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Maximum Estimation Capacity for Three Level Combined Arrays

M.L. Aggarwal, S. Roy Chowdhury[1], Anita Bansal[2], Neena Mital[3]
Deptt. of Mathematical Sciences, University of Memphis, TN 38152, USA
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SUMMARY

Chen *et al.* (1999) introduced the concept of maximum estimation capacity for two-level fractional factorial designs. Wu and Hamada (2000) compared minimum aberration designs and maximum estimation capacity designs for two-level combined arrays. We study the concept of maximum estimation capacity for 27-run and 81-run three-level combined array based on the number of eligible and clear estimable main effects and two-factor interaction effects.

Key Words : Minimum aberration, Resolution, Estimation index, Control factors, Noise factors.

1. INTRODUCTION

Box and Hunter (1961) introduced the concept of resolution for distinguishing fractional factorial designs. Fries and Hunter (1980) introduced the concept of minimum aberration for distinguishing two designs having same resolution. Franklin (1984), Deng and Tang (1999), Tang and Deng (1999), Xu and Wu (2001) and Ke and Tang (2003) discussed minimum aberration for two levels regular, non-regular and asymmetrical fractional factorial designs. Chen *et al.* (1999) introduced the concept of maximum estimation capacity for 16-run and 32-run designs. Taguchi (1959, 1987) introduced the concept of robust parameter design for improving the quality of the product and process by reducing the effects of noise induced variations. An excellent review on Taguchi's methodology is given by Ankenman and Dean (2003). The basic idea in robust design is to design an experiment that allows us to identify the settings of the control factors that make the product or process insensitive to the effects of the noise factors. Welch *et al.* (1990), Shoemaker *et al.* (1991) and Montgomery (1991) independently proposed the concept of combined array where the factors are divided into control and noise factors.

1. *Lady Shri Ram College for Women, University of Delhi, Delhi*
2. *Ramjas College, University of Delhi, Delhi*
3. *Ram Lal Anand College, University of Delhi, Delhi*

In agricultural experiments; growth and development of plants are primarily governed by the environmental conditions such as climate, which are treated as noise factors and can not be controlled. Thus the major problem is to study and analyze how these noise factors interact with other control factors like seed quality, irrigation, temperature etc. Combined array is a good approach to study these control × noise interactions in lesser number of runs. Wu and Hamada (2000) made a remark that minimum aberration criterion cannot be used for choosing good combined arrays; one should choose a design with a large number of clear control × noise interactions, control main effects, noise main effects and control × noise interactions. It may be very well dealt in the concept of maximum estimation capacity as it takes care of the aliasing pattern. They have compared the properties of minimum aberration designs and maximum estimation capacity designs for two-level combined arrays based on the number of eligible and clear estimable main effects and two-factor interactions. Evangelaras *et al.* have given a complete catalogue of non-isomorphic two-level designs under the criteria of generalized minimum aberration and maximum estimation capacity. So far maximum estimation capacity has been studied for two-level fractional factorial designs under the concept of combined array. In agricultural experiments there are situations when three levels of the factors can be maintained or experimenter is concerned about the curvature in the response function for which three-level fractional factorial designs may be useful.

In this paper we study the concept of maximum estimation capacity for three-level combined array based on the number of eligible and clear estimable main effects and two-factor interactions. This work is an extension of the concept of estimation capacity given by Wu and Hamada (2000) on two-level fractional factorial designs. A catalogue for 3^{n+m-p} combined array designs (for 27-run design $n+m=4,5,...,10$ and for 81-run design $n+m=5,6,...,9$) has been developed under the criterion of maximum estimation capacity based on the number of eligible and clear estimable main effects and two-factor interactions.

2. MAXIMUM ESTIMATION CAPACITY CRITERION

Chen *et al.* (1999) introduced the criterion of model robustness known as maximum estimation capacity as

For any $1 \le k \le {}^nC_2$, the estimation capacity $E_k(D)$ of a 2^{n-p} design D is defined as the total number of models jointly estimating all the main effects and k two-factor interactions. Mathematically

$$E_k(D) = \sum_{1 \le i_1 < ... < i_k \le f} \prod_{j=1}^{k} m_{ij}(D) \qquad \text{if } k \le f$$

where $f = 2^{n-p} - 1 - n$ is the number of alias sets that do not contain main effects. For a 2^{n-p} design D of resolution III or higher, $m_i(D), i=1,2,...,f$ is the number of two-factor interactions in the i^{th} alias set. A design that maximizes $E_k(D)$ is said to have maximum estimation capacity. Chen *et al.* (1999) also showed that a design D^* has large estimation capacity if it maximizes $\sum_{i=1}^{f} m_i(D^*)$, and $m_i(D^*)$'s as uniform as possible. They also showed that minimum aberration criterion is a good surrogate for maximum estimation capacity but they are not the same. They considered 16 and 32 runs regular two-level fractional factorial designs. Cheng and Mukerjee (1998) studied regular fractional factorial designs with minimum aberration and maximum estimation capacity. Later Mukerjee *et al.* (2000) discussed regular fractions of mixed factorials with maximum estimation capacity. Wu and Hamada (2000) compared the properties of minimum aberration and estimation capacity for two-level combined arrays based on the number of eligible and clear estimable main effects and two-factor interactions under the assumption that noise-by-noise and higher order interactions are negligible. A main effect or two-factor interaction is said to be eligible if it is not aliased with any other main effects. Eligibility is a weaker property than clear estimation because it ensures that an effect is estimable only if its two-factor interaction aliases are negligible. In robust parameter design the effect ordering principle for combined array is in the following order

(i) Control-by-noise interactions (C × N), control main effects and noise main effects

(ii) Control-by-control interactions (C × C)

(iii) Noise-by- noise interactions (N × N) and higher order interactions

The required model to estimate all the cases stated in (i) and (ii) above is of the following form

$$y = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{j=1}^{m} b_j z_j + \sum_{i=1}^{n} \sum_{j=1}^{m} b_{ij} x_i z_j + \sum_{i=1}^{n} \sum_{j>1}^{m} \gamma_{ij} x_i x_j + \varepsilon$$

In the above model $x_i (i=1,2,...,n)$ is a control factor, $z_j (j=1,2,...,m)$ is a noise factor where n + m factors influence the response in an experiment out of which 'n' are control factors and 'm' are noise factors. $x_i z_j$ is the C × N interaction and $x_i x_j$ is the C × C interaction.

Wu and Hamada (2000) considered a minimum aberration resolution IV, 2^{6-2} design for explaining the importance of estimation capacity criterion over minimum aberration. They considered three control factors A, B and C and three noise factors d, e and f with the defining relation $I = ABCd = ABef = defC$. Here all main effects are clear; all the 15 two-factor interactions are eligible but not clear and are divided into seven groups of aliased effects as follows

AB = Cd = ef   BC = Ad   Af = Be   AC = Bd
Ae = Bf   Ce = df   Cf = de

whereas, in resolution III, 2^{6-2} design with the defining relation I = ABCd = def = ABCef, the effects A, B, C, Ae, Af, Be, Bf, Ce, Cf are clear; d, e, f are eligible effects; and has the following three groups of aliased effects

Ad = BC   Bd = AC   Cd = AB

In this case there are six eligible two-factor interaction effects in three pairs of aliased effects. Even though it has resolution III, its estimation capacity is superior to the previous resolution IV minimum aberration design. Also in this design there are six clear C × N two-factor interactions whereas, in resolution IV design there is not even a single clear C × N two-factor interaction. In the case of combined array the C × N interaction have more importance than C × C interaction and the least important effects are N × N interactions. Thus one should choose a design with a large number of clear C × N interactions, control main effects, noise main effects and C × C interactions. Wu and Hamada (2000) studied combined arrays for 2^{n-p} fractional factorial designs.

Chen and Cheng (2004) introduced the concept of estimation index for two-level fractional factorial designs. For a two-level fractional factorial design 'D' there are $f = 2^{n-p} - 1$ mutually exclusive alias sets. Let $\rho_i(D)$ be the length of the shortest word in the i^{th} alias set, $i = 1, 2, ..., f$ then the estimation index of D is $\rho(D) = \max\{\rho(D) : i = 1, 2, ..., f\}$ under the assumption that higher order interactions are negligible. When $\rho(D) = 2$, it means all degrees of freedom not aliased with main effects are used for estimating two-factor interactions. If $\rho(D) > 2$ then we can add more factors in the columns where no main effect or two-factor interaction are aliased.

We redefine mutually exclusive alias sets by $f = (3^{n+m-p} - 1)/2$ (where there are 'n' control factors and 'm' noise factors) for defining estimation, as in three-level orthogonal array designs each two-factor interaction between two-factors A and B are represented by two components AB and AB^2.

3. ALGORITHM TO DEVELOP 3^{n+m-p} COMBINED ARRAY DESIGNS UNDER THE CRITERION OF MAXIMUM ESTIMATION CAPACITY

For developing the designs we have considered the Yates order for standard three-level factor allocation as shown in Table 1.

Table 1 represents the standard allocation for 81-run design. In case of 27-run design, one may consider the first 13 columns of the table. The interaction between columns C_i and C_j are obtained through C_iC_j and $C_i(C_j)^2$. For easy reference, we provide the interaction table for $L_{81}(3^{40})$ in Appendix III. The following sections describe the algorithm for developing combined array designs under the concept of maximum estimation capacity. We have considered 27-run and 81-run designs to develop 3^{n+m-p} combined arrays, as large number of runs/ trials are discouraged in practical situations. But, the algorithm is generalized and can be used for higher run designs.

3.1 Maximum Estimation Capacity for 27-Run 3^{n+m-p} Combined Array Designs

In case of 27-run designs we have considered the designs for $n + m = 4, 5, ..., 10$; $p = 1, 2, ..., 7$ with $m = 1, 2, ..., 5$ noise factors. First allocate three independent factors in column 1, 2 and 5 as per the standard allocation. Then in order to allocate remaining $n+m-3$ factors take all possible combinations in the remaining 10 columns, which give all possible

Table 1. Standard allocation of factors

Col.	1	2	3	4	5	6	7	8
	a	b	ab	ab^2	c	ac	bc	abc
Col.	9	10	11	12	13	14	15	16
	ab^2c	ac^2	bc^2	abc^2	ab^2c^2	d	ad	bd
Col.	17	18	19	20	21	22	23	24
	abd	ab^2d	cd	acd	bcd	abcd	ab^2cd	ac^2d
Col.	25	26	27	28	29	30	31	32
	bc^2d	abc^2d	ab^2c^2d	ad^2	bd^2	abd^2	ab^2d^2	cd^2
Col.	33	34	35	36	37	38	39	40
	acd^2	bcd^2	$abcd^2$	ab^2cd^2	ac^2d^2	bc^2d^2	abc^2d^2	$ab^2c^2d^2$

allocations of n+m factors. For each design calculate $m_i(D), i = 1, 2, ..., f$ where $f = \{(3^{n+m-p} - 1)/2 - (n+m)\}$: mutually exclusive alias sets that do not contain main effects. Next in order to search for non-isomorphic designs arrange $m_i(D)$'s in descending order for each design. Select all designs with unique $m_i(D)$'s. We select designs on the basis of following two criteria for working on maximum estimation capacity problem.

(a) Designs that maximizes $\sum_{i=1}^{f} m_i(D)$ i.e. the design which has maximum number of estimable two-factor interactions. It has been observed that there are number of such designs. We further screen these designs on the basis of maximum number of clear two-factor interaction effects.

(b) Next we choose designs that have maximum number of clear two-factor interaction. Again it has been observed that there are number of such designs. We further screen these designs on the basis of one that maximizes $\sum_{i=1}^{f} m_i(D)$.

Based on the above criteria we get a set of selected designs. In order to develop designs under combined array we divide the factors further into 'n' control factors and 'm' noise factors. For each selected design consider all possible allocations of control and noise factors. Corresponding to each allocation of control and noise factors construct the alias table and count the number of eligible two-factor interactions. Out of the selected set of two-factor interactions, count the number of clear $C \times N$ and clear $C \times C$ interaction effects. Under the assumption that all control and noise main effects are eligible/clear, select those designs that has maximum number of clear two-factor interactions and maximum number of clear $C \times N$ interactions. Appendix I give the selected list of designs with maximum estimation capacity. All the selected designs have estimation index 2 which means that all degrees of freedom which are not aliased with main effects are used for estimating two-factor interaction effects.

3.2 Maximum Estimation Capacity for 81-Run 3^{n+m-p} Combined Array Designs

We consider here 3^{n+m-p} designs for n + m = 5, 6, ..., 9; p = 1, 2, ..., 5 and m = 1, 2, ..., 4 noise factors. In the case of 81-run design there are four independent factors which according to standard allocation of Table 1 are allocated in columns 1, 2, 5 and 14. Then in order to allocate remaining n + m – 4 factors, take all possible combinations in the remaining 36 columns, which give all possible allocations of n + m factors. After this we follow the same steps as described above in 27-run designs to obtain the maximum estimation capacity designs. Appendix II gives the selected list of 81- run designs with maximum estimation capacity. All selected 3^{8-4} and 3^{9-5} designs have estimation index 2. We consider the following example to explain the concept.

Example

Consider, 81-run 3^{7-3} design with 4 control factors and 3 noise factors. There are 7 main effects and 42 two-factor interaction effects. The number of columns out of 40 that can be used for estimating two-factor interactions are $f = ((3^{n+m-p} - 1)/2) - (n+m) = ((3^{4+3-3} - 1)/2) - (4+3) = 33$. Allocate four independent factors in column 1, 2, 5 and 14 as per the standard allocation. To allocate next three factors in the remaining 36 columns, take all possible ${}^{36}C_3$ combinations which give all possible allocations of seven factors. For each of the ${}^{36}C_3$ designs calculate $m_i(D)$. To search for non-isomorphic designs, first arrange $m_i(D)$ in descending order then select all designs with unique $m_i(D)$'s. We further select those designs that maximize $\sum_{i=1}^{f} m_i(D)$ i.e. the design which has maximum number of estimable two-factor interactions along with maximum number of clear two-factor interaction effects according to the criteria (a). Next divide the factors into control and noise factors. Select one of the designs based on criterion (a) and allocate three noise factors. This gives ${}^{7}C_3$ combinations. Corresponding to each allocation of control and noise factors we construct the alias table and count the number of eligible two-factor interactions. Out of 35 combinations there are 5 unique combinations as shown in Table 2. For these 5 unique designs, count the number of clear $C \times N$ and clear $C \times C$ interaction effects under the assumption that all control and noise main effects are eligible/clear.

Following the criteria of estimation capacity given by Hamada and Wu (2000), it can be observed that design number 5 of Table 2 has maximum number of clear two-factor interactions along with maximum number of clear $C \times N$ two-factor interactions. This 3^{7-3} designs has been listed in Appendix II.

Table 2. Allocation of control and noise factors under criterion (a)

Design No.	Control Factors Allocation				Noise Factors Allocation			Eligible 2 f_i's	Clear C × N	Clear C × C
1.	8	14	17	20	1	2	5	36	11	7
2.	5	14	17	20	1	2	8	36	11	5
3.	5	8	14	17	1	2	20	36	10	6
4.	2	5	14	20	1	8	17	36	12	6
5.	1	8	17	20	2	5	14	36	12	12

Table 3. Allocation of control and noise factors under criterion (b)

Design No.	Control factors allocation				Noise factors allocation			Eligible 2 f_i's	Clear C × N	Clear C × C
1.	8	14	22	35	1	2	5	24	24	0
2.	5	14	22	35	1	2	8	24	18	6
3.	5	8	22	35	1	2	14	24	14	6
4.	2	5	22	35	1	8	14	26	12	10
5.	2	5	8	35	1	14	22	26	13	7
6.	1	2	5	35	8	14	22	30	15	9
7.	1	2	5	8	14	22	35	30	18	6

Similarly, design selected on the basis of criterion (b) gives 7 unique combinations of control and noise allocation as shown in Table 3. We select design number 1 and 7 on the basis of maximum number of clear two-factor interactions along with maximum number of clear C × N two-factor interactions. These 3^{7-3} designs are listed in Appendix II. Further, we prefer design number 7 over design number 2 as numbers of eligible two-factor interactions are much larger in former design.

4. CONCLUDING REMARKS

In many industrial experiments there are situations where all the factors are not controllable and are treated as noise factors. In the case of combined array the C × N interaction have more importance than C × C interaction. The combined array designs developed in this paper, on the basis of maximum estimation criterion allows the estimation on maximum number of estimable two-factor interactions and clear C × Ninteractions when factors are at three levels. The list of designs given in Appendix I and II facilitates the allocation of different number of control and noise factors which gives designs with maximum estimation capacity. Most of the selected designs have estimation index 2 which means that all degrees of freedom which are not aliased with main effects are used for estimating two-factor interaction effects.

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APPENDIX I

27-Run Designs Based on Maximum Estimation Capacity Criterion

Design	Control Factors Allocation	Noise Factors Allocation	Eligible 2 f.'s	Clear C × N	Clear C × C
3^{4-1}	2 5 8	1	12	3	3
	5 8	1 2	10	4	2
	8	1 2 5	6	6	0
3^{5-2}	1 2 3 9	5	17	1	0
	3 5 9	1 2	16	0	1
	2 3 9	1 5	15	1	2
	5 9	1 2 3	14	0	2
	3 9	1 2 5	12	3	2
	2 9	1 3 5	12	3	2
	2 3	1 5 9	11	4	0
3^{6-3}	2 3 5 9 13	1	24	0	0
	3 5 9 13	1 2	23	0	0
	2 5 9 13	1 3	23	0	0
	2 3 9 13	1 5	22	0	0
	5 9 13	1 2 3	21	0	0
	2 9 13	1 3 5	19	0	0
3^{7-4}	2 3 5 6 7 9	1	27	0	0
	1 3 5 6 7 9	2	27	0	0
	2 5 6 7 9	1 3	26	0	0
	3 5 6 7 9	1 2	26	0	0
	2 3 6 7 9	1 5	26	0	0
	2 3 5 6 9	1 7	25	0	0
	3 6 7 9	1 2 5	24	0	0
	5 6 7 9	1 2 3	24	0	0
	2 3 7 9	1 5 6	24	0	0
	2 6 7 9	1 3 5	23	0	0
3^{8-5}	2 3 5 6 7 9 12	1	32	0	0
	3 5 6 7 9 12	1 2	31	0	0
	2 3 5 6 9 12	1 7	30	0	0
	3 6 7 9 12	1 2 5	29	0	0
	5 6 7 9 12	1 2 3	29	0	0
	3 5 6 9 12	1 2 7	28	0	0
	6 7 9 12	1 2 3 5	26	0	0
	5 7 9 12	1 2 3 6	25	0	0
3^{9-6}	2 3 5 6 7 9 12 13	1	36	0	0
	3 5 6 7 9 12 13	1 2	35	0	0
	3 6 7 9 12 13	1 2 5	33	0	0
	5 6 7 9 12 13	1 2 3	33	0	0
	3 6 7 12 13	1 2 5 9	30	0	0
	6 7 9 12 13	1 2 3 5	30	0	0
3^{10-7}	2 3 4 5 6 7 8 9 11	1	27	0	0
	2 4 5 6 7 8 9 11	1 3	27	0	0
	3 4 5 6 7 8 9 11	1 2	27	0	0
	2 5 6 7 8 9 11	1 3 4	27	0	0
	4 5 6 7 8 9 11	1 2 3	27	0	0
	5 6 7 8 9 11	1 2 3 4	27	0	0

APPENDIX II

81-Run Designs Based on Maximum Estimation Capacity Criterion

Design	Control Factors Allocation	Noise Factors Allocation	Eligible 2 f_i's	Clear C × N	Clear C × C
3^{5-1}	2 5 14 22	1	20	8	12
	5 14 22	1 2	18	12	6
	14 22	1 2 5	14	12	2
3^{6-2}	2 5 14 22 23	1	27	10	14
	1 5 14 22 23	2	27	6	18
	1 5 14 23	2 22	26	12	12
	1 5 14	2 22 23	24	18	6
	2 5 8 14 18	1	30	4	14
	1 2 8 14 18	5	30	7	11
	5 8 14 18	1 2	28	8	12
	1 2 14 18	5 8	28	12	6
	8 14 18	1 2 5	24	14	6
	2 14 18	1 5 8	24	15	3
3^{7-3}	2 5 8 14 17 20	1	42	6	12
	1 8 14 17 20	2 5	40	8	12
	1 8 17 20	2 5 14	36	12	12
	2 5 8 14 22 35	1	30	9	15
	5 8 14 22 35	1 2	28	16	8
	1 2 5 8 35	14 22	30	12	12
	1 2 5 22 35	8 14	30	9	15
	1 2 5 8	14 22 35	30	18	6
	8 14 22 35	1 2 5	24	24	0
3^{8-4}	2 5 8 14 17 20 40	1	56	4	12
	2 5 14 17 20 40	1 8	54	8	10
	2 5 14 20 40	1 8 17	50	12	10
	2 5 14 40	1 8 17 20	44	16	12
	1 2 5 14 21	8 17 20	50	15	7
	8 14 22 35	1 2 5 18	32	32	0
	1 2 5 12 31 36 40	14	43	6	15
	1 2 12 14 36 40	5 31	43	10	11
	1 2 5 31 36 40	12 14	41	11	12
	1 2 12 14 40	5 31 36	43	15	6
	1 5 31 36 40	2 12 14	37	19	8
	2 5 31 36 40	1 12 14	37	17	8
	1 5 14 31 36 40	1 2 12	37	19	8
	5 31 36 40	1 2 12 14	31	31	0
	1 2 12 14	5 31 36 40	43	20	1
3^{9-5}	2 5 8 14 20 25 27 31	1	72	0	1
	1 2 5 8 14 25 27 31	20	72	1	0

Design	Control Factors Allocation	Noise Factors Allocation	Eligible 2 fi's	Clear C × N	Clear C × C
3^{9-5}	2 5 8 14 20 25 31	1 27	70	0	3
	2 5 8 14 25 27 31	1 20	70	1	1
	5 8 14 20 27 31	1 2 25	66	2	3
	5 8 14 20 27	1 2 25 31	60	6	2
	8 14 20 27 31	1 2 5 25	60	5	3
	1 2 5 14 20 21 31 38	30	66	4	8
	1 14 20 21 30 31 38	2 5	64	4	10
	1 2 14 21 30 31 38	5 20	64	8	5
	1 2 5 20 21 38	14 30 31	61	12	2
	2 5 20 21 31 38	1 14 30	60	10	5
	1 14 21 30 31 38	2 5 20	60	8	9
	5 20 21 31 38	1 2 14 30	55	10	7
	1 14 30 31 38	2 5 20 21	55	12	5
	2 5 20 30 31	1 14 21 38	55	9	9
	1 14 21 30 31	2 5 20 38	55	12	6
	1 5 20 21 38	2 14 30 31	57	13	2
	2 5 8 14 20 26 31 34	1	70	3	3
	5 8 14 20 26 31 34	1 2	68	6	1
	2 5 8 14 20 26 34	1 31	68	3	5
	5 8 14 20 26 34	1 2 31	64	8	3
	2 5 8 20 26 34	1 14 31	64	5	5
	5 8 14 20 26	1 2 31 34	59	10	5
	5 8 14 20 34	1 2 26 31	58	12	2
	1 2 5 8 26	14 20 31 34	60	6	4

Interaction Table for L_{81} (3^{40}) Based on Yates Order

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
(1)	3	4	3	6	10	8	13	12	6	12	9	8	15	28	17	31	30	20	37	22	40	39	33	26	36	35	15	30	18	17	33	24	35	27	26	20	39	23	22
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	(2)	4	1	7	8	11	9	6	12	7	13	10	16	17	29	18	15	21	22	38	23	20	26	34	27	24	30	16	31	28	34	35	25	36	33	39	21	40	37
		1	3	11	9	5	6	8	13	5	10	12	29	18	14	15	17	38	23	19	20	22	27	32	24	26	31	14	28	30	25	36	32	33	35	40	19	37	39
		(3)	1	8	13	9	12	10	9	13	8	6	17	31	18	30	28	22	40	23	39	37	36	27	35	33	18	31	17	15	35	27	36	26	24	23	40	22	20
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			(4)	9	12	6	10	13	8	10	6	9	18	30	15	28	31	23	39	20	37	40	35	24	33	36	17	28	15	18	36	26	33	24	27	22	37	20	23
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						(7)	13	10	3	2	4	1	21	22	38	23	20	25	26	34	27	24	17	29	18	15	35	25	36	33	38	39	21	40	37	30	16	31	28
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								(9)	3	1	1	4	23	39	20	37	40	27	35	24	33	36	30	15	28	31	26	33	24	27	40	22	37	20	23	17	28	15	18
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									(10)	8	13	12	24	33	26	36	35	15	28	17	31	30	37	22	40	39	20	39	23	22	28	15	30	18	17	24	35	27	26
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														1	2	3	4	5	6	7	8	9	10	11	12	13	15	16	17	18	19	20	21	22	23	24	25	26	27
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															(16)	31	28	34	35	25	36	33	39	21	40	37	3	2	4	1	7	8	11	9	6	12	7	13	10
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																	(18)	36	26	33	24	27	22	37	20	23	3	1	1	4	9	12	6	10	13	8	10	6	9
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																								4	2	2	19	40	21	38	33	32	36	25	34	14	31	16	29
																								(25)	36	33	12	7	13	10	2	3	2	4	1	8	11	9	6
																									1	3	23	19	20	22	34	27	32	24	26	18	14	15	17
																									(26)	24	9	13	8	6	3	4	4	3	1	13	9	12	10
																										2	38	37	19	21	35	34	33	32	25	29	28	14	16
																										(27)	8	10	6	9	4	3	1	1	4	12	6	10	13
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																												4	2	2	10	5	13	7	11	5	9	11	7
																												(29)	18	15	21	22	38	23	20	26	34	27	24
																													1	3	11	9	5	6	8	13	5	10	12
																													(30)	28	22	40	23	39	37	36	27	35	33
																														2	12	11	10	5	7	7	6	5	11
																														(31)	23	39	20	37	40	35	24	33	36
																															13	7	12	11	5	11	8	7	5
																															(32)	24	25	26	27	15	16	17	18
																																1	2	3	4	6	7	8	9
																																(33)	26	36	35	28	17	31	30
																																	4	2	2	5	13	7	11
																																	(34)	27	24	17	29	18	15
																																		1	3	9	5	6	8
																																		(35)	33	31	18	30	28
																																			2	11	10	5	7
																																			(36)	30	15	28	31
																																				7	12	11	5
																																				(37)	22	40	39
																																					4	2	2
																																					(38)	23	20
																																						1	3
																																						(39)	37
																																							2