fish conditions. *J. Fish. Biol.*, **34(2)**, 171-182.

Joshi, C.B., Sunder, S., Murgnandam, M., Sharma, A.P., Chahan, R.S., Das, M., Dhanze, J.R., Dhanze, R., Balkhi, M.H. and Samoon, M.S. (2004). Aquaculture management in coldwaters: Evaluation of mahseer fishery potentials and its farming feasibility for conservation in Himalayan region. Unpublished NATP Project Report, NRC on Coldwater Fisheries, ICAR, Bhimtal, Uttarakhand.

Kvalseth, T.O. (1985). Cautionary note about R^2. *Amer. Statist*., **39(4)**, 279-285.

Pauly, D. (1984). Fish population dynamics in tropical waters: A manual for use with programmable calculators. *ICLARM Stud. Rev*., **8**, 325.

Ratkowsky, D.A. (1990). *Handbook of Non-linear Regression Models*. Marcel Dekker, New York.

Seber, G.A.F. and Wild, C.J. (1989). *Non-linear Regression.* John Wiley and Sons, New York.

Singh, N. Okendro and Nayak, A.K. (2007). Analysis of covariance (ANCOVA) on evaluation of sex and seasonal differences in length-weight relationship of a fish species. *The SPSS Analyst*, October-December Issue, 1-12.