J. Ind. Soc. Agril. Statist.
62(1), 2008 : 42-48

Optimum Allocation in Multivariate Stratified Sampling in Presence of Non-response

M.G.M. Khan[1], E.A. Khan[2] and M.J. Ahsan[3]
Department of Statistics Iowa State University Ames, Iowa 50011-1210, USA
(Received: December 2007)

SUMMARY

Hansen and Hurwitz (1946) suggested a technique for eliciting responses from a subsample of the non-respondents. Khare (1987) applies this procedure of subsampling in stratified sampling and discussed the problem of optimum allocation in presence of non-response. When more than one characteristics are under study, it is not possible to use the individual optimum allocations for one reason or the other. In such situations, some criterion is needed to work out an acceptable sampling fraction which is optimum for all characteristics in some sense. In this paper, the problem of determining the optimum allocation and the optimum size of subsamples to various strata in multivariate stratified sampling in presence of non-response is formulated as a Nonlinear Programming Problem (NLPP). A solution to this problem is obtained using Lagrange multipliers technique. Explicit formulae are obtained for the optimum allocation and the optimum sizes of the subsamples.

Key words: Non-response, Multivariate stratified sampling, Optimum allocation, Optimum size of subsamples, Nonlinear Programming Problem.

1. INTRODUCTION

Non-response refers to the failure to obtain information, for some reason or the other, from some of the population units that are selected in the sample. Hansen and Hurwitz (1946) presented the classical non-response theory for eliciting responses from a subsample of the non-respondents. The technique was first developed for the surveys in which the first attempt was made by mailing the questionnaires and a second attempt was made by personal interview to a subsample of the non-respondents. They constructed the estimator for the population mean and derived the expression for its variance and also worked out the optimum sampling fraction among the non-respondents. Hansen and Hurwitz's technique was further extended by El-Badry (1956) by sending waves of questionnaires to the non-respondent units to increase the response rate. Foradari (1961) generalized El-Badry's approach for different sampling designs. Srinath (1971) suggested the selection of subsamples by making several attempts. Khare (1987) investigated the problem of optimum allocation in stratified sampling in presence of non-response for fixed cost as well as for fixed precision of the estimate.

The problem of optimum allocation in stratified random sampling for a univariate population is well known in sampling literature; see for example Cochran (1977) and Sukhatme *et al.* (1984). When more than one characteristics are under study, it is not possible to use the individual optimum allocations to various strata because an allocation, which is optimum for one characteristic, may not be optimum for other characteristics. Moreover, in the absence of a strong positive correlation between the characteristics under study the individual optimum allocations may differ a lot and there may be no obvious compromise. In such situations, some criterion is needed to work out an allocation which is optimum, in some sense, for all characteristics. Methods for solving the problem of optimum allocation in multivariate stratified sampling are proposed by various authors. Dalenius (1953), Yates

[1] *On leave from the School of Computing Information and Mathematical Sciences, The University of the South Pacific, Suva, Fiji*
[2] *Department of Medical Elementology and Toxicology, Hamdard University, New Delhi-110062, India*
[3] *Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh – 202002, India*

(1960), Folks and Antle (1965), Hartley (1965), Kish (1988), Khan *et al.* (1997) worked out the multivariate optimum allocation by minimizing the weighted average of different characters. The second approach of minimizing the total cost of the survey when the variances are subjected to fixed tolerance limits, is discussed by Dalenius (1957), Yates (1960), Kokan (1963), Kokan and Khan (1967), Chatterjee (1968), Huddleston *et al.* (1970), Chatterjee (1972), Hughes and Rao (1979), Bethal (1985), Chromy (1987), Bethal (1989) etc. Särandal *et al*. (1992) formulated the generalized optimization problem for model based sampling that is of interest for several specified allocation problems. Zayatz and Sigman (1994) studied the feasibility of the use of Chromy's algorithm in a practical situation related to the annual sample survey of manufacturers. The authors have discussed the methods above for solving the problem of optimum allocation to various strata without considering the presence of non-response.

In this paper the authors present an unbiased estimate of stratum mean of a population characteristic and derive its variance using the technique of Hansen and Hurwitz (1946) that considers a subsample of non-respondents. Then the problem of determining the optimum allocations to various strata in the presence of non-response and optimum size of subsamples among the non-respondents in multivariate stratified sampling is discussed. The problem is formulated as a Nonlinear Programming Problem (NLPP), which has a convex objective function and a single linear cost constraint. Several techniques are available for solving this type of NLPP, better known as Convex Programming Problem (CPP). We used Lagrange multiplier technique to solve the formulated NLPP and explicit formulae for the optimum allocation and the optimum size of the subsamples to various strata are obtained. To verify that the solution obtained is really the required optimum the Kuhn-Tucker (1951) necessary conditions, that are sufficient also for the formulated problem, are shown to hold at the optimal point. A numerical example is also presented to illustrate the computational details.

2. FORMULATION OF THE PROBLEM

In stratified sampling where a population of size N is divided into L strata, let N_h, $\bar{Y}_h$, S_h^2 and $P_h = N_h/N$ denote the stratum size, stratum mean, stratum variance and stratum weight of h^{th} stratum. Assume that every stratum is divided into two disjoint groups of respondents and non-respondents, with N_{h1} and $N_{h2} = N - N_{h1}$ as the sizes of the respondents and non-respondents groups in the h^{th} stratum respectively. Out of a stratified random sample of size n let n_h ; h = 1, 2 ..., L units are from h^{th} stratum. Further, out of n_h, let n_{h1} units belong to the respondents group and the remaining $n_{h2} = n_h - n_{h1}$ units belong to the non-respondents group. Suppose that at the second attempt subsamples of sizes

$$r_h = n_{h2}/k_h;\ h = 1, 2, ..., L \qquad (2.1)$$

are drawn from non-respondent units; where $k_h > 1$ and $1/k_h$, denote the sampling fraction among non-respondents in the h^{th} stratum. Unbiased estimate of N_{h1} and N_{h2} are given by $\hat{N}_{h1} = n_{h1}N_h/n_h$ and $\hat{N}_{h2} = n_{h2}N_h/n_h$, respectively.

In a multivariate stratified sampling where on every unit p characteristics are under study, let $\bar{y}_{jh1}$ and $\bar{y}_{jh2(r_h)}$, j = 1, 2, ..., p denote the sample means of j^{th} characteristic of the n_{h1} respondents at the first attempt and the r_h subsampled units at the second attempt. Following the procedure of Hansen and Hurwitz (1946) an estimator of the stratum mean $\bar{Y}_{jh}$ for j^{th} characteristic in the h^{th} stratum may be defined as a weighted mean of $\bar{y}_{jh(w)}$ of $\bar{y}_{jh1}$ (the sample mean of n_{h1} respondents) and $\bar{y}_{jh2(r_h)}$ (sample mean of r_h units from non-respondents measured at the second attempt) as

$$\bar{y}_{jh(w)} = \frac{n_{h1}\bar{y}_{jh1} + n_{h2}\bar{y}_{jh2(r_h)}}{n_h} \qquad (2.2)$$

Theorem 2.1 shows that $\bar{y}_{jh(w)}$ is unbiased for $\bar{Y}_{jh}$.

Theorem 2.1. For a given sample 's' consisting of n_{h1} respondents and n_{h2} non-respondents, $\bar{y}_{jh(w)}$ is an unbiased estimate of $\bar{Y}_{jh}$.

Proof. We have

$$E\left(\bar{y}_{jh(w)}\middle|s\right) = E\left(E\left(\bar{y}_{jh(w)}\middle|n_{h1}, n_{h2}\right)\right)$$

$$= E\left(\frac{n_{h1}\bar{y}_{jh1} + n_{h2}\bar{y}_{jh2}}{n_h}\right)$$

$$= E\left(\bar{y}_{jh}\right)$$

$$= \bar{Y}_{jh}$$

The following theorem gives the sampling variance of $\bar{y}_{jh(w)}$.

Corollary. An unbiased estimate of the overall population mean $\bar{Y}_j$ of the j^{th} characteristics may be given as $\bar{y}_{j(w)} = \sum_{h=1}^{L} P_h \bar{y}_{jh(w)}$ where $P_h = N_h/N$.

Theorem 2.2. The variance of $\bar{y}_{jh(w)}$ is given as

$$V\left(\bar{y}_{jh(w)}\right) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_{jh}^2 + \frac{W_{h2}^2 S_{jh2}^2}{r_h} - \frac{W_{h2} S_{jh2}^2}{n_h} \qquad (2.3)$$

Proof. We have

$$V\left(\bar{y}_{jh(w)} \middle| s\right) = V\left(E\left(\bar{y}_{jh(w)} \middle| n_{h1}, n_{h2}\right)\right) + E\left(V\left(\bar{y}_{jh(w)} \middle| n_{h1}, n_{h2}\right)\right)$$

$$= V\left(\bar{y}_{jh}\right) + E\left(V\left(\bar{y}_{jh(w)} \middle| n_{h1}, n_{h2}\right)\right) \qquad (2.4)$$

Now

$$V\left(\bar{y}_{jh(w)} \middle| n_{h1}, n_{h2}\right) = V\left(\frac{n_{h1}}{n_h} \bar{y}_{jh1} \middle| n_{h1}, n_{h2}\right) + V\left(\frac{n_{h2}}{n_h} \bar{y}_{jh2(r_h)} \middle| n_{h1}, n_{h2}\right)$$

$$= \left(\frac{n_{h2}}{n_h}\right)^2 \left(\frac{1}{r_h} - \frac{1}{n_{h2}}\right) s_{jh2}^2$$

$$= \left(\frac{w_{h2}^2}{r_h} - \frac{w_{h2}}{n_h}\right) s_{jh2}^2 \qquad (2.5)$$

where $w_{h2} = n_{h2}/n_h$ is the proportion of non-response in the sample and $s^2_{jh_2}$ is the sample mean square based on n_{h_2} units. Again

$$E\left(V\left(\bar{y}_{jh(w)} \middle| n_{h1}, n_{h2}\right)\right) = E\left(E\left(\frac{w_{h2}^2}{r_h} - \frac{w_{h2}}{n_h}\right) s_{jh2}^2 \middle| n_{h2}\right)$$

$$= \left(\frac{E\left(w_{h2}^2\right)}{r_h} - \frac{E\left(w_{h2}\right)}{n_h}\right) S_{jh2}^2$$

$$= \frac{W_{h2}^2 S_{jh2}^2}{r_h} - \frac{W_{h2} S_{jh2}^2}{n_h} \qquad (2.6)$$

where W_{h1} and W_{h2} are the proportion of the respondents and non-respondents, respectively, in h^{th} stratum, S_{jh2}^2 is the variance among non-respondents for j^{th} characteristic in the h^{th} stratum and are assumed to be known from past experience.

The substitution of (2.5) and (2.6) in (2.4) gives (2.3) which completes the proof.

Corollary. The variance of $\bar{y}_{j(w)} = \sum_{h=1}^{L} P_h \bar{y}_{jh(w)}$ where $P_h = N_h / N$, is given as

$$V(\bar{y}_{j(w)}) = V\left(\sum_{h=1}^{L} P_h \bar{y}_{jh(w)}\right) = \sum_{h=1}^{L} P_h^2 \, V\left(\bar{y}_{jh(w)}\right) \qquad (2.7)$$

where $V\left(\bar{y}_{jh(w)}\right)$ is given by (2.3).

Assuming a linear cost function the total cost of the sample survey could be considered as

$$C' = \sum_{h=1}^{L} c_{h0} n_h + \sum_{h=1}^{L} c_{h1} n_{h1} + \sum_{h=1}^{L} c_{h2} r_h \qquad (2.8)$$

where c_{h0} denotes the per unit cost of making the first attempt, $c_{h1} = \sum_{j=1}^{p} c_{jh1}$ denotes the per unit cost for processing the results of all the p characteristics in first attempt and $c_{h2} = \sum_{j=1}^{p} c_{jh2}$ denotes the per unit cost for obtaining and processing the results of all the p characteristics in second attempt in the h^{th} stratum. Also c_{jh1} and c_{jh2} are the per unit costs of measuring the j^{th} characteristic in first and second attempts respectively.

As n_{h1} is not known until the first attempt is made, the quantity $W_{h1} n_h$ may be used as its expected value and then the total expected cost of the survey could be given as

$$C = \sum_{h=1}^{L}\left(c_{h0} + c_{h1}W_{h1}\right)n_h + \sum_{h=1}^{L} c_{h2}r_h \qquad (2.9)$$

The optimum allocations n_h and optimum size of subsamples r_h for an individual characteristic obtained by minimizing the variance in (2.7) for given cost in (2.9), or by minimizing the cost for fixed variance are available in the sampling literature (Khare 1987).

In multivariate stratified sample surveys, as discussed in Section 1, usually a compromise criterion is needed to work out a usable allocation which is optimum, in some sense, for all characteristics. However, if the total expected cost for the survey is predetermined, using the compromise criterion used by authors like Kish (1988), Khan *et al.* (2003) and others, an optimum allocation may be worked out, which minimizes the weighted sum of the sampling variances of the estimates of various characteristics within the expected budget. Thus in a population with L strata and P characteristics, if the population means of various characteristics are of interest, it may be a reasonable criterion for obtaining a compromise allocation to minimize the weighted sum of the variances of the stratified sample means of all the characteristics, that is

$$\sum_{j=1}^{p} a_j V\left(\bar{y}_{j(w)}\right) \qquad (2.10)$$

where $a_j > 0$ is the weights assigned to the j^{th} characteristic in proportion to its importance as compared to other characteristics and $V\left(\bar{y}_{j(w)}\right)$ is as given in (2.7). Without loss of generality we can assume that $\sum_{j=1}^{p} a_j = 1$.

Using (2.3), (2.7) and (2.10) and ignoring the terms independent of n_h and r_h minimizing (2.10) is equivalent to minimize

$$\sum_{h=1}^{L} \frac{P_h^2\left(A_h^2 - W_{h2}B_h^2\right)}{n_h} + \sum_{h=1}^{L} \frac{P_h^2 W_{h2}^2 B_h^2}{r_h} \qquad (2.11)$$

where

$$A_h^2 = \sum_{j=1}^{p} a_j S_{jh}^2 \text{ and } B_h^2 = \sum_{j=1}^{p} a_j S_{jh2}^2 \qquad (2.12)$$

For a fixed expected cost C_o given by the RHS of (2.9) the problem of finding the optimum allocation n_h and the sizes of the subsample r_h may be stated as the following NLPP

$$\left.\begin{array}{ll} \text{Minimise} & z = \sum_{h=1}^{L} \frac{P_h^2(A_h^2 - W_{h2}B_h^2)}{n_h} + \sum_{h=1}^{L} \frac{P_h^2 W_{h2}^2 B_h^2}{r_h} \\ \text{subject to} & \sum_{h=1}^{L}(C_{h_0} + C_{h_1}W_{h1})n_h + \sum_{h=1}^{L} C_{h2}r_h \le C_0 \\ \text{and} & n_h,\ r_h \ \ge 0\,(h=1,2,...,L) \end{array}\right\} (2.13)$$

The restrictions $n_h \ge 0$ and $r_h \ge 0$ are obvious because negative values of n_h and r_h are of no practical use.

3. THE SOLUTION

The objective function z of the NLPP (2.13) will be minimum when the values of n_h and r_h are as large as permitted by the cost constraints. This suggests that at the optimum point the cost constraint will be active, that is, it is satisfied as an equation. If the restrictions $n_h \ge 0$ and $r_h \ge 0$ are ignored, Lagrange multipliers technique can be used to determine the optimum values of n_h and r_h, say n_h^* and r_h^*. If n_h^* and r_h^* are ³ 0 the NLPP (2.13) is completely solved, otherwise some nonlinear programming technique may be used.

Define the Lagrange function φ as

$$\varphi\left(n_h, r_h, \lambda\right) = \sum_{h=1}^{L} \frac{P_h^2\left(A_h^2 - W_{h2}B_h^2\right)}{n_h} + \sum_{h=1}^{L} \frac{P_h^2 W_{h2}^2 B_h^2}{r_h}$$

$$+\lambda\left(\sum_{h=1}^{L}\left(c_{h0} + c_{h1}W_{h1}\right)n_h + \sum_{h=1}^{L} c_{h2}r_h - C_0\right) \qquad (3.1)$$

where λ is a Lagrange multiplier.

The necessary conditions for the solution of the problem are

$$\frac{\delta L}{\delta n_h} = -\frac{\left(A_h^2 - W_{h2}B_h^2\right)P_h^2}{n_h^2} + \lambda\left(c_{h0} + c_{h1}W_{h1}\right) = 0$$

which gives

$$n_h = \frac{1}{\sqrt{\lambda}} \frac{\sqrt{P_h^2\left(A_h^2 - W_{h2}B_h^2\right)}}{\sqrt{c_{h0} + c_{h1}W_{h1}}} \text{ ; } h = 1, 2,..., L \quad (3.2)$$

Also $\frac{\delta L}{\delta r_h} = -\frac{P_h^2\, W_{h2}^2 B_h^2}{r_h^2} + \lambda c_{h2} = 0$

which gives

$$r_h = \frac{1}{\sqrt{\lambda}} \frac{P_h W_{h2} B_h}{\sqrt{c_{h2}}} \text{ ; } h = 1, 2, ..., L \quad (3.3)$$

and $\frac{\delta L}{\delta \lambda} = \sum_{h=1}^{L}\left(c_{h0} + c_{h1}W_{h1}\right)n_h + \sum_{h=1}^{L} c_{h2} r_h - C_0 = 0$

which gives

$$\frac{1}{\sqrt{\lambda}} = \frac{C_0}{[\sum_{h=1}^{L}\sqrt{P_h^2(A_h^2 - W_{h2}B_h^2)(c_{h0} + c_{h1}W_{h1})} + \sum_{h=1}^{L} P_h W_{h2} B_h \sqrt{c_{h2}}]} \quad (3.4)$$

From (3.2) and (3.4)

$$n_h^* = \frac{C_0\sqrt{P_h^2\,(A_h^2 - W_{h2}B_h^2)/(c_{h0} + c_{h1}W_{h1})}}{[\sum_{h=1}^{L}\sqrt{P_h^2\,(A_h^2 - W_{h2}B_h^2)(c_{h0} + c_{h1}W_{h1})} + \sum_{h=1}^{L} P_h W_{h2} B_h \sqrt{c_{h2}}]} \quad (3.5)$$

Also (3.3) and (3.4) give

$$r_h^* = \frac{C_0 P_h W_{h2} B_h / \sqrt{c_{h2}}}{[\sum_{h=1}^{L}\sqrt{P_h^2\,(A_h^2 - W_{h2}B_h^2)(c_{h0} + c_{h1}W_{h1})} + \sum_{h=1}^{L} P_h W_{h2} B_h \sqrt{c_{h2}}]} \quad (3.6)$$

It can be verified that in NLPP (2.13) the objective function is convex for $A_h^2 \geq W_{h2}B_h^2$ or $\sum_{j=1}^{p} a_j S_h^2 \geq W_{h2} \sum_{j=1}^{p} a_j S_{jh2}^2$ and the constraint is linear. Therefore, the K-T necessary conditions for the NLPP (2.13) are sufficient also. For NLPP (2.13) these conditions are

$$\nabla_{(n_h, r_h)}\varphi = \begin{pmatrix} -\frac{(A_h^2 - W_{h2}B_h^2)P_h^{\,2}}{n_h^2} + \lambda(c_{h0} + c_{h1}W_{h1}) \\ -\frac{P_h^{\,2} W_{h2}^2 B_h^2}{r_h^2} + \lambda c_{h2} \end{pmatrix} \geq 0 \quad (3.7)$$

$$n_h\left(-\frac{\left(A_h^2 - W_{h2}B_h^2\right)P_h^{\,2}}{n_h^2} + \lambda\left(c_{h0} + c_{h1}W_{h1}\right)\right) + r_h\left(-\frac{P_h^{\,2} W_{h2}^2 B_h^2}{r_h^2} + \lambda c_{h2}\right) = 0 \quad (3.8)$$

$$\nabla_{\lambda}\varphi = \sum_{h=1}^{L}\left(c_{h0} + c_{h1}W_{h1}\right)n_h + \sum_{h=1}^{L} c_{h2} r_h - C_0 \leq 0 \quad (3.9)$$

$$\lambda\left(\sum_{h=1}^{L}\left(c_{h0} + c_{h1}W_{h1}\right)n_h + \sum_{h=1}^{L} c_{h2} r_h - C_0\right) = 0 \quad (3.10)$$

and n_h, r_h and $\lambda \geq 0$ (3.11)

where $\nabla\varphi$ denote the gradient vector of φ.

For n_h, r_h and $\lambda > 0$ the above expressions give the same set of equations as (3.2), (3.3) and (3.4), which implies that the K-T conditions hold at the point $\left(n_h^*, r_h^*\right)$ given by (3.5) and (3.6). Hence, $\left(n_h^*, r_h^*\right)$ is optimum for NLPP (2.13).

4. A NUMERICAL ILLUSTRATION

The following hypothetical data are constructed to illustrate the use of the formula (3.5) and (3.6) for computing the values of the overall optimum allocations and the optimum sample sizes from non-respondents measured at the second attempt respectively. Consider a population of size N = 3850 divided into four strata. Let the population means of the two characteristics defined on each unit of the population are to be estimated. Table 4.1 shows the relevant information.

In addition to the above, it is assumed that the relative value of the variances of the non-respondents and respondents, that is, $S_{jh2}^2 / S_{jh}^2 = 0.25$ for j = 1, 2 and

Table 4.1. Data for four strata and two characteristics

h	N_h	P_h	S_{1h}^2	S_{2h}^2	W_{h1}	W_{h2}	c_{h0}	c_{h1}	c_{h2}
1	1214	0.32	4817.72	8121.15	0.70	0.30	1	2	3
2	822	0.21	6251.26	7613.52	0.80	0.20	1	3	4
3	1028	0.27	3066.16	1456.40	0.75	0.25	1	4	5
4	786	0.20	6207.25	6977.72	0.72	0.28	1	5	6

$h = 1, 2, ..., 4$. Further, let the total amount available for the survey be $C_0 = 5000$ units and both the characteristics be equally important, that is, $a_1 = a_2 = 0.5$.

Substituting the values from Table 4.1 in (2.12) we get

$$A_1^2 = 6469.44 \qquad A_2^2 = 6932.39$$

$$A_3^2 = 2261.28 \qquad A_4^2 = 6592.49$$

and

$$B_1^2 = 1617.38 \qquad B_2^2 = 1733.10$$

$$B_3^2 = 565.32 \qquad B_4^2 = 1624.12$$

This gives the optimum sample and subsample sizes n^*_h and r^*_h ($h = 1, 2, 3$ and 4) using (3.5) and (3.6) as

$$n_1^* = 541, \; n_2^* = 313, \; n_3^* = 211, \; n_4^* = 247$$

and $r_1^* = 76$, $r_2^* = 30$, $r_3^* = 24$, $r_4^* = 31$ respectively.

Since the expected value of n_{h2} is $W_{h2}n_h$, from (2.1) we have the expected values of $k_h = W_{h2}n_h/r_h$ as

$$k_1 = 2.14,\; k_2 = 2.09,\; k_3 = 2.20, \text{ and } k_4 = 2.23$$

Thus, the values of the optimum sampling fractions among non-respondents for the four strata are

$1/k_1 = 0.467$, $1/k_2 = 0.478$, $1/k_3 = 0.455$ and $1/k_4 = 0.448$ respectively.

ACKNOWLEDGEMENTS

The authors are grateful to the learned referee for his valuable comments and suggestions that helped in improving this paper.

REFERENCES

Bethal, J. (1985). An optimum allocation algorithm for multivariate surveys. *Proceedings of the Section on Survey Methods Research,* American Statistical Association, 209-212.

Bethal, J. (1989). Sample allocation in multivariate surveys. *Survey Methodology*, **15**, 47-57.

Chatterji, S. (1968). Multivariate stratified surveys. *J. Amer. Statist. Assoc.*, **63**, 530-534.

Chatterjee, S. (1972). A study of optimum allocation in multivariate surveys. *Skand. Akt*., **55**, 73-80.

Chromy, J. (1987). Design optimization with multiple objectives. *Proc. of the Section on Survey Methods Research,* American Statistical Association, 194-199.

Cochran, W.G. (1977). *Sampling Techniques*. 2nd ed., John Wiley, New York

Dalenius, T. (1953). The multivariate sampling problem. *Skand. Aktuar-Tidskr*, **36**, 92-102.

Dalenius, T. (1957). *Sampling in Sweden: Contributions to the Methods and Theories of Sample Survey Practice*. Almqvist Och Wiksell, Stockholm.

El-Badry, M.A. (1956). A sampling procedure for mailed questionnaires. *J. Amer. Statist. Assoc.,* **51**, 209-227.

Folks, J.L. and Antle, C.E. (1965). Optimum allocation of sampling units to the strata when there are R responses of interest. *J. Amer. Statist. Assoc.*, **60**, 225-233.

Foradari, G.T. (1961). Some Nonresponse Sampling Theory for Two Stage Designs. North Carolina State College, Mimeographed Series No. **297**, Raleigh.

Hansen, M.H., Hurwitz, W.N. (1946). The problem of nonresponse in sample surveys. *J. Amer. Statist. Assoc.,* **40**, 517-529.

Hartley, H.O. (1965). Multipurpose optimum allocation in stratified sampling. *Proc. of the Section on Social Statistics*, American Statistical Association, 258-261.

Hughes, E. and Rao, J.N.K. (1979). Some problems of optimum allocation in sample surveys involving inequality constraints. *Comm. Statist - Theory Methods*. **A8(15)**, 1551-1574.

Huddleston, H.F., Claypool, P.L. and Hocking, R.R. (1970). Optimum sample allocation to strata using convex programming. *Appl. Stat*., **19**, 273-278.

Khan, M.G.M., Ahsan, M.J. and Jahan, N. (1997). Compromise allocation in multivariate stratified sampling: An integer solution. *Naval Research Logistics*, **44**, 69-79.

Khan, M.G.M., Khan, E.A. and Ahsan, M.J. (2003). An optimal multivariate stratified sampling design using dynamic programming. *Australian & New Zealand J. Statist.,* **45(1)**, 107-113.

Khare, B.B. (1987). Allocation in stratified sampling in presence of nonresponse. *Metron*, **XLV(1-2)**, 213-221.

Kish, L. (1988). Multipurpose sample designs. *Survey Methodology*, **14**, 19-32.

Kokan, A.R. (1963). Optimum allocation in multivariate surveys. *J. Roy. Statist. Soc*., **A126**, 557-565.

Kokan, A.R. and Khan, S.U. (1967). Optimum allocation in multivariate Surveys: An analytical solution. *J. Roy. Statist. Soc.,* **B29**, 115-125.

Kuhn, H.W., Tucker, A.W. (1951). Nonlinear Programming. *Proc. of the Second Berkley Symposium on Mathematical Statistics and Probability*, University of California Press, Berkley, 481-492.

Särndal, C.E., Swensson, B., and Wretman, J. (1992). *Model Assisted Survey Sampling*. Springer Series in Statistics.

Srinath, K.P. (1971). Multiple sampling in nonresponse problems. *J. Amer. Statist. Assoc.,* **66**, 583-586.

Sukhatme, P.V., Sukhatme, B.V., Sukhatme, S. and Asok, C. (1984). *Sampling Theory of Surveys with Applications*. 3rd ed., Iowa State University Press, Ames, Iowa.

Yates, F. (1960). *Sampling Methods for Census and Surveys.* 2nd ed., Charles Griffin & Co. Ltd., London.

Zayatz, L., and Sigman, R. (1994). Feasibility study of the use of chromy's algorithm in poisson sample selection for the annual survey of manufacturers. *Proc. of the Section on Survey Method Research,* American Statistical Association, 641-646.