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On Estimating the Ratio of Proportions of two Categories of a Population using Auxiliary Information

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SUMMARY

Wynn (1976) introduced a difference type estimator for estimating the population proportion using simple random sampling. Later Singh *et al*. (1986) modified the Wynn (1976) estimator and also suggested another difference type estimator for the ratio of two population proportions using auxiliary information. In this paper we propose a linear weighted estimator for the ratio of two population proportions. The proposed estimator is more efficient than usual ratio estimator, Wynn-type (1976) estimator and Singh *et al.* (1986) estimator. A numerical study is also conducted to evaluate the performance of different estimators.

Key words : Ratio of proportions, Bias, Mean square error, Efficiency.

1. INTRODUCTION

Consider a population subdivided into various categories with respect to two variables y and x. The objective is to estimate the ratio of the proportions of population units falling in two specific categories relative to the variables y and x. We assume that an auxiliary variable z which is strongly associated with both y and x is also available. For example in epidemiology research one may be interested in the prevalence of disease A relative to disease B using information provided by some auxiliary characteristic (such as extent of smoking) that is strongly associated with both A and B.

Let $\Omega = (\Omega_1, \Omega_2, ..., \Omega_N)$ be a finite population of size N. Let $A = \{A_1, A_2, ..., A_a\}$ be a partition of Ω according to characteristic y and N_{i00} be the number of population units in the i^{th} subclass $A_i\,(i = 1, 2, ...a)$ of Ω such that $\sum_{i=1}^{a} N_{i00} = N$. Let $B = \{B_1, B_2 ....., B_b\}$ be another partition of Ω according to another characteristic x and N_{0j0} be the number of population units in the j^{th} subclass $B_j\,(j = 1, 2...,b)$ of Ω such that $\sum_{j=1}^{b} N_{0j0} = N$. Let $C = \{C_1, C_2, ..., C_c\}$ be another partition of Ω according to an auxiliary variate z, and N_{00k} be the number of population units in the k^{th} subclass C_k (k = 1, 2, ..., c) of Ω such that $\sum_{k=1}^{c} N_{00k} = N$. We draw a simple random sample of size n without replacement from Ω. Let, n_{i00}, n_{0j0} and n_{00k} be sample quantities analogous to N_{i00}, N_{0j0} and N_{00k} for y, x and z respectively. Let us also define N_{ijo} to be the number of population units that belong to $A_i \cap B_j$. We can similarly define N_{i0k}, N_{0jk} and corresponding sample quantities. Let $P_{i00} = \frac{N_{i00}}{N}$, $P_{0j0} = \frac{N_{0j0}}{N}$, $P_{00k} = \frac{N_{00k}}{N}$, $P_{ij0} = \frac{N_{ij0}}{N}$, $P_{0jk} = \frac{N_{0jk}}{N}$, $P_{i0k} = \frac{N_{i0k}}{N}$ and $p_{i00} = \frac{n_{i00}}{n}$, $p_{0j0} = \frac{n_{0j0}}{n}$, $p_{00k} = \frac{n_{00k}}{n}$,

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$p_{ij0} = \frac{n_{ij0}}{n}$, $p_{0jk} = \frac{n_{0jk}}{n}$, $p_{i0k} = \frac{n_{i0k}}{n}$ be the population and sample proportions respectively for $i = 1, 2, ..., a;\ j = 1, 2, ..., b;\ k = 1, 2, ..., c.$

We are interested in estimating $R = P_{i00} / P_{0j0}$ for i = 1, 2, ..., a; j = 1, 2, ..., b by using the known value of P_{00k}. We define the following terms

$$\xi_0 = \frac{p_{i00} - P_{i00}}{P_{i00}},\ \xi_1 = \frac{p_{0j0} - P_{0j0}}{P_{0j0}},\ \xi_2 = \frac{p_{00k} - P_{00k}}{P_{00k}},$$

therefore $E(\xi_v) = 0\ (v = 0, 1, 2)$ and

$$E(\xi_0^2) = \frac{Var(p_{i00})}{P_{i00}^2} = \theta \frac{P_{i00}(1 - P_{i00})}{P_{i00}^2}$$

$$E(\xi_1^2) = \frac{Var(p_{0j0})}{P_{0j0}^2} = \theta \frac{P_{0j0}(1 - P_{0j0})}{P_{0j0}^2}$$

$$E(\xi_2^2) = \frac{Var(p_{00k})}{P_{00k}^2} = \theta \frac{P_{00k}(1 - P_{00k})}{P_{00k}^2}$$

$$E(\xi_0 \xi_1) = \frac{Cov(p_{i00}, p_{0j0})}{P_{i00} P_{0j0}} = \theta \frac{P_{ij0}(-P_{i00} P_{0j0})}{P_{i0k} P_{0j0}}$$

$$E(\xi_0 \xi_2) = \frac{Cov(p_{i00}, p_{00k})}{P_{i00} P_{00k}} = \theta \frac{P_{i0k} - P_{i00} P_{00k}}{P_{i00} P_{00k}}$$

$$E(\xi_1 \xi_2) = \frac{Cov(p_{0j0}, p_{00k})}{P_{0j0} P_{00k}} = \theta \frac{P_{0jk} - P_{0j0} P_{00k}}{P_{0j0} P_{00k}}$$

where $\theta = \frac{N - n}{n(N - 1)}$

Now we discuss a few estimators for proportion and ratio of proportions suggested by various authors.

(i) Usual Ratio Estimator

The usual ratio estimator $\hat{R}_0 = \frac{p_{i00}}{p_{0j0}}$ is used to estimate the ratio of population proportions $R = \frac{P_{i00}}{P_{0j0}}$. The bias and mean square error (MSE) of $\hat{R}_0$, to first order of approximation, are given by

$$B(\hat{R}_0) \approx \theta \frac{1}{P_{0j0}^2} \left[P_{i00} - P_{ij0} \right] \tag{1.1}$$

and $$MSE(\hat{R}_0) \approx \theta \frac{R}{P_{0j0}^2} \left[P_{i00} + P_{0j0} - 2P_{ij0} \right] \tag{1.2}$$

(ii) Wynn-type Estimator

Wynn (1976) considered the following difference-type estimator for the population proportion P_{100} as

$$\hat{P}_{i00} = p_{i00} + (P_{00k} - p_{00k}) \tag{1.3}$$

Singh *et al.* (1986) modified the estimator $\hat{P}_{i00}$ and called it Wynn-type estimator to estimate the ratio of two proportions. Their estimator is given by

$$\hat{R}_W = \frac{p_{i00}}{p_{0j0}} + (P_{00k} - p_{00k})$$

for $(i = 1, 2, ..., a;\ j = 1, 2, ..., b;\ k = 1, 2, ..., c)$ (1.4)

The bias and MSE of $\hat{R}_w$, to first order of approximation, are given by

$$B(\hat{R}_W) \approx B(\hat{R}_0) \tag{1.5}$$

and

$$MSE(\hat{R}_W) \approx MSE(\hat{R}_0) + \theta \left[P_{00k}(1 - P_{00k}) \right] - 2R\theta \left[\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}} \right] \tag{1.6}$$

(iii) Singh *et al.* Estimator

Singh *et al.* (1986) also introduced the following difference-type estimator for R as

$$\hat{R}_S = \frac{p_{i00}}{p_{0j0}} + d(P_{00k} - p_{00k}) \tag{1.7}$$

where d is the constant.

The estimator $\hat{R}_S$ has the flexibility of using any value of k between 1 and c.

The bias and MSE of $\hat{R}_S$, to first order of approximation, are given by

$$B(\hat{R}_S) \approx B(\hat{R}_0) \quad (1.8)$$

and

$$MSE(\hat{R}_S) \approx MSE(\hat{R}_0) + d^2\theta\left[P_{00k}(1-P_{00k})\right] - 2dR\theta\left[\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right] \quad (1.9)$$

Using (1.9), the MSE of $\hat{R}_S$ is minimum for $d = R\dfrac{\left(\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right)}{P_{00k}(1-P_{00k})}$, and this minimum is given by

$$MSE(\hat{R}_S)_{min} \approx MSE(\hat{R}_0) - R^2\theta\frac{\left(\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right)^2}{P_{00k}(1-P_{00k})} \quad (1.10)$$

2. PROPOSED ESTIMATOR

We propose the following linear weighted estimator for $R = \dfrac{P_{i00}}{P_{0j0}}$ as

$$\hat{R}_P = \alpha\frac{p_{i00}}{p_{0j0}} + \beta_k(P_{00k} - p_{00k})\frac{P_{00k}}{p_{00k}} \quad (2.1)$$

where $(i = 1, 2, ..., a;\ j = 1, 2, ..., b;\ k = 1, 2, ..., c)$ and α, β_k are suitably chosen constants whose values are to be determined later. This estimator generalizes the Singh *et al.* (1986) estimator by exploiting the features of a ratio estimator by using the term $\dfrac{P_{00k}}{p_{00k}}$.

From (2.1), we have

$$\hat{R}_P = \alpha\frac{P_{i00}(1+\xi_0)}{P_{0j0}(1+\xi_1)} + \beta_k\left\{P_{00k} - P_{00k}(1+\xi_2)\right\}(1+\xi_2)^{-1}$$

Retaining up to second order terms in, ξ's we have

$$(\hat{R}_P - R) \approx (\alpha-1)R + \alpha R\left(\xi_0 - \xi_1 - \xi_0\xi_1 + \xi_1^2\right) - \beta_k P_{00k}\left(\xi_2 - \xi_2^2\right) \quad (2.2)$$

Using (2.2), the bias and MSE of $\hat{R}_P$ are given by

$$B(\hat{R}_P) \approx (\alpha-1)R + \alpha\theta\frac{1}{P_{0j0}^2}\left(P_{i00} - P_{ij0}\right) + \beta_k\theta\left(1 - P_{00k}\right) \quad (2.3)$$

and

$$MSE(\hat{R}_P) \approx E\left[(\alpha-1)R + \alpha R(\xi_0 - \xi_1) - \beta_k P_{00k}\xi_2\right]^2$$

or

$$MSE(\hat{R}_P) \approx (\alpha-1)^2R^2 + \alpha^2 MSE(\hat{R}_0) + \beta_k^2\theta P_{00k}(1-P_{00k}) - 2\alpha\beta_k R\theta\left(\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right) \quad (2.4)$$

The optimum values of α and β_k are given by

$$\alpha^* = \frac{R^2}{R^2 + MSE(\hat{R}_S)_{min}}$$

and

$$\beta_k^* = \alpha^* R\frac{\left(\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right)}{P_{00k}(1-P_{00k})}$$

Substituting the optimum values of α and β_k in (2.4), we get the minimum MSE of $\hat{R}_p$ which is given by

$$MSE(\hat{R}_P)_{min} \approx \frac{R^2 MSE(\hat{R}_S)_{min}}{R^2 + MSE(\hat{R}_S)_{min}} \quad (2.5)$$

Expression (2.5) provides only an ideal optimum MSE since the optimum values of α and b_k i.e. α^* and β_k^* involve unknown parameters. In practice one can either use reasonable values of these parameters known from prior studies (see Srivastava 1967, Murthy 1967 and Lui 1990) or one can estimate these parameters from the sample as given below

Let $$\hat{\alpha}^* = \frac{\hat{R}^2}{\hat{R}^2 + M\hat{S}E(\hat{R}_S)_{min}}$$

and $$\hat{\beta}_k^* = \hat{\alpha}^* \hat{R} \frac{\left(\frac{\hat{P}_{i0k}}{\hat{P}_{i00}} - \frac{\hat{P}_{0jk}}{\hat{P}_{0j0}}\right)}{P_{00k}(1 - P_{00k})}$$

where P_{00k} is known.

Proceeding as in Upadhyaya *et al.* (2006a, b), it can be shown that $E(\hat{\alpha}^*) = \alpha$ and $E(\hat{\beta}_k^*) = \beta_k + o(n^{-1})$.

Using the estimated values of α^* and β_k^* in (2.1), the estimator becomes

$$\hat{R}_P^* = \hat{\alpha}^* \frac{p_{i00}}{p_{0j0}} + \hat{\beta}_k^* (P_{00k} - p_{00k}) \frac{P_{00k}}{p_{00k}} \quad (2.6)$$

Writing $\hat{\alpha}^* = \alpha^*(1+\xi_3)$ and $\hat{\beta}_k^* = \beta_k^*(1+\xi_4)$ where, $E(\xi_3) = E(\xi_4) = 0(n^{-1})$, we get

$$\hat{R}_P^* = \alpha^*(1+\xi_3)\frac{P_{i00}(1+\xi_0)}{P_{0j0}(1+\xi_1)} + \beta_k^*(1+\xi_4)\{P_{00k} - P_{00k}(1+\xi_2)\}(1+\xi_2)^{-1} \quad (2.7)$$

where $\xi_3 = \frac{(\hat{\alpha}^* - \alpha^*)}{\alpha^*}$ and $\xi_4 = \frac{(\hat{\beta}_k^* - \beta_k^*)}{\beta_k^*}$

From (2.7), we have

$$(\hat{R}_P^* - R) \approx (\alpha^* - 1)R + \alpha^* R\left(\xi_0 - \xi_1 + \xi_3 - \xi_0\xi_1 + \xi_1^2\right) - \beta_k^* P_{00k}\left(\xi_2 - \xi_2^2 + \xi_2\xi_4\right)$$

From above expression, the MSE of $\hat{R}_P^*$, to first degreeL of approximation, is given by

$$MSE(\hat{R}_P^*) = E(\hat{R}_P^* - R)^2$$
$$\approx E[(\alpha^* - 1)\, R + \alpha^* R(\xi_0 - \xi_1) - \beta_k^* P_{00k}\xi_2]^2 \quad (2.8)$$

Squaring and then substituting the expected values of ξ's, α^* and β^* in (2.8), we get

$$MSE(\hat{R}_P^*) \approx \frac{R^2 MSE(\hat{R}_S)_{min}}{R^2 + MSE(\hat{R}_S)_{min}} \quad (2.9)$$

The expression given in (2.9) is exactly the same as the one given in (2.5).

3. COMPARISON OF ESTIMATORS

We now compare the proposed estimator $(\hat{R}_P)$ with the usual ratio estimator $(\hat{R}_0)$, Wynn-type (1976) estimator $(\hat{R}_w)$ and Singh *et al.* (1986) estimator $(\hat{R}_s)$.

(i) By (1.2) and (2.5)

$$MSE(\hat{R}_P)_{min} < MSE(\hat{R}_0)$$

if $[MSE(\hat{R}_0) - \frac{R^2 MSE(\hat{R}_S)_{min}}{R^2 + MSE(\hat{R}_S)_{min}}] > 0$

or if $MSE(\hat{R}_0)R^2 + MSE(\hat{R}_0)MSE(\hat{R}_S)_{min} - R^2 MSE(\hat{R}_S)_{min} > 0$

or if $MSE(\hat{R}_0)MSE(\hat{R}_S)_{min} + R^4\theta \frac{\left(\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right)^2}{P_{00k}(1 - P_{00k})} > 0$

(ii) By (1.6) and (2.5)

$$MSE(\hat{R}_P)_{min} < MSE(\hat{R}_w)$$

if $[MSE(\hat{R}_W) - \frac{R^2 MSE(\hat{R}_S)_{min}}{R^2 + MSE(\hat{R}_S)_{min}}] > 0$

or if $MSE(\hat{R}_w)MSE(\hat{R}_S)_{min} + R^2[MSE(\hat{R}_w) - MSE(\hat{R}S)_{min}] > 0$

or if $MSE(\hat{R}_W)MSE(\hat{R}_S)_{min} + R^2\theta \left(P_{00k}(1 - P_{00k}) - 2R\left\{\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right\} + R^2 \frac{\left\{\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right\}^2}{P_{00k}(1 - P_{00k})} \right) > 0$

or if $MSE(\hat{R})\ MSE(\hat{R}_S)_{min}$

$$+R^2\theta\left(\sqrt{P_{00k}(1-P_{00k})}-R\frac{\left(\frac{P_{i0k}}{P_{i00}}-\frac{P_{0jk}}{P_{0j0}}\right)}{\sqrt{P_{00k}(1-P_{00k})}}\right)^2>0$$

(iii) By (1.10) and (2.5)

$$MSE(\hat{R}_P)_{min} < MSE(\hat{R}_S)_{min}$$

if $[MSE(\hat{R}_s)_{min} - \frac{R^2 MSE(\hat{R}_s)_{min}}{R^2 + MSE(\hat{R}_s)_{min}}] > 0$

or if $\left(MSE(\hat{R}_S)_{min}\right)^2 > 0$

All of the above conditions in (i)-(iii) are obviously true, establishing superiority of the proposed estimator over the competing estimators.

4. NUMERICAL EXAMPLE

We reproduce below the example discussed by Singh *et al.* (1986). Although the population size used in this example is rather small, we decided to use the same data set so that the comparison with Singh *et al*. (1986) is unbiased.

Data: [source: (Cochran, 1977, p. 182)]

The variables are defined as

y : number of paralytic polio cases in 'placebo' group

x : number of paralytic polio cases in 'not inoculated' group and

z : number of children in placebo group.

Table 1. Joint frequencies for y and z

z\y	0-2	3-5	6-8	> 8	N_{00k}
1-4.9	20	2	1	-	23
5-9.9	1	3	1	1	6
10-14.9	1	1	1	-	3
15-19.9	-	-	-	1	1
20-24.9	-	-	-	1	1
N_{i00}	22	6	3	3	N = 34

Table 2. Joint frequencies for x and z

z/x	0-2	3-5	6-8	> 8	N_{00k}
1-4.9	20	1	2	-	23
5-9.9	2	2	2	-	6
10-14.9	1	1	-	1	3
15-19.9	-	-	-	1	1
20-24.9	-	-	1	-	1
N_{0j0}	23	4	5	2	N = 34

Table 3. Joint frequencies for y and x

y\x	0-2	3-5	6-8	> 8	N_{100}
0-2	19	2	1	-	22
3-5	2	2	2	-	6
6-8	2	-	-	1	3
>8	-	-	2	1	3
N_{0j0}	23	4	5	2	N = 34

The percent relative efficiency (PRE) is obtained by using the following expression

$$PRE = \frac{MSE(\hat{R}_0)}{MSE(\hat{R}_v)} \times 100 \ \ (v = 0,\ W,\ S,\ P)$$

The results are given in the following table

Table 4. PRE of estimators $\hat{R}_v (v = 0,\ W,\ S,\ P)$ with respect to $\hat{R}_0$ based on category i = j = 1 and various choices of k

Estimator\k	1	2	3	4	5
$\hat{R}_0$	100.000	100.000	100.000	100.000	100.000
$\hat{R}_W$	75.027	65.695	84.878	93.779	93.779
$\hat{R}_S$	101.541	102.585	100.010	100.000	100.000
$\hat{R}_P$	106.173	107.217	104.643	104.643	104.640

Results in the above table clearly show the gain in efficiency in using the proposed estimator. Similar results were observed for other choices of i and j (i = 1, 2, 3, 4; j = 1, 2, 3, 4) with various choices of k. Table 5 shows PRE values for a different choice of i and j.

Table 5. PRE of estimators $\hat{R}_v$ (v = O, W, S, P) with respect to $\hat{R}_0$ based on category (i = 1, j = 2) and various choices of k_0.

Estimator/k	1	2	3	4	5
$\hat{R}_0$	100.000	100.000	100.000	100.000	100.000
$\hat{R}_W$	102.811	98.038	99.102	99.989	99.989
$\hat{R}_S$	130.465	120.085	106.517	100.000	100.000
$\hat{R}_P$	214.177	203.797	190.229	183.712	183.712

The estimator in (2.1) can be generalized even more by making use of all of the known proportions in various categories relative to the auxiliary variable z. Doing so, the generalized estimator is

$$\hat{R}_P^{(k)} = \gamma \frac{p_{i00}}{p_{0j0}} + \sum_{k=1}^{c} \delta_k \left(P_{00k} - p_{00k}\right) \frac{P_{00k}}{p_{00k}} \quad (4.1)$$

for $(i = 1, 2, ..., a; \; j = 1, 2, ..., b; \; k = 1, 2, ..., c)$ where γ and δ_k are suitably chosen constants.

Retaining terms up to power two in ξ's, we have

$$(\hat{R}_P^{(k)} - R) \approx (\gamma - 1)R + \gamma R\left(\xi_0 - \xi_1 - \xi_0\xi_1 + \xi_1^2\right) + \sum_{k=1}^{c} \delta_k P_{00k}\left(\xi_2 - \xi_2^2\right) \quad (4.2)$$

We can find the bias and minimum MSE of $\hat{R}_P^{(k)}$, to first degree of approximation, just as we did before.

$$B(\hat{R}_P^{(k)}) \approx (\gamma - 1)R + \gamma\theta \frac{1}{P_{0j0}^2}\left(P_{i00} - P_{ij0}\right) + \theta \sum_{k=1}^{c} \delta_k \left(1 - P_{00k}\right) \quad (4.3)$$

and

$$MSE(\hat{R}_P^{(k)}) \approx E\left[(\gamma - 1)R + \gamma R(\xi_0 - \xi_1) - \sum_{k=1}^{c} \delta_k P_{00k}\xi_2\right]^2$$

or $$MSE(\hat{R}_P^{(k)}) \approx (\gamma - 1)^2 R^2 + \gamma^2 MSE(\hat{R}_0) + \theta \sum_{k=1}^{c} \delta_k^2 P_{00k}(1 - P_{00k}) - 2\gamma R\theta \sum_{k=1}^{c} \delta_k \left(\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right) \quad (4.4)$$

From (4.4), we get the optimum values of γ and δ_k as given by

$$\gamma^* = \frac{R^2}{R^2 + A} \quad \text{and} \quad \delta_k^* = \gamma^* R \frac{\left(\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right)}{P_{00k}(1 - P_{00k})}$$

where $$A = MSE(\hat{R}_0) - R^2\theta \sum_{k=1}^{c} \frac{\left(\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right)^2}{P_{00k}(1 - P_{00k})}$$

Substituting in (4.4) the optimum values of γ and δ_k, i.e. γ^* and δ_k^*, we get the minimum MSE of $\hat{R}_P^{(k)}$ as given by

$$MSE(\hat{R}_P^{(k)})_{min} \approx \frac{R^2 A}{R^2 + A} \quad (4.5)$$

Note from (2.5) and (4.5) that

$$MSE(\hat{R}_P^{(k)})_{min} < MSE(\hat{R}_p)_{min}$$

if $$\left[\frac{R^2 MSE(\hat{R}_S)_{min}}{R^2 + MSE(\hat{R}_S)_{min}} - \frac{R^2 A}{R^2 + A}\right] > 0$$

or if $$R^4[MSE(\hat{R}_S)_{min} - A] > 0$$

or if $$R^6\theta\left(\sum_{k=1}^{c} \frac{\left(\frac{P_{i0k}}{P_{i00}} - \frac{P_{0jk}}{P_{0j0}}\right)^2}{P_{00k}(1 - P_{00k})} - \frac{\left(\frac{P_{i0r}}{P_{i00}} - \frac{P_{0jr}}{P_{0j0}}\right)^2}{P_{00r}(1 - P_{00r})}\right) > 0$$

The above condition is obviously true for any value of r = 1, 2, ..., c.

The optimum parameter values are given in Tables 6 and 7.

Table 6. Optimum values for d, α, β_k, δ_k, γ, A and R based on (i = 1, j = 1) for k = 1, 2 ... 5

Optimum\k	1	2	3	4	5
d^*	0.17275	–0.27316	0.02345	0.00000	0.00000
α^*	0.95637	0.95680	0.95573	0.95573	0.95573
β_k^*	0.16521	–0.26136	0.02246	0.00000	0.00000
δ_k^*	0.16540	–0.26153	0.02250	0.00000	0.00000
γ* = 0.95744, A= 0.04067, R = 0.95652					

Similarly we can find the optimum values for other choices of i and j. For example, these values for (i = 1, j = 2) are given in Table 7 below.

Table 7. Optimum values for d, α, β_k, δ_k, γ, A and R based on (i = 1, j = 2) for k = 1, 2 ... 5

Optimum\k	1	2	3	4	5
d^*	16.56320	–0.17204	–13.98390	0.00000	0.00000
α^*	0.60915	0.58924	0.55994	0.54433	0.54433
β_k^*	10.08940	–10.13630	–7.83013	0.00000	0.00000
δ_k^*	11.4197	–11.86030	–9.64131	0.00000	0.00000
γ* = 0.68946, A = 13.6250, R = 5.5					

The percent relative efficiency (PRE) of the estimator based on simultaneous use of all k values, and for $(i=1,\ \ j=1)$, is given by

$$PRE(\hat{R}_P^{(k)}) = \frac{MSE(\hat{R}_0)}{MSE(\hat{R}_P^{(k)})_{min}} \times 100$$

$$= \frac{0.042382}{0.038936} \times 100 = 108.851\%$$

The above values are obtained by using Equations (1.2), (4.5) and Table 6. For (i = 1, j = 2), this is given by

$$PRE(\hat{R}_P^{(k)}) = \frac{MSE(\hat{R}_0)}{MSE(\hat{R}_P^{(k)})_{min}} \times 100$$

$$= \frac{25.3230}{9.39385} \times 100 = 269.411\%$$

The above values are obtained by using the Equations (1.2), (4.5) and Table 7.

Thus there is additional gain in simultaneously using information for all values of k.

5. CONCLUSION

From Tables 4 and 5, we observed that the performance of proposed estimator $(\hat{R}_P)$ is better than the usual ratio estimator $(\hat{R}_0)$, Wynn-type (1976) estimator $(\hat{R}_W)$ and Singh *et al.* (1986) estimator $(\hat{R}_S)$ regardless of which sub-category relative to z is utilized. This was clearly expected based on the efficiency comparison in Section 3. The precision of $\hat{R}_P$ is highest for k = 2 in Table 4 and for k = 1 in Table 5. Wynn-type (1976) estimator shows the poorest performance, even worse than the usual ratio estimator except in Table 5 for k = 1. It is also observed that the efficiency of the generalized estimator $\left(\hat{R}_P^{(k)}\right)$ is even better.

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