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Shrinkage Estimation of Proportion of Population Possessing Stigmatizing Character in Unrelated Question Randomized Response Technique

Housila P. Singh, S. Shukla and N. Mathur
Vikram University, Ujjain
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SUMMARY

This paper is devoted to the estimation of population proportion π_A of population belonging to some sensitive group A using prior or a guessed value π_0 of π_A. A class of shrinkage estimators for π_A using prior information π_0 is suggested under unrelated question randomized response model and analyzed its properties.

Key words : Bias, Mean squared error, Guessed value, Randomized response model.

1. INTRODUCTION

Warner (1965) has given a skillful interviewing procedure known as randomized response (RR) technique for estimating the proportion π_A of population members of a sensitive group A. Horvitz *et al.* (1967) suggested a variant originally due to Simmons to the effect that the confidence of respondent in anonymity provided by randomized response method might be enhanced if one of the questions referred to a non-sensitive, innocuous attribute, say Y, unrelated to the sensitive attribute A. For instance, the sensitive question may be A : "Do you habitually consume illicit drugs?" or Y: "Do you like the game of cricket?". Since the second question relates to an innocuous characteristic that is unrelated to the drug addiction, one can reasonably expect that the respondent's privacy is being sufficiently well protected (Chaudhuri and Mukerjee (1988)). The model developed by Horvitz *et al.* (1967) is known as unrelated question randomized response model (U-model). The theoretical framework for this model was given by Greenberg *et al.* (1969).

If π_Y is the proportion in the population with the attribute Y, $\pi_Y + \pi_A \gtrless 1$. Suppose that the statement Y with probability $(1-p)$ and θ denotes the probability of "yes" answer, we have

$$\theta = p\pi_A + (1-p)\pi_Y \tag{1.1}$$

$$\Rightarrow \pi_A = \frac{\{\theta - (1-p)\pi_Y\}}{p} \tag{1.2}$$

Assume that a simple random sample of size *n* is drawn with replacement from the population and each interviewee is asked to report only "yes" or "no" regarding belonging to A (chosen with probability p) or to Y (chosen with probability $(1-p)$). If n_1 be the number of "yes" responses in the sample and $\hat{\theta} = n_1/n$, an unbiased estimator of π_A is defined by

$$\hat{\pi}_A = \frac{\{\hat{\theta} - \pi_Y(1-p)\}}{p} \tag{1.3}$$

The variance of $\hat{\pi}_A$ is given by

$$V(\hat{\pi}_A) = \frac{\theta(1-\theta)}{np^2} \tag{1.4}$$

$$= \frac{\pi_A(1-\pi_A)}{n} + \frac{(1-p)}{np^2} K_p(\pi_Y, \pi_A) \tag{1.5}$$

where

$$K_p(\pi_Y, \pi_A) = p(1-2\ \pi_Y)\pi_A + \pi_Y[1-(1-p)\pi_Y]$$

This U-model was further modified by various authors including Moor (1971), Folsom *et al.* (1973), Greenberg *et al.* (1969), Abul-Ela *et al.* (1967), Mangat *et al.* (1992), Singh *et al.* (1994) and Singh (1994).

This paper suggests a modification of usual unbiased estimator in (1.3) when a "guessed value" or "prior value" of π_A is available.

2. SHRINKAGE ESTIMATOR WITH KNOWN π_Y

In practice the experimenter often possesses some knowledge of the experimental conditions, based on familiarity with the behaviour of the system/population under consideration or from past experience or from some extraneous source, and is thus in a position to give an adequate guess or an initial estimate, say π_0, of the value of the population proportion π_A. Motivated by Thompson (1968), we suggest a shrinkage estimator for π_A as

$$\hat{\pi}_s = \delta\hat{\pi}_A + (1-\delta)\pi_0,\ 0 \le \delta \le 1 \tag{2.1}$$

where δ is a constant specified by the experimenter according to his belief on π_0. A value of π_0 near to zero implies a strong belief in π_0 and a value near to "1" implies a strong belief in sample estimate $\hat{\pi}_A$.

It is well known that $\hat{\theta}$ follows a binomial distribution with parameter n and θ. Thus the bias and mean squared error of $\hat{\pi}_s$ are respectively, given by

$$B(\hat{\pi}_s) = (1-\delta)(\pi_0 - \pi_A) \tag{2.2}$$

and

$$MSE(\hat{\pi}_s) = \delta^2\frac{\theta(1-\theta)}{np^2} + (1-\delta)^2(\pi_0 - \pi_A)^2 \tag{2.3}$$

It follows from (1.4) (or (1.5)) and (2.3) that the relative efficiency of $\hat{\pi}_s$ with respect to $\hat{\pi}_A$ is

$$RE(\hat{\pi}_s,\ \hat{\pi}_A) = \left[\delta^2 + \frac{np^2}{\theta(1-\theta)}(\pi_0 - \pi_A)^2(1-\delta)^2\right]^{-1} \tag{2.4}$$

which is greater than 'one' if

$$\frac{np^2}{\theta(1-\theta)}(\pi_0 - \pi_A)^2 \le \left\{\frac{(1+\delta)}{(1-\delta)}\right\} \tag{2.5}$$

The ranges of δ can be calculated for different values of n, p, π_A, π_Y and π_0 are their findings are displayed in Table 3.1.

The "optimum" value of δ which minimizes the MSE of $\hat{\pi}_s$ in (2.3) is given by

$$\delta = \frac{(\pi_0 - \pi_A)^2}{\left[(\pi_0 - \pi_A)^2 + \frac{\theta(1-\theta)}{np^2}\right]} \tag{2.6}$$

The value of δ depends on the unknown parameter π_A. Since π_0 is the guessed value or prior value of π_A, one may replace π_A by $\alpha\ \pi_0$, where α is a positive constant. Thus, the value of δ in (2.6) reduce to

$$\hat{\delta}_1 = \frac{(\alpha-1)^2\pi_0^2}{\left[(\alpha-1)^2\pi_0^2 + \frac{f_p(\pi_Y, \pi_0, \alpha)}{np^2}\right]} \tag{2.7}$$

where

$$f_p(\pi_Y, \pi_0, \alpha) = \{p\alpha\pi_0 + (1-p)\pi_Y\}\{1 - p\alpha\pi_0 - (1-p)\pi_Y\} \tag{2.8}$$

Substitution of (2.7) in (2.1) yields a shrinkage estimator for π_A as

$$\hat{\pi}_{s1} = \pi_0 + \frac{(\alpha-1)^2\pi_0^2(\hat{\pi}_A - \pi_0)}{\left[(\alpha-1)^2\pi_0^2 + \frac{1}{n}\left\{\alpha\pi_0(1-\alpha\ \pi_0) + \frac{(1-p)}{p^2}K_p(\pi_Y, \pi_0, \alpha)\right\}\right]} \tag{2.9}$$

where

$$K_p(\pi_Y, \pi_0, \alpha) = [p(1-2\pi_Y)\alpha\pi_0 + \pi_Y\{1-(1-p)\pi_Y\}] \tag{2.10}$$

The bias and MSE of $\hat{\pi}_{s1}$ are respectively given by

$$B(\hat{\pi}_{s1}) = \frac{\left[\alpha\pi_0(1-\alpha\ \pi_0)+\left\{(1-p)/p^2\right\}K_p(\pi_Y,\pi_0,\alpha)\right](\pi_A-\pi_0)}{\left[n(\alpha-1)^2\pi_0^2+\left\{\alpha\pi_0(1-\alpha\pi_0)+\frac{(1-p)}{p^2}K_p(\pi_Y,\pi_0,\alpha)\right\}\right]} \quad (2.11)$$

and $\text{MSE}(\hat{\pi}_{s1})=\dfrac{M}{D}$ (2.12)

where

$$M=\left[(\alpha-1)^4\pi_0^4\left\{\theta(1-\theta)/p^2\right\}+\left\{\alpha\pi_0(1-\alpha\pi_0)+\left\{(1-p)/p^2\right\}K_p(\pi_Y,\pi_0,\alpha)\right\}^2\left\{(\pi_0-\pi_A)^2/n\right\}\right]$$

and

$$D=n\left[(\alpha-1)^2\pi_0^2+\frac{1}{n}\left\{\alpha\pi_0(1-\alpha\pi_0)+\frac{(1-p)}{p^2}K_p(\pi_Y,\pi_0,\alpha)\right\}\right]^2$$

The relative efficiency of $\hat{\pi}_{s1}$ with respect to $\hat{\pi}_A$ is

$$\text{RE}(\hat{\pi}_{s1},\hat{\pi}_A)=\frac{D}{nU} \quad (2.13)$$

where

$$U=\left[(\alpha-1)^4\pi_0^4+\frac{p^2(\pi_0-\pi_A)^2}{\theta(1-\theta)n}\left\{\alpha\pi_0(1-\alpha\pi_0)+\frac{(1-p)}{p^2}K_p(\pi_Y,\pi_0,\alpha)\right\}^2\right]$$

which is greater than 'unity' if

$$\left[\alpha^2\pi_0^2\left\{\frac{(2n-1)}{n}+\frac{p^2(\pi_0-\pi_A)^2}{\theta(1-\theta)}\right\}+\alpha\pi_0\left\{\frac{1}{n}-\frac{p^2(\pi_0-\pi_A)^2}{\theta(1-\theta)}-4\pi_0\right\}+\left\{2\pi_0^2+\left(\frac{1-p}{p^2}\right)\left\{\frac{1}{n}-\frac{p^2(\pi_0-\pi_A)^2}{\theta(1-\theta)}\right\}K_p(\pi_Y,\pi_0,\alpha)\right\}\right]>0 \quad (2.14)$$

The ranges of α can be calculated for which $\text{RE}(\hat{\pi}_{s1},\hat{\pi}_A)$ is greater than one. For different values of n, p, π_A, π_Y and π_0 are calculated and displayed in Table 3.3.

3. NUMERICAL ILLUSTRATION AND CONCLUSIONS

For the sake of convenience and tangible idea about the performance of the proposed estimator $\hat{\pi}_s$ over conventional unbiased estimator $\hat{\pi}_A$, we have computed the ranges of (δ,α) and percent relative efficiency of $\hat{\pi}_s$ over $\hat{\pi}_A$ for different values of n = 5,10,50; p = 0.7; π_A = 0.05,0.10,0.25 π_Y = 0.05,0.10(0.30)0.70; δ = 0.25(0.25)0.90 $\pi_0=\pi_A/20,\pi_A/10,\pi_A/8,\pi_A/4(\pi_A/4)\pi_A$

It is observed from Table 3.1 that the proposed estimator $\hat{\pi}_s$ is better than the estimator $\hat{\pi}_A$ for full range of δ (i.e. $0<\delta\le 1$) when π_A and sample size n (≤ 10) are small. The range of δ decreases as (n, π_A, p) increase, while it increases as π_Y increases. It is also observed that when sample size n is large and the guessed value π_0 departs much from the true value, the range of dominance of δ becomes shorter.

The range of α for which the estimator $\hat{\pi}_s$ is better than $\hat{\pi}_A$ is given in Table 3.3.

From Table 3.2, we have made the following observations:

(i) The PRE decreases as sample size n increases and it is less than 100% for n = 50 and $0.10\le\pi_A\le 0.25$. The PRE increases as the guessed value π_0 increases towards π_A and it is maximum when $\pi_0=\pi_A$.

(ii) The proposed estimator $\hat{\pi}_s$ is always better than the unbiased estimator $\hat{\pi}_A$ when π_A is small (i.e. $\pi_A=0.05$).

(iii) The suggested estimator $\hat{\pi}_s$ is more efficient than $\hat{\pi}_A$ with substantial gain in efficiency when ¼ ≤ δ < 1 except in few cases particularly when the sample size is large (i.e. n = 50).

(iv) When δ approaches unity (i.e. $3/4 \le \delta < 1$), the estimator $\hat{\pi}_s$ is better than $\hat{\pi}_A$.

(v) For fixed (δ, p), the gain in efficiency increases as π_Y increases.

(vi) Larger gain in efficiency is observed for smaller sample sizes (i.e. $n \le 10$).

Thus, we see that there is enough scope of choosing δ in order to generate estimators better than $\hat{\pi}_A$ from $\hat{\pi}_s$. We also conclude that the estimator $\hat{\pi}_s$ is useful for small values of n and π_A even if the difference ($\pi_A - \pi_0$) (>0) is large. In practice such sample sizes are desirable when the survey procedure like RRT is much expensive. Even the estimator $\hat{\pi}_s$ is better than $\hat{\pi}_A$ for large values of n when $3/4 \le \delta < 1$.

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Table 3.1. The range of δ for which the estimator $\hat{\pi}_s$ is better than $\hat{\pi}_A$

$\pi_0 \downarrow$ n$\rightarrow$	P = 0.7, $\pi_Y = 0.05$								
	$\pi_A = 0.05$			$\pi_A = 0.10$			$\pi_A = 0.25$		
	5	10	50	5	10	50	5	10	50
$\pi_A/20$	0.00~1	0.00~1	0.08~1	0.00~1	0.00~1	0.48~1	0.00~1	0.28~1	0.80~1
$\pi_A/10$	0.00~1	0.00~1	0.02~1	0.00~1	0.00~1	0.44~1	0.00~1	0.23~1	0.78~1
$\pi_A/8$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.41~1	0.00~1	0.21~1	0.77~1
$\pi_A/4$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.28~1	0.00~1	0.06~1	0.70~1
$\pi_A/2$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.43~1
$3\pi_A/4$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
π_A	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_Y = 0.10$									
$\pi_A/20$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.42~1	0.00~1	0.26~1	0.79~1
$\pi_A/10$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.38~1	0.00~1	0.21~1	0.77~1
$\pi_A/8$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.35~1	0.00~1	0.18~1	0.76~1
$\pi_A/4$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.21~1	0.00~1	0.03~1	0.68~1
$\pi_A/2$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.40~1
$3\pi_A/4$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
π_A	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_Y = 0.40$									
$\pi_A/20$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.18~1	0.00~1	0.14~1	0.74~1
$\pi_A/10$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.13~1	0.00~1	0.09~1	0.71~1
$\pi_A/8$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.10~1	0.00~1	0.06~1	0.70~1
$\pi_A/4$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.61~1
$\pi_A/2$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.30~1
$3\pi_A/4$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
π_A	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_Y = 0.70$									
$\pi_A/20$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.05~1	0.00~1	0.08~1	0.71~1
$\pi_A/10$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.02~1	0.68~1
$\pi_A/8$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.66~1
$\pi_A/4$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.57~1
$\pi_A/2$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.24~1
$3\pi_A/4$	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
π_A	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1

Table 3.2. The Percent Relative Efficiencies of $\hat{\pi}_S$ with respect to $\hat{\pi}_A$

$\pi_0 \downarrow$	$\delta = 0.25$, $p = 0.7$, $\pi_Y = 0.05$								
	$\pi_A = 0.05$			$\pi_A = 0.10$			$\pi_A = 0.25$		
$n \rightarrow$	5	10	50	5	10	50	5	10	50
$\pi_A/20$	781.49	517.00	139.45	449.61	261.55	60.18	176.18	93.22	19.56
$\pi_A/10$	824.73	555.55	153.84	485.37	286.08	66.77	193.86	103.18	21.76
$\pi_A/8$	847.22	576.15	161.86	504.63	299.56	70.47	203.67	108.76	23.00
$\pi_A/4$	968.06	693.96	212.54	616.63	381.91	94.41	265.03	144.48	31.15
$\pi_A/2$	1240.18	1012.49	410.12	936.34	661.83	197.83	494.03	292.11	68.41
$3\pi_A/4$	1491.80	1397.30	927.36	1359.16	1181.35	577.21	1025.86	754.95	242.55
π_A	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00
	$\delta = 0.25$, $p = 0.7$, $\pi_Y = 0.10$								
$\pi_A/20$	879.80	606.70	174.18	498.27	295.08	69.23	185.37	98.38	20.69
$\pi_A/10$	922.35	647.93	191.69	536.10	322.00	76.76	203.84	108.85	23.02
$\pi_A/8$	944.26	669.77	201.40	556.37	336.73	80.98	214.07	114.71	24.34
$\pi_A/4$	1059.46	791.92	262.21	672.80	425.96	108.25	277.95	152.19	32.95
$\pi_A/2$	1304.25	1100.78	489.66	992.25	719.10	224.56	513.81	306.04	72.27
$3\pi_A/4$	1514.16	1437.07	1021.13	1387.53	1224.88	632.10	1046.78	777.83	254.57
π_A	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00
	$\delta = 0.25$, $p = 0.7$, $\pi_Y = 0.40$								
$\pi_A/20$	1159.55	909.25	333.44	697.76	446.17	114.86	229.22	123.45	26.31
$\pi_A/10$	1193.22	951.35	362.88	740.56	481.77	126.93	251.28	136.35	29.26
$\pi_A/8$	1210.07	972.95	378.93	763.01	500.96	133.67	263.45	143.54	30.93
$\pi_A/4$	1293.72	1085.86	475.14	885.97	612.59	176.62	338.46	189.24	41.80
$\pi_A/2$	1447.67	1321.83	779.65	1178.04	932.20	349.20	602.27	370.95	91.09
$3\pi_A/4$	1558.99	1520.03	1266.77	1468.50	1356.97	844.12	1131.42	875.13	311.19
π_A	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00
	$\delta = 0.25$, $p = 0.7$, $\pi_Y = 0.70$								
$\pi_A/20$	1260.88	1040.37	433.65	805.19	537.96	147.18	255.88	139.06	29.89
$\pi_A/10$	1288.88	1079.05	468.67	848.38	577.23	162.28	279.99	153.42	33.23
$\pi_A/8$	1302.76	1098.65	487.56	870.79	598.17	170.68	293.24	161.41	35.12
$\pi_A/4$	1370.29	1198.26	597.83	990.57	717.34	223.70	374.36	211.98	47.42
$\pi_A/2$	1489.06	1392.51	916.89	1256.44	1034.34	428.45	651.70	409.19	102.89
$3\pi_A/4$	1570.74	1542.54	1348.78	1497.62	1407.56	950.35	1173.21	926.17	344.99
π_A	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00	1600.00

Table 3.2 continued...

$\delta = 0.50$, $p = 0.7$, $\pi_Y = 0.05$									
$\pi_A/20$	358.30	324.48	184.86	311.45	255.01	104.09	210.75	143.07	40.08
$\pi_A/10$	362.17	330.88	195.65	318.68	264.84	112.63	221.49	153.15	44.15
$\pi_A/8$	364.06	334.04	201.28	322.27	269.84	117.24	227.04	158.51	46.42
$\pi_A/4$	372.95	349.32	231.84	339.79	295.34	144.30	256.46	188.74	60.64
$\pi_A/2$	387.51	375.77	302.49	370.80	345.57	223.77	320.32	267.12	114.70
$3\pi_A/4$	396.80	393.65	370.17	392.28	384.85	334.20	376.58	355.75	246.63
π_A	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00
$\delta = 0.50$, $p = 0.7$, $\pi_Y = 0.10$									
$\pi_A/20$	366.65	338.44	209.47	321.11	268.21	115.71	216.46	148.37	42.20
$\pi_A/10$	369.81	343.86	220.22	327.73	277.59	124.80	227.14	158.60	46.46
$\pi_A/8$	371.35	346.52	225.78	331.01	282.32	129.69	232.65	164.02	48.82
$\pi_A/4$	378.54	359.27	255.28	346.88	306.22	158.02	261.69	194.46	63.64
$\pi_A/2$	390.17	380.81	319.50	374.51	352.08	238.02	323.92	272.15	119.44
$3\pi_A/4$	397.50	395.02	376.30	393.31	386.84	341.84	377.81	357.96	252.01
π_A	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00
$\delta = 0.50$, $p = 0.7$, $\pi_Y = 0.40$									
$\pi_A/20$	383.80	368.86	281.28	349.75	310.72	164.15	240.32	171.75	52.32
$\pi_A/10$	385.40	371.83	290.11	354.31	317.99	174.71	250.57	182.42	57.44
$\pi_A/8$	386.17	373.27	294.54	356.54	321.60	180.28	255.80	188.02	60.27
$\pi_A/4$	389.75	380.01	316.69	367.13	339.24	211.03	282.86	218.78	77.80
$\pi_A/2$	395.38	390.86	358.13	384.69	370.51	286.13	337.82	292.37	140.81
$3\pi_A/4$	398.83	397.68	388.64	396.06	392.20	363.80	382.40	366.29	273.94
π_A	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00
$\delta = 0.50$, $p = 0.7$, $\pi_Y = 0.70$									
$\pi_A/20$	388.39	377.44	307.97	360.46	328.04	190.77	252.58	184.56	58.51
$\pi_A/10$	389.55	379.64	315.40	364.15	334.20	201.57	262.50	195.34	64.12
$\pi_A/8$	390.11	380.70	319.10	365.95	337.24	207.21	267.53	200.98	67.21
$\pi_A/4$	392.69	385.63	337.19	374.41	351.89	237.59	293.30	231.54	86.25
$\pi_A/2$	396.72	393.49	369.42	388.21	377.09	306.79	344.33	302.26	152.86
$3\pi_A/4$	399.17	398.35	391.89	396.98	394.01	371.76	384.46	370.08	284.86
π_A	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00

Table 3.2 continued...

$\delta = 0.75$, $p = 0.7$, $\pi_Y = 0.05$									
$\pi_A/20$	175.51	173.30	157.42	172.33	167.21	135.10	161.65	148.20	88.99
$\pi_A/10$	175.74	173.75	159.29	172.88	168.24	138.51	163.17	150.77	93.79
$\pi_A/8$	175.85	173.96	160.20	173.14	168.73	140.21	163.90	152.04	96.28
$\pi_A/4$	176.36	174.96	164.52	174.35	171.04	148.53	167.37	158.11	109.61
$\pi_A/2$	177.14	176.51	171.63	176.24	174.72	163.47	173.00	168.47	139.28
$3\pi_A/4$	177.62	177.46	176.20	177.39	177.00	173.97	176.56	175.35	166.29
π_A	177.78	177.78	177.78	177.78	177.78	177.78	177.78	177.78	177.78
$\delta = 0.75$, $p = 0.7$, $\pi_Y = 0.10$									
$\pi_A/20$	176.00	174.26	161.46	173.05	168.57	139.65	162.47	149.59	91.54
$\pi_A/10$	176.18	174.61	162.99	173.53	169.47	142.79	163.92	152.06	96.32
$\pi_A/8$	176.27	174.78	163.74	173.75	169.91	144.35	164.62	153.28	98.80
$\pi_A/4$	176.66	175.57	167.24	174.80	171.93	151.93	167.92	159.09	112.01
$\pi_A/2$	177.28	176.79	172.94	176.44	175.13	165.28	173.26	168.96	140.98
$3\pi_A/4$	177.65	177.53	176.54	177.44	177.11	174.48	176.63	175.49	166.89
π_A	177.78	177.78	177.78	177.78	177.78	177.78	177.78	177.78	177.78
$\delta = 0.75$, $p = 0.7$, $\pi_Y = 0.4$									
$\pi_A/20$	176.95	176.13	169.81	174.98	172.28	153.30	165.55	154.90	102.27
$\pi_A/10$	177.03	176.29	170.60	175.27	172.83	155.50	166.73	156.97	106.93
$\pi_A/8$	177.07	176.37	170.98	175.40	173.09	156.57	167.30	157.99	109.31
$\pi_A/4$	177.26	176.74	172.73	176.03	174.31	161.69	169.96	162.80	121.75
$\pi_A/2$	177.55	177.32	175.50	177.00	176.22	170.25	174.21	170.79	147.59
$3\pi_A/4$	177.72	177.66	177.20	177.58	177.39	175.83	176.87	175.98	169.13
π_A	177.78	177.78	177.78	177.78	177.78	177.78	177.78	177.78	177.78
$\delta = 0.75$, $p = 0.7$, $\pi_Y = 0.70$									
$\pi_A/20$	177.19	176.60	172.06	175.64	173.55	158.47	166.95	157.37	107.84
$\pi_A/10$	177.25	176.72	172.63	175.85	173.97	160.25	168.00	159.24	112.37
$\pi_A/8$	177.28	176.78	172.91	175.96	174.18	161.12	168.51	160.16	114.68
$\pi_A/4$	177.41	177.04	174.17	176.44	175.12	165.23	170.87	164.48	126.60
$\pi_A/2$	177.61	177.45	176.16	177.18	176.59	171.97	174.64	171.61	150.70
$3\pi_A/4$	177.74	177.70	177.37	177.63	177.48	176.29	176.98	176.20	170.14
π_A	177.78	177.78	177.78	177.78	177.78	177.78	177.78	177.78	177.78

Table 3.2 continued...

$\delta = 0.90$, $p = 0.7$, $\pi_Y = 0.05$									
$\pi_A/20$	123.28	123.10	121.71	123.02	122.60	119.27	122.10	120.78	111.14
$\pi_A/10$	123.30	123.14	121.89	123.07	122.68	119.69	122.24	121.05	112.28
$\pi_A/8$	123.31	123.16	121.97	123.09	122.73	119.89	122.31	121.18	112.84
$\pi_A/4$	123.35	123.24	122.36	123.19	122.92	120.81	122.61	121.77	115.48
$\pi_A/2$	123.41	123.36	122.97	123.34	123.22	122.27	123.08	122.70	119.78
$3\pi_A/4$	123.44	123.43	123.33	123.43	123.40	123.16	123.36	123.27	122.52
π_A	123.46	123.46	123.46	123.46	123.46	123.46	123.46	123.46	123.46
$\delta = 0.90$, $p = 0.7$, $\pi_Y = 0.10$									
$\pi_A/20$	123.32	123.18	122.09	123.08	122.71	119.82	122.18	120.92	111.76
$\pi_A/10$	123.33	123.21	122.22	123.12	122.79	120.19	122.31	121.18	112.85
$\pi_A/8$	123.34	123.22	122.29	123.14	122.82	120.36	122.37	121.30	113.39
$\pi_A/4$	123.37	123.28	122.60	123.22	122.99	121.17	122.66	121.87	115.89
$\pi_A/2$	123.42	123.38	123.07	123.35	123.25	122.43	123.10	122.74	119.98
$3\pi_A/4$	123.45	123.44	123.36	123.43	123.40	123.20	123.37	123.28	122.57
π_A	123.46	123.46	123.46	123.46	123.46	123.46	123.46	123.46	123.46
$\delta = 0.90$, $p = 0.7$, $\pi_Y = 0.40$									
$\pi_A/20$	123.39	123.33	122.82	123.24	123.02	121.31	122.45	121.46	114.10
$\pi_A/10$	123.40	123.34	122.88	123.26	123.06	121.52	122.55	121.67	114.99
$\pi_A/8$	123.40	123.35	122.91	123.27	123.09	121.63	122.60	121.76	115.42
$\pi_A/4$	123.42	123.38	123.06	123.32	123.18	122.11	122.83	122.21	117.45
$\pi_A/2$	123.44	123.42	123.28	123.40	123.34	122.85	123.18	122.90	120.71
$3\pi_A/4$	123.45	123.45	123.41	123.44	123.43	123.31	123.39	123.32	122.76
π_A	123.46	123.46	123.46	123.46	123.46	123.46	123.46	123.46	123.46
$\delta = 0.90$, $p = 0.7$, $\pi_Y = 0.70$									
$\pi_A/20$	123.41	123.37	123.00	123.29	123.12	121.81	122.57	121.70	115.16
$\pi_A/10$	123.42	123.38	123.05	123.31	123.16	121.97	122.66	121.88	115.96
$\pi_A/8$	123.42	123.38	123.07	123.32	123.17	122.05	122.71	121.97	116.34
$\pi_A/4$	123.43	123.40	123.17	123.35	123.25	122.42	122.90	122.36	118.15
$\pi_A/2$	123.44	123.43	123.33	123.41	123.36	123.00	123.21	122.97	121.04
$3\pi_A/4$	123.45	123.45	123.43	123.45	123.43	123.34	123.40	123.33	122.84
π_A	123.46	123.46	123.46	123.46	123.46	123.46	123.46	123.46	123.46

Table 3.3. The range of α for which the estimator $\hat{\pi}_s$ is better than $\hat{\pi}_A$

$\pi_0 \downarrow$ $n \rightarrow$	$\alpha = 0.25$, $p = 0.7$, $\pi_Y = 0.05$								
	$\pi_A = 0.05$			$\pi_A = 0.10$			$\pi_A = 0.25$		
	5	10	50	5	10	50	5	10	50
$\pi_A/20$	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/10$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/8$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/4$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/2$	0.01~1	0.00~1	0.00~1	0.01~1	0.01~1	0.00~1	0.03~1	0.02~1	0.02~1
$3\pi_A/4$	0.01~1	0.01~1	0.00~1	0.02~1	0.01~1	0.01~1	0.06~1	0.05~1	0.04~1
π_A	0.01~1	0.01~1	0.00~1	0.02~1	0.02~1	0.01~1	0.09~1	0.08~1	0.07~1
	$\alpha = 0.25$, $p = 0.7$, $\pi_Y = 0.10$								
$\pi_A/20$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/10$	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/8$	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/4$	0.01~1	0.01~1	0.00~1	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/2$	0.01~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1	0.03~1	0.02~1	0.01~1
$3\pi_A/4$	0.02~1	0.01~1	0.00~1	0.02~1	0.01~1	0.01~1	0.06~1	0.05~1	0.04~1
π_A	0.02~1	0.01~1	0.00~1	0.03~1	0.02~1	0.01~1	0.10~1	0.08~1	0.07~1
	$\alpha = 0.25$, $p = 0.7$, $\pi_Y = 0.40$								
$\pi_A/20$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/10$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.02~1	0.00~1	0.00~1
$\pi_A/8$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.02~1	0.00~1	0.00~1
$\pi_A/4$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.03~1	0.01~1	0.00~1
$\pi_A/2$	0.04~1	0.02~1	0.00~1	0.05~1	0.02~1	0.01~1	0.06~1	0.03~1	0.01~1
$3\pi_A/4$	0.05~1	0.02~1	0.01~1	0.05~1	0.03~1	0.01~1	0.09~1	0.06~1	0.04~1
π_A	0.05~1	0.03~1	0.01~1	0.06~1	0.04~1	0.02~1	0.13~1	0.10~1	0.08~1
	$\alpha = 0.25$, $p = 0.7$, $\pi_Y = 0.70$								
$\pi_A/20$	0.07~1	0.03~1	0.00~1	0.06~1	0.03~1	0.00~1	0.03~1	0.00~1	0.00~1
$\pi_A/10$	0.07~1	0.03~1	0.00~1	0.06~1	0.03~1	0.00~1	0.03~1	0.00~1	0.00~1
$\pi_A/8$	0.07~1	0.03~1	0.01~1	0.06~1	0.03~1	0.00~1	0.04~1	0.00~1	0.00~1
$\pi_A/4$	0.07~1	0.03~1	0.01~1	0.06~1	0.03~1	0.00~1	0.05~1	0.01~1	0.00~1
$\pi_A/2$	0.07~1	0.03~1	0.01~1	0.07~1	0.04~1	0.01~1	0.08~1	0.04~1	0.01~1
$3\pi_A/4$	0.07~1	0.04~1	0.01~1	0.08~1	0.04~1	0.01~1	0.11~1	0.07~1	0.04~1
π_A	0.07~1	0.04~1	0.01~1	0.08~1	0.05~1	0.02~1	0.15~1	0.11~1	0.08~1

Table 3.3 continued...

$\alpha = 0.50$, $p = 0.7$, $\pi_Y = 0.05$									
$\pi_A/20$	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/10$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/8$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/4$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/2$	0.01~1	0.00~1	0.00~1	0.01~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1
$3\pi_A/4$	0.01~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1	0.05~1	0.03~1	0.02~1
π_A	0.01~1	0.01~1	0.00~1	0.02~1	0.01~1	0.01~1	0.07~1	0.05~1	0.04~1
$\alpha = 0.50$, $p = 0.7$, $\pi_Y = 0.10$									
$\pi_A/20$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/10$	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/8$	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/4$	0.01~1	0.01~1	0.00~1	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/2$	0.02~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1	0.03~1	0.02~1	0.00~1
$3\pi_A/4$	0.02~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1	0.05~1	0.03~1	0.02~1
π_A	0.02~1	0.01~1	0.00~1	0.03~1	0.02~1	0.01~1	0.07~1	0.05~1	0.04~1
$\alpha = 0.50$, $p = 0.7$, $\pi_Y = 0.40$									
$\pi_A/20$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/10$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.02~1	0.00~1	0.00~1
$\pi_A/8$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.02~1	0.00~1	0.00~1
$\pi_A/4$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.03~1	0.01~1	0.00~1
$\pi_A/2$	0.05~1	0.02~1	0.00~1	0.05~1	0.02~1	0.00~1	0.05~1	0.03~1	0.00~1
$3\pi_A/4$	0.05~1	0.02~1	0.01~1	0.05~1	0.03~1	0.01~1	0.08~1	0.05~1	0.02~1
π_A	0.05~1	0.03~1	0.01~1	0.06~1	0.03~1	0.01~1	0.10~1	0.06~1	0.04~1
$\alpha = 0.50$, $p = 0.7$, $\pi_Y = 0.70$									
$\pi_A/20$	0.07~1	0.03~1	0.00~1	0.06~1	0.03~1	0.00~1	0.03~1	0.00~1	0.00~1
$\pi_A/10$	0.07~1	0.03~1	0.00~1	0.06~1	0.03~1	0.00~1	0.03~1	0.00~1	0.00~1
$\pi_A/8$	0.07~1	0.03~1	0.01~1	0.06~1	0.03~1	0.00~1	0.04~1	0.00~1	0.00~1
$\pi_A/4$	0.07~1	0.03~1	0.01~1	0.07~1	0.03~1	0.00~1	0.05~1	0.01~1	0.00~1
$\pi_A/2$	0.07~1	0.03~1	0.01~1	0.07~1	0.03~1	0.01~1	0.07~1	0.03~1	0.00~1
$3\pi_A/4$	0.07~1	0.04~1	0.01~1	0.08~1	0.04~1	0.01~1	0.10~1	0.06~1	0.02~1
π_A	0.07~1	0.04~1	0.01~1	0.08~1	0.04~1	0.01~1	0.12~1	0.07~1	0.04~1
π_A	0.09~1	0.05~1	0.01~1	0.10~1	0.05~1	0.01~1	0.13~1	0.08~1	0.04~1

Table 3.3 continued...

$\alpha = 0.75$, $p = 0.7$, $\pi_Y = 0.05$									
$\pi_A/20$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/10$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/8$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/4$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/2$	0.01~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1
$3\pi_A/4$	0.01~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1	0.04~1	0.02~1	0.01~1
π_A	0.02~1	0.01~1	0.00~1	0.03~1	0.01~1	0.00~1	0.06~1	0.03~1	0.01~1
$\alpha = 0.75$, $p = 0.7$, $\pi_Y = 0.10$									
$\pi_A/20$	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/10$	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/8$	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/4$	0.01~1	0.01~1	0.00~1	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/2$	0.02~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1	0.03~1	0.01~1	0.00~1
$3\pi_A/4$	0.02~1	0.01~1	0.00~1	0.03~1	0.01~1	0.00~1	0.05~1	0.02~1	0.01~1
π_A	0.02~1	0.01~1	0.00~1	0.03~1	0.02~1	0.00~1	0.06~1	0.04~1	0.01~1
$\alpha = 0.75$, $p = 0.7$, $\pi_Y = 0.4$									
$\pi_A/20$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.02~1	0.00~1	0.00~1
$\pi_A/10$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.02~1	0.00~1	0.00~1
$\pi_A/8$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.02~1	0.00~1	0.00~1
$\pi_A/4$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.03~1	0.01~1	0.00~1
$\pi_A/2$	0.05~1	0.02~1	0.00~1	0.05~1	0.02~1	0.00~1	0.05~1	0.02~1	0.00~1
$3\pi_A/4$	0.05~1	0.02~1	0.00~1	0.05~1	0.03~1	0.01~1	0.07~1	0.04~1	0.01~1
π_A	0.05~1	0.03~1	0.01~1	0.06~1	0.03~1	0.01~1	0.08~1	0.05~1	0.02~1
$\alpha = 0.75$, $p = 0.7$, $\pi_Y = 0.70$									
$\pi_A/20$	0.07~1	0.03~1	0.00~1	0.06~1	0.03~1	0.00~1	0.03~1	0.00~1	0.00~1
$\pi_A/10$	0.07~1	0.03~1	0.01~1	0.06~1	0.03~1	0.00~1	0.03~1	0.00~1	0.00~1
$\pi_A/8$	0.07~1	0.03~1	0.01~1	0.06~1	0.03~1	0.00~1	0.04~1	0.00~1	0.00~1
$\pi_A/4$	0.07~1	0.03~1	0.01~1	0.07~1	0.03~1	0.00~1	0.05~1	0.01~1	0.00~1
$\pi_A/2$	0.07~1	0.03~1	0.01~1	0.07~1	0.03~1	0.01~1	0.07~1	0.03~1	0.00~1
$3\pi_A/4$	0.07~1	0.04~1	0.01~1	0.08~1	0.04~1	0.01~1	0.09~1	0.04~1	0.01~1
π_A	0.07~1	0.04~1	0.01~1	0.08~1	0.04~1	0.01~1	0.10~1	0.05~1	0.02~1

Table 3.3 continued...

$\alpha = 0.90$, $p = 0.7$, $\pi_Y = 0.05$									
$\pi_A/20$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/10$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/8$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/4$	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/2$	0.01~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1	0.03~1	0.01~1	0.00~1
$3\pi_A/4$	0.02~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1	0.04~1	0.02~1	0.00~1
π_A	0.02~1	0.01~1	0.00~1	0.03~1	0.01~1	0.00~1	0.06~1	0.03~1	0.01~1
$\alpha = 0.90$, $p = 0.7$, $\pi_Y = 0.10$									
$\pi_A/20$	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/10$	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1	0.00~1	0.00~1	0.00~1
$\pi_A/8$	0.01~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/4$	0.01~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1	0.01~1	0.00~1	0.00~1
$\pi_A/2$	0.02~1	0.01~1	0.00~1	0.02~1	0.01~1	0.00~1	0.03~1	0.01~1	0.00~1
$3\pi_A/4$	0.02~1	0.01~1	0.00~1	0.03~1	0.01~1	0.00~1	0.05~1	0.02~1	0.00~1
π_A	0.02~1	0.01~1	0.00~1	0.03~1	0.02~1	0.00~1	0.06~1	0.03~1	0.01~1
$\alpha = 0.90$, $p = 0.7$, $\pi_Y = 0.40$									
$\pi_A/20$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.02~1	0.00~1	0.00~1
$\pi_A/10$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.02~1	0.00~1	0.00~1
$\pi_A/8$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.02~1	0.00~1	0.00~1
$\pi_A/4$	0.04~1	0.02~1	0.00~1	0.04~1	0.02~1	0.00~1	0.03~1	0.00~1	0.00~1
$\pi_A/2$	0.05~1	0.02~1	0.00~1	0.05~1	0.02~1	0.00~1	0.05~1	0.02~1	0.00~1
$3\pi_A/4$	0.05~1	0.02~1	0.00~1	0.06~1	0.03~1	0.01~1	0.07~1	0.03~1	0.00~1
π_A	0.05~1	0.03~1	0.01~1	0.06~1	0.03~1	0.01~1	0.08~1	0.04~1	0.01~1
$\alpha = 0.90$, $p = 0.7$, $\pi_Y = 0.70$									
$\pi_A/20$	0.07~1	0.03~1	0.00~1	0.06~1	0.03~1	0.00~1	0.03~1	0.00~1	0.00~1
$\pi_A/10$	0.07~1	0.03~1	0.01~1	0.06~1	0.03~1	0.00~1	0.03~1	0.00~1	0.00~1
$\pi_A/8$	0.07~1	0.03~1	0.01~1	0.06~1	0.03~1	0.00~1	0.04~1	0.00~1	0.00~1
$\pi_A/4$	0.07~1	0.03~1	0.01~1	0.07~1	0.03~1	0.00~1	0.05~1	0.01~1	0.00~1
$\pi_A/2$	0.07~1	0.04~1	0.01~1	0.07~1	0.04~1	0.01~1	0.07~1	0.03~1	0.00~1
$3\pi_A/4$	0.07~1	0.04~1	0.01~1	0.08~1	0.04~1	0.01~1	0.09~1	0.04~1	0.01~1
π_A	0.07~1	0.04~1	0.01~1	0.08~1	0.04~1	0.01~1	0.10~1	0.05~1	0.01~1