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Designs for Competition Experiments in Complete Blocks using Sequences with Minimum Number of Triplets

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SUMMARY

Designs for competition experiments are used for studying the competition effects among treatments applied to neighboring experimental units. In these experiments it is assumed that the response of treatments applied to an experimental unit is influenced by the response of treatments applied to its immediate left and right neighbour positions, the situation being represented by a triplet of treatment symbols. To save resources these triplets are arranged into sequences of symbols. Each symbol in that sequence is used both for studying the treatment effect as well as the neighbour effects. Each block consists of one or more sequences of these treatment triplets arranged in a line (row). In this article a method of construction of designs making use of minimum number treatment triplets in complete blocks, randomisation of designs making use of these sequences along with the analysis and an illustration are presented.

Key words: Serial designs, Competition effects, Sequences of treatments.

1. INTRODUCTION

The designs for competition experiments are used for studying the interference/competition among treatments applied to neighboring experimental units in experiments on agro forestry, intercropping, varietal trials and laboratory experiments. In these designs it has been assumed that the treatments applied to a plot affect the response of the treatments in the neighboring plots. Hanson *et al.* (1961) defined a competition environment, an additive model (ignoring interaction effects) and the analysis based on this model was developed and used for the estimating competition effects on soybean. In this field experiment competition resulted from genotypic or plant type differences, due to spacing and width of rows. Competition effects are defined with reference to an experimental unit and to a competing environment. An experimental unit indicated a plant, a single row of field plot or like units and an attempt to recognise and minimise the competition effects among these units in soybean was made. A set of p competing units were selected and a competing structure was developed using these units. All competing types of genotypes (i, j), i, j = 1, 2,..., p made up p(p + 1)/2 environments. Effects like average competition, specific competition were defined and its significance tested. Competitive advantages or disadvantages or different pairs were also studied. But difference due to positions or directions (i.e. difference of the environments like (i, j) and (j, i)) were not studied and procedure for randomisation were not specified. The analysis of variance is attempted by the method of fitting of constants.

A detailed discussion of occurrence of competition in agricultural field experiments, mainly in rice was made by Gomez and Gomez (1994). The methodology adopted by them was similar to the layout and analysis of split plot designs. Variation between row positions and interaction effect between row position and adjacent varieties were made use of in studying the competition effects. A layout consisting of three strip plots each containing eleven sub plots is developed to measure varietal competition in three rice varieties. Here also, the estimation of directional effects, methods of randomisation and a layout for a general case are not

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given. Dyke and Shelley (1976) introduced the term 'serial designs' and constructed 'sequences of treatments' that allow estimation of competition effects among neighboring plots. Their constructions were based on computer programmes. Four treatments (four fungicide applications) along with the neighbour effects were represented by 36 treatment triplets and then arranged in sequences (blocks) consisting of 38 units and estimations were made. They worked out the treatment effects, left hand and right hand neighbour effects and interactions. Similar type of works were carried out by Lin *et al.* (1985), Subrahmanyam (1991), Rawlings (1974) etc. Azais (1987) and Monad and Bailey (1993) attempted a theoretical basis for these experiments.

In the present study, it is assumed that the units receiving treatments are arranged in a row (line) within a block and a treatment applied to a unit is affected by the treatments applied to its immediate left and right neighboring positions for light, water, nutrients etc. Further, it is assumed that a treatment is affected only by the treatments in its immediate left and right neighbour positions and these effects need not be equal. Any specific treatment is called as a test treatment and those treatments which impose a competition effect on this test treatment (appearing to its immediate left and right neighbour positions) are called as its left and right neighbour respectively. The three effects are designated as test treatment effect, left neighbour effect and right neighbour effect respectively. Let us assume that there are 's' treatments denoted by the symbols 1, 2, 3, ...,s and suppose that a test treatment 'i' is placed in between two immediate neighbouring treatments 'j' and 'k'. This situation can be represented by a triplet of the form 'jik'; j, i, k = 1, 2,.., s. For 's' symbols there are s^3 such triplets. By a triplet of symbols 'ijk' we mean that among the s treatments, treatments with numbers i, j and k were applied to the three units in the respective positions.

2. FORMATION OF SEQUENCES AND TRIPLET CLASSES

The observations are recorded from the treatment applied to the middle unit (denoted by the middle symbol) of each triplet, but to save resources sequences making use of these triplets were developed, wherever possible so that a treatment (symbol) can be used to estimate test treatment effect as well as the neighbour effects. Two triplets (or sequences) may be combined to a new sequence when the last ordered pair of symbols of one is made same as the first ordered pair of symbols of the second triplet (or sequence). In a sequence the symbols at the beginning and end are called as border symbols and the others are called as inner symbols. The border symbols were used only to provide the neighbour effects while the inner symbols were used both for measuring test treatment effects as well as neighbour effects. The border symbols in a sequence are distinguished from the inner symbols by putting it within parenthesis. Thus, sequences using symbols 1, 2, 3, 4 etc. say, (1) 2 3 (4) and (3) 4 5 (6) may be combined as (1) 2 3 4 5 (6). Two sequences (1) 2 3 4 (5) and (6) 7 8 9 (2) which cannot be combined, are made to a jointed sequence as (1) 2 3 4 (5, 6) 7 8 9 (2) indicating that those symbols in parenthesis were used only to provide the neighbour effect and no observation is recorded from it.

In serial designs, we can attempt developing sequences using the whole s^3 triplets or subset of the s^3 triplets. The s^3 triplets may be classified into five distinct classes as 'iii, iij, jii, jij and jik' where $(i \neq j \neq k)$. Selected triplets from the above classes can be combined to form sequences. One of these sequences or more than one sequence in a jointed form will make up blocks of a serial design. In a design all the test treatments and the neighbour effects will appear in a single block or in different blocks. Further, to get the neighbour effect of a treatment with itself we have to duplicate a treatment in a sequence. (By duplication (or triplication) we mean repeating a symbol twice (or thrice) at its place of occurrence in a sequence). To estimate all the neighbour effects or contrasts among these effects we must have all the treatments appearing at least once in the left position and once in the right position of a given test treatment. In other words, all possible ordered pairs of treatments have to appear at least once in the design. Compared to a randomized complete block design, these serial designs make use of only one third of the experimental units. But the serial designs proposed by Dyke and Shelley (1976), Subrahmanyam (1991) were non random and in the former case the sequence was developed using a computer programme. We begin with some ideas on treatment effects, linear model used and estimability of the effects based on these five triplet classes in Section 3. Estimability of the effects in triplet classes is discussed in Section 4. A method of construction of designs for estimation of competition effects by making use of minimum number of triplets and complete symmetric digraphs is given in Section 5.The procedure of analysis of data from these designs is given in Section 6. In Section 7, we illustrate the procedure using a simulative data.

3. MODEL AND NORMAL EQUATIONS

For a set of 's' treatments in 'b' blocks making use of 'n' design points (represented by 'n' inner symbols of the sequences used), the linear model with fixed effects assumed is

$$Y = \mu 1_n + X_\beta \beta + X_t t + X_l l + X_\rho \rho + e \qquad (3.1)$$

where

Y : n × 1 observational vector, n is the total number of observations

1_n : n × 1 vector of ones

X_β : n × b matrix of observation versus blocks

The test treatment effect, left and right neighbour effects are represented by vectors t, l and ρ respectively, defined as

X_t : n × s matrix of observations versus test treatments

X_l : n × s^2 matrix of observation versus left neighbour effects

X_ρ : n × s^2 matrix of observation versus right neighbour effects

μ : General mean

t : s × 1 vector of test treatment effects (t_i ; i = 1, 2, ...s)

β : b × 1 vector of block effects

l : $s^2 \times 1$ vector of left neighbour effects (l_{iv} ; i = 1, 2, ... s and u = 1, 2, ...s and expressed in the order ($l_{11}, l_{22}, \ldots l_{ss}, l_{21}, l_{31}, \ldots l_{s1}, l_{12}, l_{32}, \ldots l_{s2}, \ldots l_{13}, \ldots l_{s-1, s}$), l_{ui} is the left neighbour effect on the i^{th} test treatment due to the u^{th} left neighbour

ρ : $s^2 \times 1$ vector of right neighbour effects (ρ_{iv} ; i = 1, 2, ...s and v = 1, 2, ...s and expressed in the order ($\rho_{11}, \rho_{22}, \ldots \rho_{ss}, \rho_{12}, \rho_{13}, \ldots \rho_{1s}, \rho_{21}, \rho_{23}, \ldots, \rho_{2s}, \rho_{31}, \rho_{32}, \ldots, \rho_{s, s-1}$), ρ_{iv} is the right neighbour effect on the i^{th} test treatment due to the v^{th} right neighbour

e : the vector of independently and identically distributed error components with zero expectation and unit variance. *i.e.* E(e) = 0 and D(e) = σ^2 I

Applying the usual least square method of estimation we have the following normal equations

$$\begin{pmatrix} n & 1'_n X_\beta & 1'_n X_t & 1'_n X_l & 1'_n X_\rho \\ X'_\beta 1_n & X'_\beta X_\beta & X'_\beta X_t & X'_\beta X_l & X'_\beta X_\rho \\ X'_t 1_n & X'_t X_\beta & X'_t X_t & X'_t X_l & X'_t X_\rho \\ X'_l 1_n & X'_l X_\beta & X'_l Y_t & X'_l X_l & X'_l X_\rho \\ X'_\rho 1_n & X'_\rho X_\beta & X'_\rho X_t & X'_\rho X_l & X'_\rho X_\rho \end{pmatrix} \times \begin{pmatrix} \mu \\ \beta \\ t \\ l \\ \rho \end{pmatrix} = \begin{pmatrix} G \\ B \\ T \\ L \\ R \end{pmatrix} \qquad (3.2)$$

where G denote the grand total of all the n observations, B is the b × 1 vector of block totals, T is the s × 1 vector of totals of the test treatments, L is the $s^2 \times 1$ vector of totals due to the i^{th} test treatment with the u^{th} as its left neighbour corresponding to l_{ui}'s and R is the $s^2 \times 1$ vector of totals due to the i^{th} test treatment with the v^{th} as its right neighbour corresponding to ρ_{iv}'s for i, u, v = 1,2,..., s.

Equation (3.2) contains ($2s^2$ + b + s + 1) normal equations. Obtaining general solutions to these normal equations is involved. In view of the inherent relationship of the X matrices it can be observed that only ($2s^2$ + b – s – 1) equations can be independent. Thus, to estimate the contrasts among the effects a minimum of ($2s^2$ – s) = s(2s – 1) independent design points are needed. The rank of the above matrix will reduce by 2(s + 1) and one set of solutions can be obtained by putting that much number of linear independent constraints on the parameters suitably. Further, the left and right effects need to be adjusted in the estimation of sum of squares.

4. ESTIMABILITY OF EFFECTS IN TRIPLET CLASSES

The treatment effects used in the model (3.1) viz. t_i, l_{ui} and ρ_{iv} are to be estimated in a serial design based on blocks developed using sequences, selecting triplets from the five classes mentioned in Section 2. But it can be seen from Table 4.1 that none of the triplet class alone will provide all the effects and contrasts of these effects are estimable by choosing selected groups of the triplets belonging to more than one of these classes. A list of possible groupings of these triplets in which all effects are present, are given in Table 4.2. To construct designs using these selected triplet classes the possibility of developing sequences is to be established, and further the number of treatment triplets required based on these classes should not be too large (As in the case of Groups

7, 8, 9, 10 etc.). Group 3 is the most useful one containing classes iij, jij and jii with $s + s(s-1) + s(s-1) = s(2s-1)$ triplets through which designs can be developed with minimum number of distinct triplets (for $s \geq 3$) and this is the minimum number of distinct triplets required to estimate contrasts among the neighbour effects i.e. $s(2s-1)$. Further, it is proved that triplets of this group will always form sequences, which is illustrated in the next section using results on graph theory.

Table 4.1. Estimability of treatment effects in triplet classes

Effects	Triplet classes ($i \neq j \neq k$)				
	iii	iij	jii	jij	jik
t_i	Y	Y	Y	Y	Y
l_{ii}	Y	Y	N	N	N
l_{ui}, ($u \neq 1$)	N	N	Y	Y	Y
ρ_{ii}	Y	N	Y	N	N
ρ_{iv} ($i \neq v$)	N	Y	N	Y	Y

Note : Y indicates those triplets that will provide the respective effect and N that will not.

Table 4.2. Groups of triplet classes of treatments providing all parameters (t_i, l_{ui} and ρ_{iv}) and the number of distinct triplets in each group

Group	Triplet classes	Total number of distinct triplets	Remarks
1	iii and jii	s^2	$s^2 < s(2s-1)$, hence design cannot be constructed (for $s > 2$)
2	iij and jii	$2s(s-1)$	$2s(s-1) < s(2s-1)$ Hence design cannot be constructed
3	iii, iij and jii	$s(2s-1)$	Useful in minimum number of triplets
4	iii and jik	$s(s^2-3s+3)$	Useful for $s > 3$ but number of triplets needed is more than group 3
5	iij, jii and jij	$3s(s-1)$	do
6	iij, jii and jik	$s^2(s-1)$	do
7	iii, iij, jii and jij	$s(3s-2)$	do
8	iii, iij, jii and jik	$s(s^2-s+1)$	do
9	iij, jii, jij and jik	s^3-s	Useful for $s > 3$, but there are very large number of triplets
10	All triplets	s^3	do

5. EXISTENCE OF SEQUENCES USING THE TRIPLETS BELONGING TO THE CLASSES iii, ijj AND jii

The possibility of existence of sequences using these triplets is verified using Graph theory. This theory will automatically give a method for the randomization of such sequences also.

Consider a digraph with 's' vertices. A Complete Symmetric Digraph (CSD) is one in which there are $(s-1)$ arcs from each vertex to the rest. For a sequence used in a serial design, each symbol assigned to a test treatment can be assumed as a vertex of a digraph, and the directed arc from this vertex to other vertices, as the left and right neighbor effects respectively, so that the whole set up can be represented by a CSD with 's' vertices. In other words given a CSD with 's' vertices it is possible to develop a sequence of symbols in which each symbol will have all the symbols as left and right neighbour a fixed number of times.

Theorem 5.1. Given a CSD with s vertices, one can develop a sequence of symbols in which each symbol will have all other symbols as left and right neighbour exactly once.

Proof. The proof can be evolved by induction. First let us prove that for $s = 2$ symbols a CSD will give such a sequence. For $s = 2$, let the two vertices of the CSD be 1 and 2 and there will be two arcs one from 1 to 2 and other from 2 to 1, so the path through which one can move over these arcs once, starting from a vertex may be written in the form of a sequence as 121 or as 212 (Fig. 5.1).

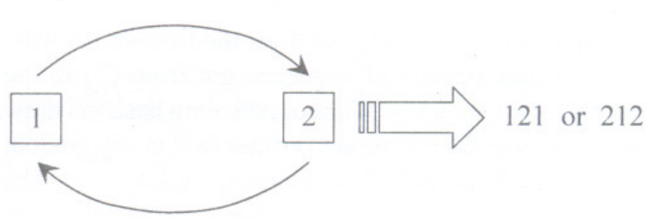

Fig. 5.1. Developing sequences using digraphs – for two symbols 1 and 2

Similarly for $s = 3$ symbols let the three vertices of the CSD be 1, 2 and 3. When we move through all the directed paths once we can develop sequences of vertices in that path as 1231321 or 1321231 etc.(Fig. 5.2). Obviously in these sequences all symbols will have the remaining symbols as left and right neighbour exactly once.

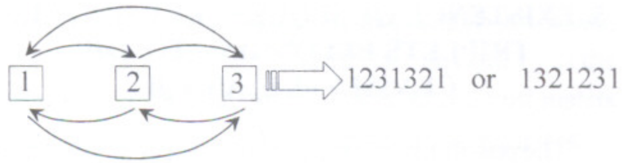

Fig. 5.2. Developing sequences using digraphs – for three symbols 1, 2 and 3

Now border symbols can be included on these sequences like (2) 1 2 (1), or (1)2 1(2), or (2)123132(1) etc. so as to measure the remaining neighbour effects. Here the last vertex reached in the sequence of the arcs is made as the right border and the vertex from which the last arc of the sequence originated will be the left border.

Now, the process can be extended by including a new vertex to a CSD. Let us assume that there exists C_k, a CSD for s = k, representing a sequence of arcs ending at the vertex i such that, this sequence has every other symbol as its left and right neighbour exactly once. i.e. assume that there exist a CSD, C_k for the k vertices in the box shown in Fig. 5.3. From C_k we can obtain a CSD of k + 1 symbols say, C_{k+1} by the following method.

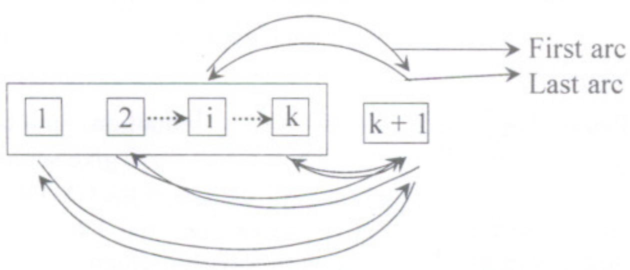

Fig. 5.3. Developing sequences using digraphs – Method of extension from a CSD with k vertices to a new $(k+1)^{th}$ vertex

In continuation of C_k we draw the first arc from the vertex i (last symbol of sequence got from C_k) to the additional vertex k + 1. Without returning back to i draw arcs to each of the remaining vertices (≠ i) in any random order, one by one each time returning back to k + 1. The last arc will be from the $(k+1)^{th}$ vertex to the i^{th} vertex from which we have started the extension. By definition, the resulting graph is a CSD for (k + 1) vertices. Now a sequence of the vertices can be written according to the order of the paths of movement and it is combined with the sequence obtained for C_k, which will provide the required sequence with all neighbour effects. As an illustration for four symbols, we can include the symbol 4 with the previous sequence 1231321 by joining a sequence like 1424341 with this new symbol to get a sequence for 4 vertices as 1231321424341. Incorporating the border symbols it will become (4)123132142434(1).

Obviously, the sequence generated will contain all the test treatments repeated (s – 1) times, the neighbour effect of a treatment with itself equal to zero and the neighbour effect of a treatment with rest of the treatments is one. So the theorem gives a very easy method of constructing sequences providing all left and right neighbour effects exactly once, except for one on itself, without selecting any initial collection of triplets. The sequence so formed will not contain triplets of the type iij or jii but it can be introduced into the sequence simply by duplicating each of the symbols at all its places of occurrence in that sequence, i.e. in (s – 1) positions. By duplicating the symbols, the original arcs of the underlying CSD remain unaltered while (s – 1) additional loops are getting added at each vertex. The result can be stated as follows.

Theorem 5.2. A sequence containing triplets of the classes iij and jii can be obtained by duplicating each of the inner symbols of a sequence, obtained from a complete symmetric digraph.

Proof. In a sequence obtained from a CSD all treatment symbols will have all other treatments as its left and right neighbour exactly at one place. When all these symbols are duplicated it will be a sequence containing triplets of the class iij and jii (for all i and j) and the neighbour properties remain unaltered. By this process, the number of times each test treatment is replicated is 2(s – 1), in the inner positions. Each treatment will have itself as left neighbour and right neighbour (s – 1) times. Each treatment will have every other treatment as its left and right neighbour exactly once.

In a similar manner in the original sequence obtained from Theorem 5.1 let us double the symbols at all places except for one place where it is triplicated, for each of the s symbols. The resulting sequence will contain the triplets of the type iii also.

Theorem 5.3. If there exists a sequence containing all the triplets belonging to the classes iij and jii (j ≠ i) we can always extend it to include the triplets of the classes iii also.

Proof. By making use of Theorem 5.1, a number of sequences can be developed by choosing different order of selection of the symbols (vertices) in the underlying CSD. The following theorem is introduced in this regard.

Theorem 5.4. A necessary and sufficient condition for a sequence of ordered triplets to give all the left and right neighbours for a given symbol with rest of the symbols (and not for a given symbol with itself) exactly once, is that the diagram represented by the sequence is a complete symmetric digraph.

Proof. To prove the necessary part consider a sequence with the above properties denoted by a, b, c,...(Any sequence with n + 2 symbols can be considered as made up of n ordered triplets). Since a has got b as right neighbour at one place in the sequence, we can associate it as an arc from a to b. Similarly a will have b as a left neighbour in one position in the sequence which represent an arc from b to a. By this argument we can find 2(s – 1) arcs, (s – 1) of them, to and (s – 1) of them, from any vertex to each of the remaining vertices in a collection of s vertices, (denoted by s symbols of the sequence). Hence the sequence represents a CSD.

Now the sufficiency part is already proved in Theorem 5. 1.

Blocks of the designs for competition experiments are now the sequences constructed by the above method. Any randomly chosen vertex or path of the underlying CSD can be chosen to construct the sequence, containing the s(2s – 1) triplets belonging to the classes iii, iij and jii. This will provide a restricted randomisation for sequences, rather than using any computer programme for the construction of sequences. (The randomization as in the case of other block designs is not possible as it will not keep the neighbour properties). Designs using these sequences will give complete blocks with all the test treatments replicated equally, the neighbour effect of all the treatments with itself will have same number of replications, and neighbour effects of a given treatment with the rest also will have an equal number of replications.

6. ANALYSIS OF DESIGNS USING TRIPLETS OF THE CLASSES iii, ijj AND jii

It has been proved that sequences using s(2s – 1) triplets of the classes iii, iij and jii ($i \neq j \neq k$; i, j, k =1, 2,.., s) can always be constructed. It will provide complete blocks for serial designs giving s(2s – 1) inner symbols and two borders.

Analysis of designs using the above classes of triplets is attempted below. One set of solutions to equations (3.2) are obtained by putting 2(s + 1) linear independent constraints on the parameters. The left and right effects are to be adjusted in deriving the sum of squares.

Expressing the $s^2 \times 1$ vectors l and ρ as

$$l' = (l_1', l_2')$$

and

$$\rho' = (\rho_1', \rho_2')$$

where

$$l_1' = (l_{11}, l_{22}, l_{33}, \ldots, l_{ss})$$
$$l_2' = (l_{21}, l_{31}, \ldots, l_{s1}, l_{12}, l_{32}, \ldots, l_{s2}, \ldots, l_{s-1,s})$$
$$\rho_1' = (\rho_{11}, \rho_{22} \ldots, \rho_{ss}) \text{ and}$$
$$\rho_2' = (\rho_{12}, \rho_{13}, \ldots, \rho_{1s}, \rho_{21}, \rho_{23}, \ldots, \rho_{2s} \ldots, \rho_{s, s-1})$$

The normal equations (3.2) for complete block designs using blocks with above triplet classes can be deduced in the following way

$$\begin{pmatrix} G \\ B \\ T \\ L_1 \\ L_2 \\ R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} bs(2s-1) & s(2s-1)1_b' & b(2s-1)1_s' & bs1_s' & b1'_{s(s-1)} & bs1_s' & b1'_{s(s-1)} \\ s(2s-1)1_b & s(2s-1)I_b & b(2s-1)J_{b\times s} & sJ_{b\times s} & J_{b\times s(s-1)} & sJ_{b\times s} & J_{b\times s(s-1)} \\ b(2s-1)1_s & (2s-1)J_{s\times b} & b(2s-1)I_s & bsI_s & bI_s \otimes 1_{s-1} & b_sI_s & bI \otimes 1 \\ bs1_s & sJ_{s\times b} & bsI_s & bsI_s & O & bI_s & bI_s \otimes 1 \\ b1_{s(s-1)} & J_{s(s-1)\times b} & bI_s \otimes 1_{s-1} & O & bI_{s(s-1)} & bI_s \otimes 1_{s-1} & O \\ bs1_s & sJ_{s\times b} & bsI_s & bI_s & bI_s \otimes 1_{s-1} & bsI_s & O \\ b1_{s(s-1)} & J_{s(s-1)\times b} & bI_s \otimes 1_{s-1} & bI_s \otimes 1_{s-1} & O & O & bI_{s(s-1)} \end{pmatrix} \begin{pmatrix} \mu \\ \beta \\ t \\ l_1 \\ l_2 \\ \rho_1 \\ \rho_2 \end{pmatrix} \quad (6.1)$$

L_1, L_2, R_1 and R_2 are the corresponding observational totals, 1_b is the $b \times 1$ column vector of ones and J_{bxs} is the $b \times s$ matrix with all elements unity, O is a null matrix and $\otimes$ indicates Kronecker product of two matrices.

We make use of the following 2(s +1) constraints on the parameters

$$\sum_i t_i = 0, \sum_j \beta_j = 0$$

$$sl_{ii} + \sum_{u \neq i} l_{ui} = 0, \forall i, u = 1, 2, \ldots s$$

$$s\rho_{ii} + \sum_{v \neq i} \rho_{iv} = 0, \forall i, v = 1, 2, \ldots s \quad (6.2)$$

Under the above constraints the normal equations will reduce to

$$G = bs(2s-1)\mu$$
$$B_j = s(2s-1)(\mu + \beta_j)$$
$$T_i = b(2s-1)(\mu + t_i)$$
$$L_{ii} = bs(\mu + t_i + l_{ii}) + b(\rho_{ii} + \sum_{v \neq i} \rho_{iv})$$
$$L_{ui} = b(\mu + t_i + l_{ui}) + b\rho_{ii}$$
$$R_{ii} = bs(\mu + t_i + \rho_{ii}) + b(\rho_{ii} + \sum_{u \neq i} l_{ui})$$
$$R_{ii} = b(\mu + t_i + \rho_{iv}) + bl_{ii} \quad (6.3)$$

for u, i, v = 1, 2, ...s, u ≠ i, v ≠ i

These equations lead to the following estimates

$$\hat{\mu} = \frac{G}{bs(2s-1)} \quad (6.4)$$

$$\hat{\beta}_j = \frac{B_j}{s(2s-1)} - \frac{G}{bs(2s-1)} \quad (6.5)$$

$$\hat{t}_i = \frac{T_i}{b(2s-1)} - \frac{G}{bs(2s-1)} \quad (6.6)$$

$$\hat{l}_{ii} = \frac{1}{b(2s-1)}[sL_{ii} + (s-1)R_{ii} - sT_i] \quad (6.7)$$

$$\hat{\rho}_{ii} = \frac{1}{b(2s-1)}[sR_{ii} + (s-1)L_{ii} - sT_i] \quad (6.8)$$

$$\hat{l}_{ui} = \frac{1}{b}L_{ui} - \frac{1}{b(2s-1)}[sR_{ii} + (s-1)(L_{ii} - T_i)] \quad (6.9)$$

and

$$\hat{\rho}_{iv} = \frac{1}{b}R_{iv} - \frac{1}{b(2s-1)}[sL_{ii} + (s-1)(R_{ii} - T_i)] \quad (6.10)$$

for all u, i, v =1,2, ... s (u ≠ i, v ≠ i)

6.1 Computation of Sum of Squares

The total sum of squares, block sum of squares, treatment sum of squares (as a total of three effects) and the error sum of squares are not affected by the rest, and thus these sum of squares can be estimated orthogonally in the usual manner :

Total sum of squares with bs(2s – 1) – 1 degrees of

$$\text{freedom} = \sum Y^2 - \frac{G^2}{bs(2s-1)} \quad (6.11)$$

Block sum of squares with (b – 1) degrees of freedom

$$= \frac{\sum B_j^2}{s(2s-1)} - \frac{G^2}{bs(2s-1)} \quad (6.12)$$

Treatment sum of squares with s(2s – 1) – 1 degrees of

$$\text{freedom} = \frac{\sum Y_{uiv.}^2}{b} - \frac{G^2}{bs(2s-1)} \quad (6.13)$$

where $Y_{uiv.}$ denote the total of the observations due to the treatment triplet 'uiv' over the blocks; $\sum Y^2$ is the sum of squares of all observations. The error sum of squares with (b – 1)[s(2s – 1) – 1] degrees of freedom can now be obtained by subtracting block and treatment sum of squares from the total sum of squares.

6.2 Splitting of Treatment Sum of Squares

In order to identify the effect of test treatments and the effect of neighbours on each test treatment, the treatment sum of squares (6.13) computed is to be split into various components. As the effect of test treatment permit independent estimation, this sum of squares can be computed easily while the sum of squares due to neighbour effects need adjustment. The sum of squares due to test treatment can be obtained as

$$\frac{\sum T_i^2}{b(2s-1)} - \frac{G^2}{bs(2s-1)} \qquad (6.14)$$

The reduced normal equations for l_{ui}'s after eliminating the other parameters will come out as

$$L_{ii}^* = L_{ii} + \frac{s-1}{s} R_{ii} - T_i$$

$$= \frac{b}{s}[(s-1)l_{ii} - \sum_{u \neq i} l_{ui}]$$

$$= \frac{b}{s}(2s-1)l_{ii} \qquad (6.15)$$

and $$L_{ii}^* = L_{ui} - \frac{R_{ii}}{s} \text{ (for } u \neq i \text{)}$$

Thus the sum of squares due to left effects adjusted for the right neighbour effects within the i^{th} test treatment

$= \sum_u L_{ui}^* l_{ui}$ with (s – 1) degrees of freedom for i=1, 2,...,s (6.16)

Further, the total sum of squares due to left effects adjusted for the right neighbour effects with s(s – 1) degrees of freedom can also be obtained by taking the sum of equations (6.16) over all the test treatments (i) as

$$[(3s - s^2 - 1)\frac{b}{s}]\sum l_{ii}^2 + b\sum\sum l_{ui}^2 \text{ (for } u \neq i) \qquad (6.17)$$

Similarly the sum of squares due to right neighbour effects (adjusted for the left neighbour effects) within the i^{th} test treatment is given as

$\sum_v R_{iv}^* \rho_{iv}$ with (s – 1) degrees of freedom for i = 1, 2,..., s (6.18)

where $$R_{ii}^* = R_{ii} + \frac{s-1}{s} L_{ii} - T_i \text{ and}$$

$$R_{iv}^* = R_{iv} - \frac{L_{ii}}{s} \text{ (for } v \neq i) \qquad (6.19)$$

Total sum of squares due to right effects adjusted for the left neighbour effects with s(s – 1) degrees of freedom can also be obtained (as in the case of equation 6.17) as

$$[(3s - s^2 - 1)\frac{b}{s}]\sum \rho_{ii}^2 + b\sum\sum \rho_{iv}^2 \text{ (for } v \neq i \text{)} \qquad (6.20)$$

The sum of squares due to left effects (unadjusted) within the test treatment with (s – 1) degrees of freedom

$$= \frac{L_{ii}^2}{bs} + \sum_{u \neq i} \frac{L_{ui}^2}{b} - \frac{T_i^2}{b(2s-1)} \qquad (6.21)$$

Similarly the sum of squares due to right effects (unadjusted) within the i^{th} test treatment with (s – 1) degrees of freedom

$$= \frac{R_{ii}^2}{bs} + \sum_{v \neq i} \frac{R_{iv}^2}{b} - \frac{T_i^2}{b(2s-1)} \qquad (6.22)$$

The unadjusted sum of squares due to left or right effects(for all test treatments) is also equal to the sum of squares due to L_{ui}'s (or R_{iv}'s) minus sum of squares due to test treatment effects. (6.23)

Further, we can algebraically verify that the sum of squares due to left effects (adjusted for right effects) within a given test treatment + sum of squares due to right effects (unadjusted) for the same test treatment = sum of squares due to left effects (unadjusted) + sum of squares due to right effects (adjusted for left effects) within the same test treatment. The analysis of variance of this design is summarised in Table 6.1.

Test of significance and comparison of treatment effects can now be made as in the case of designs with fixed effects models.

7. ILLUSTRATION

The procedure of analysis is illustrated for the case of s = 3 using a sample of size 45 simulated by using a random number generating algorithm following a normal distribution with mean zero and variance unity. It is assumed that the test treatments were arranged in 15 triplets of the type iii, iij and jii in three replications. The data used were given in Table 7.1 and the methods of computations were as discussed below.

Table 6.1. Analysis of variance of design for competition experiments in complete blocks using s(2s – 1) triplets of the classes iii, iij and jii in complete blocks

Source	Degrees of freedom	Sum of squares (given by equations)
Total	bs(2s – 1) – 1	6.11
Blocks	(b – 1)	6.12
Treatments	s(2s – 1) – 1	6.13
Test treatments	(s – 1)	6.14
Left effects (adjusted) within the 1st treatment	(s –1)	6.16
Right effects (unadjusted) within the 1st treatment	(s – 1)	6.22
⋮	⋮	⋮
Left effects (adjusted) within the sth treatment	(s – 1)	6.16
Right effects (unadjusted) within the sth treatment	(s – 1)	6.22
Total of left effects (adjusted) within test treatments	s(s – 1)	Sum of eqns.6.16 or 6.17
Total of right effects (unadjusted) within test treatments	s(s – 1)	Sum of eqns.6.22 or 6.23
Right effects (adjusted) within the 1st treatment	(s – 1)	6.18
Left effects (unadjusted) within the 1st treatment	(s – 1)	6.21
⋮	⋮	⋮
Right effects (adjusted) within the sth treatment	(s – 1)	6.18
Left effects (unadjusted) within the sth treatment	(s – 1)	6.21
Total of right effects (adjusted) within test treatments	s(s – 1)	Sum of eqns.6.18 or 6.20
Total of left effects (unadjusted) within test treatments	s(s – 1)	Sum of eqns.6.21 or 6.23
Error	(b – 1) × [s(2s – 1) – 1]	(by subtraction)

From Table 7.1 the following subtotals can be obtained

L_{11} =1709.529	L_{21} =579.493	L_{31} =588.422
L_{12} =610.217	L_{22} =1799.447	L_{32} = 599.839
L_{13} =627.031	L_{23} =635.770	L_{33} =1911.550
R_{11} =1745.039	R_{12} =572.895	R_{13} =559.506
R_{21} =597.531	R_{22} =1807.126	R_{32} =640.647
R_{31} =629.419	R_{23} =604.846	R_{33} =1904.292
T_1 =2877.440	T_2 = 3009.503	T_3 =3174.350

Correction factor $= (9061.301)^2 / 45 = 1824603.907$

Total sum of squares = 4511.590

Block sum of squares = 822.741
Treatment sum of squares = 3197.886; and
Error sum of squares = 490.966

Sum of squares due to test treatments

= (2877.4402 + 3009.5032 + 3174.3502)/15

– correction factor = 2950.625

7.1 Computation of Sum of Squares due to Right Effects (Unadjusted) using (6.22)

Sum of squares within test treatment 1 = 125.719

Within test treatment 2 = 9.482
Within test treatment 3 = 21.040
Thus Sum of squares due to right effects (unadjusted)

= 125.719 + 9.482 + 21.040 = 156.241

Table. 7.1. Simulated data using 15 triplets of three treatments (1, 2 and 3)

Triplets	Blocks			Total
	1	2	3	
111	193.331	188.865	194.928	577.124
112	186.998	189.374	196.523	572.895
113	183.688	185.712	190.106	559.506
221	196.933	197.102	203.496	597.531
222	191.919	199.704	205.447	597.070
223	193.973	195.895	214.978	604.846
331	204.458	212.112	212.849	629.419
332	211.555	213.437	215.655	640.647
333	211.083	207.734	222.667	641.484
211	192.242	193.298	193.953	579.493
311	185.674	196.794	205.954	588.422
122	198.430	205.932	205.855	610.217
322	187.155	202.346	210.338	599.839
133	203.604	211.058	212.369	627.031
233	208.458	207.578	219.741	635.777
Block total	2949.501	3006.941	3104.859	9061.301= G

(This sum of squares can also be worked out as sum of squares due to R_{ii} and R_{iv}'s – Sum of squares due to test treatments = 3106.865 – 2950.625 = 156.241)

7.2 Computation of Sum of Squares due to Left Effects (Unadjusted) using (6.21)

Sum of squares within test treatment 1 = 92.991

Within test treatment 2 = 28.818

Within test treatment 3 = 26.109

Sum of squares due to left effects (unadjusted)

= 92.991 + 28.818 + 26.109 =147.918

7.3 Left and Right Neighbour Effects (using equations 6.7, 6.8, 6.9 and 6.10)

l_{11} = –0.911 l_{21} = – 0.122 l_{31} = 2.855

l_{12} = 3.321 l_{22} = –1.061 l_{32} = –0.138

l_{13} = –3.474 l_{23} = – 0.558 l_{33} = 1.344

ρ_{11} = 1.457 ρ_{12} = 0.047 ρ_{13} = –4.416

ρ_{21} = –0.396 ρ_{22} = – 0.549 ρ_{23} = 2.043

ρ_{31} = –3.162 ρ_{32} = 0.581 ρ_{33} = 0.860

7.4 Adjusted Sum of the Squares (using equations 6.15 and 6.19)

L^*_{11} = – 4.556 L^*_{21} = – 2.187 L^*_{31} = 6.742

L^*_{12} = 7.842 L^*_{22} = –5.305 L^*_{32} = –2.536

L^*_{13} = –7.733 L^*_{23} = 1.006 L^*_{33} = 6.721

R^*_{11} = 7.282 R^*_{12} = 3.053 R^*_{13} = –10.336

R^*_{21} = –2.285 R^*_{22} = –2.746 R^*_{23} = 5.030

R^*_{31} = – 7.764 R^*_{32} = 3.464 R^*_{33} = 4.301

Thus, the sum of squares due to left effects adjusted for right effects within the i[th] test treatment = $\Sigma L^*_{ui} l_{ui}$

Within test treatment 1 = 23.658
Within test treatment 2 = 32.024
Within test treatment 3 = 35.336
Total = 91.018

(The total sum of squares due to left effects (adjusted for right effects) can also be worked out using equation 6.17 as = –1 × 3.762 + 3 × 31.593 = 91.018)

Similarly the sum of squares due to (right effects adjusted for left effects) were

Within test treatment 1 = 56.396
Within test treatment 2 = 12.687
Within test treatment 3 = 30.258
Total = 99.341

The Analysis of variance to test the null hypothesis of no difference among the test treatments, left neighbour effects (within the test treatments) and right neighbour effects (within the test treatments) is made in Table 7.2.

Table 7.2. Analysis of variance of design for competition experiment

Source	Degrees of 822 freedom	Sum of squares	Mean	F
Total		44	4511.590	
Blocks	2	741	411.371	23.460
Treatments	14	3197.883	228.420	13.026
Test treatments	2	2950.625	1475.313	84.137
(left effects adjusted)	6	91.018	15.170	< 1
(right effects unadjusted)	6	153.241	–	
(left effects unadjusted)	6	147.910	–	
(right effects adjusted)	6	99.341	16.556	< 1
Error	28	490.966	17.535	

The analysis of variance table for this simulated data reveal the significance of the test treatment effects. The left and right neighbour effects were found to be not significant.

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