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Estimation of Parameters of Morgenstern Type Bivariate Logistic Distribution by Ranked Set Sampling

Manoj Chacko and P. Yageen Thomas
University of Kerala, Trivandrum 695 581
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SUMMARY

Ranked set sampling is applicable whenever ranking of a set of sampling units can be done easily by a judgement method or based on the measurement of an auxiliary variable on the units selected. In this work, we derive different estimators of the parameters associated with the distribution of the study variate *Y*, based on ranked set sample obtained by using an auxiliary variable *X* correlated with *Y* for ranking the sample units, when (*X*, *Y*) follows a Morgenstern type bivariate logistic distribution. The theory developed in this paper is illustarted using a real data. Efficiency comparison among these estimators are also made.

Key words: Ranked set sample, Morgenstern type bivariate logistic distribution, Best linear unbiased estimator, Concomitants of order statistics.

1. INTRODUCTION

The concept of ranked set sampling was first introduced by McIntyre (1952) as a process of improving the precision of the sample mean as an estimator of the population mean. This is applicable whenever ranking of a set of sampling units can be done easily by a judgement method see, Chen *et al.* (2004). Ranking by judgement method is not recommendable if the judgement method is not mathematically much relevant with the problem of study. In certain situations, one may prefer exact measurement of some easily measurable variable associated with the study variable rather than making ranking by a crude judgement method. Suppose the variable of interest say *Y*, is difficult or much expensive to measure, but an auxiliary variable *X* correlated with *Y* is readily measurable and can be ordered exactly. In biological studies, such as in the root zone analysis of bamboo plants (*Bambusa arundinacea*), the shoot height of the plant is a correlated character with root weight. Clearly shoot height can be measured very easily whereas root weight measurement requires uprooting of the sampled plants. Hence, in such situations, we can choose the most desired plants with respect to their shoot length value and on which we measure the root weight for further analysis as presented in any ranked set sampling (RSS). Thus, as an alternative to McIntyre (1952) method of ranked set sampling, Stokes (1977) used an auxiliary variable for ranking of the sampling units, which is as follows: Choose n independent bivariate samples, each of size n. In the first sample, the *Y* variate associated with smallest ordered *X* is measured, in the second sample, the *Y* variate associated with the second smallest, *X* is measured. This process is continued until the*Y* variate associated with the largest is measured.

Stokes (1977) suggested the ranked set sample mean as an estimator for the mean of the study variate *Y*, when an auxiliary variable *X* is used for ranking the sample units, under the assumption that (*X*, *Y*) follows a bivariate normal distribution. Barnett and Moore (1977) improved it by deriving the Best Linear Unbiased Estimator (BLUE) of the mean of the study variate *Y*, based on ranked set sample obtained on the study variate *Y*. Chacko and Thomas (2007) obtained the BLUE of the parameter involved in the study variate *Y*, under the assumption that (*X*, *Y*) follows bivariate Pareto distribution. Unbalanced RSS arising from Morgentern type bivariate exponential distribution have been considered by Chacko and Thomas (2008).

Chacko and Thomas (2006) used the concomitants of record values arising from a Morgenstern type bivariate logistic distribution to estimate some of its parameters. Sampling to get a given number of record values will require several selection (uncertain number) of units and moreover the obtained concomitants of record values are correlated, which makes one to determine the variance and covariance of concomitants of record values to use them for inference problem. However, in case of Stokes method of ranked set sampling, the number of units to be selected is definite and there exists no correlation between one observation to another as they are drawn from independent samples so that handling the observations in a ranked set sample for inference problem will be very easy.

In this work, we consider the case when (X, Y) follows Morgenstern type bivariate logistic distribution (MTBLD) with cumulative distribution function (cdf) defined by (Kotz *et al*. 2000):

$$F_{X,Y}(x,y) = \left(1+\exp\left\{-\frac{x-\theta_1}{\sigma_1}\right\}\right)^{-1}\left(1+\exp\left\{-\frac{y-\theta_2}{\sigma_2}\right\}\right)^{-1} \left(1+\alpha\left[\frac{\exp\left\{-\frac{x-\theta_1}{\sigma_1}\right\}}{1+\exp\left\{-\frac{x-\theta_1}{\sigma_1}\right\}}\right]\left[\frac{\exp\left\{-\frac{y-\theta_2}{\sigma_2}\right\}}{1+\exp\left\{-\frac{y-\theta_2}{\sigma_2}\right\}}\right]\right) \quad (1.1)$$

In Section 2, we derive unbiased estimators of the parameters θ_2 and σ_2 involved in MTBLD defined by (1.1) based on a ranked set sample. In Section 3, we derive the BLUE of θ_2 and σ_2 involved in MTBLD based on the ranked set sample and have made an efficiency comparison with corresponding unbiased estimators developed in Section 2. In Section 4, we illustrate the methods developed in Section 2 and Section 3 using real data.

2. UNBIASED ESTIMATORS OF θ_2 AND σ_2

Let (X, Y) be a bivariate random variable which follows a MTBLD with cdf defined by (1.1). Suppose n sampling units each of size n are taken. Let $X_{(r)r}$ be the rth order statistic of the auxiliary variate X in the rth sample and let $Y_{[r]r}$ be the measurement made on the variate associated with $X_{(r)r}$, $r = 1, 2, ..., n$. By using the approach of Scaria and Nair (1999) for obtaining means and variances of concomitants of order statistics arising from Morgenstern family of distributions, we get the mean and variance of $Y_{[r]r}$ for $1 \le r \le n$ as

$$E[Y_{[r]r}] = \theta_2 - \alpha\left(\frac{n-2r+1}{n+1}\right)\sigma_2 \quad (2.1)$$

$$Var[Y_{[r]r}] = \left(\frac{\pi^2}{3} - \alpha^2\left(\frac{n-2r+1}{n+1}\right)^2\right)\sigma_2^2 \quad (2.2)$$

Since $Y_{[r]r}$ and $Y_{[s]s}$ for $r \neq s$ are drawn from two independent samples, we have

$$Cov[Y_{[r]r}, Y_{[s]s}] = 0, r \neq s \quad (2.3)$$

If we write

$$\xi_r = -\alpha(n-2r+1)/(n+1) \quad (2.4)$$

and
$$\delta_r = \frac{\pi^2}{3} - \alpha^2\left(\frac{n-2r+1}{n+1}\right)^2 \quad (2.5)$$

then from (2.1) and (2.2), we can write

$$E[Y_{[r]r}] = \theta_2 + \xi_r\sigma_2 \quad (2.6)$$

$$Var[Y_{[r]r}] = \delta_r\sigma_2^2 \quad (2.7)$$

In the following theorem, we propose estimators θ_2^* and σ_2^* of θ_2 and σ_2 involved in (1.1) and prove that they are unbiased estimators for θ_2 and σ_2.

Theorem 2.1. Let $Y_{[r]r}$, $r = 1, 2, ..., n$ be the ranked set sample observations on a study variate Y obtained out of ranking made on an auxiliary variate Y, when (X, Y) follows MTBLD as defined in (1.1). Then the ranked set sample mean given by

$$\theta_2^* = \frac{1}{n}\sum_{r=1}^{n} Y_{[r]r} \quad (2.8)$$

is an unbiased estimator of θ_2 and

$$\sigma_2^* = \frac{1}{\sum_{r=1}^{[n/2]}\xi_r}\sum_{r=1}^{[n/2]} T_r \quad (2.9)$$

is an unbiased estimator of σ_2, where $T_r = (Y_{[r]r} - Y_{[n-r+1]\overline{n-r+1}})/2$ and [.] is the usual greatest integer function. The variances of the above estimators are given by

$$\mathrm{Var}[\theta_2^*] = \frac{\sigma_2{}^2}{n}\left[\frac{\pi^2}{3} - \frac{\alpha^2}{n}\sum_{r=1}^{n}\left(\frac{n-2r+1}{n+1}\right)^2\right] \quad (2.10)$$

and

$$\mathrm{Var}[\sigma_2^*] = \frac{\sigma_2^2}{2(\sum_{r=1}^{[n/2]}\xi_r)^2}\sum_{r=1}^{[n/2]}\left[\frac{\pi^2}{3} - \alpha^2\left(\frac{n-2r+1}{n+1}\right)^2\right] \quad (2.11)$$

Proof.

$$\mathrm{E}[\theta_2^*] = \frac{1}{n}\sum_{r=1}^{n}\mathrm{E}[Y_{[r]r}] = \frac{1}{n}\sum_{r=1}^{n}\left[\theta_2 - \alpha\frac{n-2r+1}{n+1}\sigma_2\right] \quad (2.12)$$

Note that

$$\sum_{r=1}^{n}(n-2r+1) = 0 \quad (2.13)$$

Applying (2.13) in (2.12), we get

$$\mathrm{E}[\theta_2^*] = \theta_2$$

Thus θ_2^* is an unbiased estimator of θ_2. The variance of θ_2^* is given by

$$\mathrm{Var}[\theta_2^*] = \frac{1}{n^2}\sum_{r=1}^{n}\mathrm{Var}(Y_{[r]r})$$

Thus using (2.2) in the above sum, we get

$$\mathrm{Var}[\theta_2^*] = \frac{\sigma_2^2}{n}\left[\frac{\pi^2}{3} - \frac{\alpha^2}{n}\sum_{r=1}^{n}\left(\frac{n-2r+1}{n+1}\right)^2\right]$$

From (2.9) we have

$$\mathrm{E}[\sigma_2^*] = \frac{1}{\sum_{r=1}^{[n/2]}\xi_r}\sum_{r=1}^{[n/2]}\mathrm{E}[T_r]$$

$$= \frac{1}{2\sum_{r=1}^{[n/2]}\xi_r}\sum_{r=1}^{[n/2]}\mathrm{E}[Y_{[r]r} - Y_{[n-r+1]\overline{n-r+1}}]$$

On using (2.1) in the above equation and simplifying, we get

$$\mathrm{E}[\sigma_2^*] = \sigma_2$$

The variance of σ_2^* is given by

$$\mathrm{Var}[\sigma_2^*] = \frac{1}{(\sum_{r=1}^{[n/2]}\xi_r)^2}\sum_{r=1}^{[n/2]}Var[\mathrm{T}_r]$$

$$= \frac{1}{4(\sum_{r=1}^{[n/2]}\xi_r)^2}\sum_{r=1}^{[n/2]}Var[\mathrm{Y}_{[r]r}] + Var[\mathrm{Y}_{[n-r+1]\overline{n-r+1}}]$$

On using (2.2) in the above equation and simplifying, we get

$$\mathrm{Var}[\sigma_2^*] = \frac{\sigma_2^2}{2(\sum_{r=1}^{[n/2]}\xi_r)^2}\sum_{r=1}^{[n/2]}\left[\frac{\pi^2}{3} - \alpha^2\left(\frac{n-2r+1}{n+1}\right)^2\right]$$

Thus the theorem is proved.

We compare the variance of θ_2^* with the Cramer Rao Lower Bound $\pi^2\sigma_2^2/(3n)$ of any unbiased estimator of θ_2 involved in the marginal distribution of *Y* in (1.1). Clearly the ratio of $\pi^2\sigma_2^2/(3n)$ with variance of θ_2^* denoted by $e_1(\theta_2^*)$ is given by

$$e_1(\theta_2^*) = \frac{\frac{\pi^2}{3}}{\left[\frac{\pi^2}{3} - \frac{\alpha^2}{n}\sum_{r=1}^{n}\left(\frac{n-2r+1}{n+1}\right)^2\right]} \quad (2.14)$$

Since

$$\left[\frac{\pi^2}{3}-\frac{\alpha^2}{n}\sum_{r=1}^{n}\left(\frac{n-2r+1}{n+1}\right)^2\right] = n\sigma_2^2\text{Var}(\theta_2^*)\geq 0$$

We have $\frac{\alpha^2}{n}\sum_{r=1}^{n}\left(\frac{n-2r+1}{n+1}\right)^2 \leq \frac{\pi^2}{3}$ and hence $e_1(\theta_2^*)\geq 1$. Thus we conclude that there is some gain in efficiency of the estimator θ_2^* due to ranked set sampling. It is to be noted that $\text{Var}(\theta_2^*)$ is a decreasing function of α^2 and hence its variance is least when $\alpha = \pm 1$. Thus the gain in efficiency of the estimator θ_2^* is increasing as $|\alpha|$ is increasing.

Again on simplifying (2.14), we get

$$e_1(\theta_2^*) = \frac{\frac{\pi^2}{3}}{\frac{\pi^2}{3}-\alpha^2[\frac{2}{3}(\frac{2+1/n}{1+1/n})-1]}$$

Clearly

$$\lim_{n\to\infty} e_1(\theta_2^*) = \lim_{n\to\infty}\frac{\pi^2/3}{\pi^2/3-\alpha^2\left[\frac{2}{3}(\frac{2+1/n}{1+1/n})-1\right]}$$

$$= \frac{\pi^2/3}{\pi^2/3-\alpha^2/4}$$

Clearly the maximum asymptotic value for $e_1(\theta_2^*)$ is attained when $|\alpha|=1$ and in this case $e_1(\theta_2^*)$ tends to $4\pi^2/(4\pi^2-3)$.

3. BEST LINEAR UNBIASED ESTIMATORS OF θ_2 AND σ_2

In this section we provide better estimators θ_2^* than and σ_2^* of θ_2 and σ_2 respectively by deriving the BLUE $\hat{\theta}_2$ and $\hat{\sigma}_2$ of θ_2 and σ_2 respectively provided the parameter α is known. Suppose n sampling units each of size n are taken from the population with cdf defined by (1.1). Let $Y_{[n]} = (Y_{[1]1}, Y_{[2]2}, \ldots, Y_{[n]n})'$. Then from (2.6), (2.7) and (2.3), the mean vector and the dispersion matrix of $Y_{[n]}$ are given by

$$\text{E}[Y_{[n]}] = \theta_2\mathbf{1}+\sigma_2\xi \tag{3.1}$$

$$\text{D}[Y_{[n]}] = \sigma_2^2\mathbf{G} \tag{3.2}$$

where $\xi = (\xi_1, \xi_2, \ldots, \xi_n)'$ and $\mathbf{G} = \text{diag}(\delta_1, \delta_2, \ldots, \delta_n)$ in which ξ_r and δ_r are as defined in (2.4) and (2.5) respectively and 1 is a column vector of ones. If the parameter involved in ξ and $\mathbf{G}$ is known, then (3.1) and (3.2) together defines a generalized Gauss-Markov set up and hence the BLUEs $\hat{\theta}_2$ and $\hat{\sigma}_2$ of θ_2 and σ_2 are obtained as

$$\hat{\theta}_2 = \Delta^{-1}\left[\xi'\mathbf{G}^{-1}(\xi\mathbf{1}'-\mathbf{1}\xi)\mathbf{G}^{-1}\right]Y_{[n]} \tag{3.3}$$

and

$$\hat{\sigma}_2 = \Delta^{-1}\left[1'\mathbf{G}^{-1}(\mathbf{1}\xi'-\xi\mathbf{1}')\mathbf{G}^{-1}\right]Y_{[n]} \tag{3.4}$$

with variances given by

$$\text{Var}(\hat{\theta}_2) = \sigma_2^2(\xi'\mathbf{G}^{-1}\xi)/\Delta \tag{3.5}$$

and

$$\text{Var}(\hat{\sigma}_2) = \sigma_2^2(\mathbf{1}'\mathbf{G}^{-1}\mathbf{1})/\Delta \tag{3.6}$$

where $\Delta = (\xi'\mathbf{G}^{-1}\xi)(1'\mathbf{G}^{-1}1)-(\xi'\mathbf{G}^{-1}\mathbf{1})^2$. On substituting the values of ξ and $\mathbf{G}$ in (3.3) and (3.4) and simplifying, we get

$$\hat{\theta}_2 = \sum_{r=1}^{n}\left\{\frac{\delta_r^{-1}(\sum_{i=1}^{n}\xi_i^2\delta_i^{-1})-\xi_r\delta_r^{-1}(\sum_{i=1}^{n}\delta_i^{-1})}{(\sum_{i=1}^{n}\delta_i^{-1})(\sum_{i=1}^{n}\xi_i^2\delta_i^{-1})-(\sum_{i=1}^{n}\xi_i\delta_i^{-1})^2}\right\}Y_{[r]r} \tag{3.7}$$

and

$$\hat{\sigma}_2 = \sum_{r=1}^{n}\left\{\frac{\xi_r\delta_r^{-1}(\sum_{i=1}^{n}\delta_i^{-1})-\delta_r^{-1}(\sum_{i=1}^{n}\xi_i\delta_i^{-1})}{(\sum_{i=1}^{n}\delta_i^{-1})(\sum_{i=1}^{n}\xi_i^2\delta_i^{-1})-(\sum_{i=1}^{n}\xi_i\delta_i^{-1})^2}\right\}Y_{[r]r} \tag{3.8}$$

Table 3.1. Variances and efficiences of the estimators

n	α	Var(θ_2^*)	Var($\hat{\theta}_2$)	Var(σ_2^*)	Var($\hat{\sigma}_2$)	e_1	e_2
2	0.25	1.641	1.641	236.370	236.371	1.000	1.000
	0.50	1.631	1.631	58.718	58.718	1.000	1.000
	0.75	1.614	1.614	25.819	25.819	1.000	1.000
	1.00	1.589	1.589	14.304	14.304	1.000	1.000
4	0.25	0.819	0.819	81.934	65.387	1.00001	1.253
	0.50	0.810	0.810	20.249	16.038	1.0002	1.263
	0.75	0.794	0.794	8.826	6.899	1.001	1.280
	1.00	0.772	0.770	4.828	3.699	1.003	1.305
6	0.25	0.546	0.546	47.548	36.558	1.00002	1.301
	0.50	0.538	0.538	11.725	8.922	1.0002	1.314
	0.75	0.526	0.525	5.091	3.802	1.001	1.339
	1.00	0.509	0.506	2.769	2.009	1.005	1.379
8	0.25	0.409	0.409	33.146	25.158	1.00002	1.317
	0.50	0.403	0.403	8.163	6.123	1.0003	1.333
	0.75	0.393	0.392	3.537	2.597	1.002	1.362
	1.00	0.379	0.377	1.918	1.360	1.006	1.410
10	0.25	0.327	0.327	25.345	19.123	1.00002	1.325
	0.50	0.322	0.322	6.237	4.646	1.0004	1.342
	0.75	0.314	0.313	2.699	1.964	1.002	1.374
	1.00	0.302	0.300	1.460	1.024	1.007	1.427
12	0.25	0.273	0.273	20.482	15.403	1.00002	1.330
	0.50	0.268	0.268	5.038	3.738	1.0004	1.348
	0.75	0.261	0.260	2.178	1.577	1.002	1.381
	1.00	0.251	0.249	1.177	0.819	1.007	1.437
14	0.25	0.234	0.234	17.170	12.887	1.00002	1.332
	0.50	0.230	0.230	4.221	3.125	1.0004	1.351
	0.75	0.223	0.223	1.824	1.316	1.002	1.385
	1.00	0.214	0.213	0.984	0.682	1.008	1.444
16	0.25	0.204	0.204	14.773	11.073	1.00003	1.334
	0.50	0.201	0.201	3.631	2.684	1.0004	1.353
	0.75	0.195	0.195	1.568	1.129	1.002	1.389
	1.00	0.187	0.186	0.845	0.583	1.008	1.449
18	0.25	0.182	0.182	12.959	9.705	1.00003	1.335
	0.50	0.179	0.179	3.184	2.351	1.0004	1.355
	0.75	0.173	0.173	1.374	0.988	1.002	1.391
	1.00	0.166	0.165	0.741	0.510	1.008	1.453
20	0.25	0.164	0.164	11.540	8.637	1.00003	1.336
	0.50	0.161	0.161	2.835	2.091	1.0004	1.356
	0.75	0.156	0.156	1.223	0.878	1.002	1.393
	1.00	0.149	0.148	0.659	0.453	1.009	1.456

The variances given by (3.5) and (3.6) can also be simplified as

$$\mathrm{Var}(\hat{\theta}_2) = \frac{\sum_{i=1}^{n} \xi_i^2 \delta_i^{-1}}{(\sum_{i=1}^{n} \delta_i^{-1})(\sum_{i=1}^{n} \xi_i^2 \delta_i^{-1}) - (\sum_{i=1}^{n} \xi_i \delta_i^{-1})^2} \sigma_2^2$$

and
$$\mathrm{Var}(\hat{\sigma}_2) = \frac{\sum_{i=1}^{n} \delta_i^{-1}}{(\sum_{i=1}^{n} \delta_i^{-1})(\sum_{i=1}^{n} \xi_i^2 \delta_i^{-1}) - (\sum_{i=1}^{n} \xi_i \delta_i^{-1})^2} \sigma_2^2$$

We have computed $\mathrm{Var}(\theta_2^*)$, $\mathrm{Var}(\hat{\theta}_2)$, efficiency $e(\hat{\theta}_2 \mid \theta_2^*) = \mathrm{Var}(\theta_2^*) / \mathrm{Var}(\hat{\theta}_2)$ of $\hat{\theta}_2$ relative to θ_2^*, $Var(\sigma_2^*)$, $Var(\hat{\sigma}_2)$, efficiency $e(\hat{\sigma}_2 \mid \sigma_2^*) = \mathrm{Var}(\sigma_2^*) / \mathrm{Var}(\hat{\sigma}_2)$ of $\hat{\sigma}_2$ relative to σ_2^* for $\alpha = 0.25(0.25)1$ and $n = 2(2)20$ and the same are given in Table 3.1. From this table one can easily see that $\hat{\theta}_2$ is relatively more efficient than θ_2^*. Further, we observe from the table that for fixed α, both $e(\hat{\theta}_2 \mid \theta_2^*)$ and $e(\hat{\sigma}_2 \mid \sigma_2^*)$ increasing with *n*. Also from the table we notice that the precision obtained for the BLUE $\hat{\sigma}_2$ is more than that obtained for $\hat{\theta}_2$.

Remark 3.1. If we have a situation with α unknown, we introduce an estimator (moment type) for α as follows. For MTBLD the correlation coefficient between the two variables is given by $\rho = 3\alpha/\pi^2$. If *r* is the sample correlation coefficient between $X_{(i)i}$ and $Y_{[i]i}$, $i = 1, 2, ..., n$ then the moment type estimator for α is obtained by equating with the population correlation coefficient ρ and is obtained as

$$\hat{\alpha} = \begin{cases} -1 \text{ if} & r \le -3/\pi^2 \\ 1 \text{ if} & r \ge 3/\pi^2 \\ r\pi^2/3 & \text{otherwise} \end{cases} \tag{3.9}$$

4. AN ILLUSTRATION

In this section, as an application of theory developed on RSS in the previous sections, we consider a bivariate data set from Platt *et al.* (1969) relating to 396 Confir (*Pinus Palustrine*) trees. In Chen *et al.* (2004) also, the above bivariate data set is reproduced in which the first component *X* of a bivariate observation represents the diameter in centimetres of the Confir tree at breast height and the second component *Y* represents height in feet of the tree. Clearly *X* can be measured easily but it is somewhat difficult to measure *Y*. Also observations, such ar girth (function of diameter) or height follows normal distribution. It is well known that logistic distribution is having more or less similar properties of a normal distribution (Malik 1985, p. 123) and hence it is known as an alternative model to normal distribution. Assume that (*X*, *Y*) follows Morgenstern type bivariate logistic distribution. We select 10 random samples each of size 10 from the 396 tree data and rank the sampling units of each sample according to the *X* variate values (diameter of the tree). From the i^{th} sample, we measure the *Y* variate (height of the tree) corresponding to the ith order statistic of the *X* variate. The obtained RSS observations are reported in Table 4.1.

Table 4.1. RSS observations

i	$X_{(i)i}$	$Y_{[i]i}$
1	6.3	11
2	10.1	28
3	3.8	6
4	4.5	10
5	6.0	16
6	15.9	28
7	38.6	42
8	17.8	38
9	41.4	177
10	51.7	219

The sample correlation between *X* and *Y* is 0.883. Thus, from (3.9), an estimate of α is taken as 1. We have obtained the RSS estimators θ_2^* and $\hat{\sigma}_2$ derived in Section 2, the BLUEs $\hat{\theta}_2$ and $\hat{\sigma}_2$ based on RSS;

$\sigma_2^{-2}Var(\theta_2^*)$, $\sigma_2^{-2}Var(\hat{\theta}_2)$, $\sigma_2^{-2}Var(\sigma_2^*)$ and $\sigma_2^{-2}Var(\hat{\sigma}_2)$ and are given in the Table 4.2 .

Table 4.2. Estimators and their variances

Estimator	Estimate	variance / σ_2^2
θ_2^*	57.500	0.3017
$\hat{\theta}_2$	60.745	0.2997
σ_2^*	95.062	1.9443
$\hat{\sigma}_2$	108.062	1.0236

The usual traditional estimators of involved in MTBLD is the sample mean $\overline{Y}$ and its variance is $\pi^2\sigma_2^2/(3n)$. For n = 10, this variance is $0.3287\sigma_2^2$, which is clearly larger than $\mathrm{Var}(\theta_2^*)$ and $\mathrm{Var}(\hat{\theta}_2)$. This establishes the advantage of estimating the mean height of trees more closely to the true value of the mean using RSS.

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