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Nonlinear Time-Series Modelling: A Mixture-ARCH Approach

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SUMMARY

In the class of nonlinear time-series models, mixture models may be employed to describe those data sets that depict sudden bursts, outliers and flat stretches at irregular time epochs. In this paper, mixture autoregressive conditional heteroscedastic (MAR-ARCH) model is thoroughly studied. As an illustration, weekly wholesale onion price data during April, 1998 to November, 2001 is considered. After eliminating trend, seasonal fluctuations are studied by fitting Box-Jenkins airline model to residual series where appropriate filters are determined by using OCSB auxiliary regression. Presence of ARCH is examined by applying Lagrange multiplier (LM) test. Estimation of parameters is carried out using Expectation Maximization (EM) algorithm and the best model is selected on the basis of Bayesian Information Criterion (BIC). Relevant computer programs for estimation of parameters along with their standard errors and computation of BIC are developed in MATLAB, version 5.3. It is concluded that, for data under consideration, a two-component MAR-ARCH is the best in the class of MAR-ARCH family. Further, identified MAR-ARCH model is also shown to perform better than three-component MAR model identified earlier in terms of having fewer numbers of parameters and lower BIC value.

Key words : MAR-ARCH model, EM algorithm, Airline model, Bayesian information criterion, Naive approach, Forecasting.

1. INTRODUCTION

Linear Gaussian time-series models have dominated development of time-series model building for past seven decades or so. However, these are not able to describe volatility due to clustering of outliers, which is present in many real data sets. To handle such a situation, Engle (1982) introduced autoregressive conditional heteroscedastic (ARCH) models in which study of autocorrelation of squared residual series is required, unlike linear models. Ghosh and Prajneshu (2003) have shown that AR(p)-ARCH(q) model provided a good description to these types of data.

However, some other realistic features, such as flat stretches, bursts, occasional outliers and sharp changes can not be successfully tackled through ARCH approach. This results in conditional distributions to be multimodal, which is not so in a linear set up. To this end, *mixture* nonlinear time-series models, pioneered by Le *et al.* (1996), may be employed. A heartening feature of this family is that it encompasses well-known self-exciting threshold autoregressive (SETAR) family (Tong (1995)). Recently, Ghosh *et al.* (2006) have thoroughly studied two families of mixture nonlinear time-series models, *viz.* Gaussian mixture transition distribution (GMTD) models and Mixture autoregressive (MAR) models and compared their performance for modelling as well as forecasting of weekly wholesale onion price data. It is concluded that, for the data under consideration, MAR family is superior to GMTD family.

The purpose of the present paper is to study a more general family of nonlinear time-series models by combining MAR and ARCH families, along similar lines as Wong and Li (2001). The main advantage of doing this is that heteroscedastic nature of mixture components distribution can be captured which, in turn, allows one to consider more general structure of heteroscedastic conditional variance. Resultant model is then applied to weekly wholesale onion price data as an illustration. Estimation of parameters is carried out using EM algorithm and the best model is selected on the basis of Bayesian Information Criterion (BIC). One-step and two-step ahead predictive distributions are obtained using a naive approach. The distributions are found to be unimodal and bimodal when volatility function is respectively low and high. Properties of stationarity along with stability of the fitted model is studied. Both point and interval forecasts are carried out and it is shown that, for data set under consideration, two-component MAR-ARCH model performs best not only in MAR-ARCH family, but also among other mixture families.

2. MIXTURE AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTIC MODEL

For modelling of nonlinear time-series data, MAR model suffers from a limitation, analogous to that of AR model, that squared autocorrelation structure is not able to model intricacies of complex conditional variances satisfactorily. Accordingly, MAR model is generalized to MAR-ARCH model, which consists of a mixture of *K* autoregressive components with autoregressive conditional heteroscedasticity. The model is defined by

$$F\left(y_t \middle| y^{t-1}\right) = \sum_{k=1}^{K} \alpha_k \Phi\left[e_{k,t}(h_{k,t})^{-1/2}\right] \quad (1)$$

where $e_{k,t} = y_t - \phi_{k0} - \phi_{k1} y_{t-1} - \ldots - \phi_{kp_k}$, $h_{k,t} = \beta_{k0} + \beta_{k1} e^2{}_{k,t-1} + \ldots + \beta_{kp_k} e^2{}_{k,t-q_k}$ and is denoted as MAR-ARCH $(K; p_1, p_2, \ldots, p_k; q_1, q_2, \ldots, q_k)$. Here $F(y_t|y^{t-1})$ is conditional cumulative distribution function of Y_t given past information, evaluated at y_t, y^t is information up to time t; $\Phi(.)$ is (conditional) cumulative distribution function of Standard Gaussian distribution; $\alpha_1 + \ldots + \alpha_K = 1, \alpha_k > 0$ *(k=1, 2, ..., K)*. Let $p = \max(p_1, p_2, \ldots, p_k)$, $q = \max(q_1, q_2, \ldots, q_K)$ and $\phi(.)$ be the probability density function of a standard normal distribution. To avoid possibility of zero or negative conditional variance, following condition for β_{ki}'s must be imposed:

$$\beta_{k0} > 0, k = 1, 2, \ldots, K; \qquad \beta_{ki} \geq 0$$
$$i = 1, 2, \ldots, q_k, \qquad k = 1, 2, \ldots, K$$

Shape of conditional distribution of series changes over time as conditional means and variances of components, which depend on past values of time-series in different ways, differ. Conditional variance of y_t is given by

$$\mathrm{Var}(y_t|y^{t-1}) = \sum_{k=1}^{K} \alpha_k h_{k,t} + \sum_{k=1}^{K} \alpha_k \mu_{k,t}^2 - \left(\sum_{k=1}^{K} \alpha_k \mu_{k,t}\right)^2 \quad (2)$$

The first term allows modelling of dependence of conditional variance on past 'errors', while second and third terms model change of conditional variance due to difference in conditional means in the components.

3. ESTIMATION OF PARAMETERS

The EM algorithm, which is most readily available procedure for estimating mixture models, is employed for estimation of parameters. Suppose that observation $Y = (y_1, \ldots, y_n)$ is generated from MAR-ARCH model (1). Let $Z = (Z_1, \ldots, Z_n)$ be unobserved random variable, where Z_t is a K-dimensional vector with component k equal to one, if y_t comes from k^{th} component of conditional distribution function, and zero otherwise. Denote k^{th} component of Z_t as $Z_{k,t}$. Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_{K-1})$, $\theta_k = (\phi_{k0}, \phi_{k1}, \ldots, \phi_{kp_k})'$, $\beta_k = (\beta_{k0}, \beta_{k1}, \ldots, \beta_{kq_k})'$, k=1, 2, ..., K and $\theta = (\alpha', \theta_1', \beta_1', \ldots, \theta_k', \beta_k')'$ where $'$ denotes transpose of a vector or a matrix. The (conditional) log-likelihood is given by

$$l = \sum_{t=p+q+1}^{n} l_t = \sum_{t=p+q+1}^{n} \left\{ \sum_{k=1}^{K} Z_{k,t} \log(\alpha_k) - \sum_{k=1}^{K} (Z_{k,t}/2) \log(h_{k,t}) - \sum_{k=1}^{K} (Z_{k,t} e_{k,t}^2 / 2_{hk,t}) \right\} \quad (3)$$

where N = n – p – q. First order derivatives of log-likelihood with respect to θ were derived by Wong and Li (2001). Iterative EM procedure estimates parameters by maximizing log-likelihood function (3). It comprises an E-step and an M-step described as follows :

E-step: Suppose that θ is known. The missing data Z are replaced by their conditional expectation, conditional on the parameters and on observed data Y. In this case, conditional expectation of k^{th} component of Z_t is just conditional probability that observation Y_t comes from k^{th} component of mixture distribution, conditional on θ and Y. Let $\tau_{k,t}$ be conditional expectation of $Z_{k,t}$. Then E-step equations are:

$$\tau_{k,t} = \alpha_k (h_{k,t})^{-1/2} \phi[e_{k,t}(h_{k,t})^{-1/2}] \left[\sum_{t=1}^{K} \alpha_\ell (h_{\ell,t})^{-1/2} \phi\left[e_{\ell,t}(h_{\ell,t})^{-1/2}\right]\right]^{-1} \quad (4)$$

M-step: Suppose missing data are known. The estimates of parameters θ can be obtained by maximizing log likelihood θ. This can be done on replacing $Z_{k,t}$ by $\tau_{k,t}$ in first order derivatives of log-likelihood (3). The parameter estimates of α are

$$\hat{\alpha} = 1/(n-p-q) \sum_{t=p+q+1}^{n} \tau_{k,t}; k = 1,\ldots,K \qquad (5)$$

Newton-Raphson method is used for parameter estimates of θ_k's and β_k's. Starting with initial values $\theta_k^{(0)}$ and $\beta_k^{(0)}$, values of θ_k and β_k in subsequent iterations are given by

$$\theta_k^{(i+1)} = \theta_k^{(i)} + \left\{ \partial^2 \ell / \partial^2 \theta_k \Big|_{\theta^{(i)}\beta^{(i)}} \right\} \frac{\partial \ell}{\partial \theta_k}\Big|_{\theta^{(i)}\beta^{(i)}} \qquad (6)$$

and

$$\beta_k^{(i+1)} = \beta_k^{(i)} + \left\{ \partial^2 \ell / \partial^2 \beta_k \Big|_{\theta^{(i+1)}\beta^{(i)}} \right\} \frac{\partial \ell}{\partial \beta_k}\Big|_{\theta^{(i+1)}\beta^{(i)}} \qquad (7)$$

where $\theta_k^{(i)}$ and $\beta_k^{(i)}$ are values in i^{th} iteration. The parameter estimates $\hat{\theta}_k$ and $\hat{\beta}_k$ in a particular M-step are obtained by iterating (6) and (7) until convergence is achieved. Final estimates of parameter vector θ are obtained by iterating E-steps and M-steps until convergence is achieved. The standard errors of parameter estimates can be computed by Missing information principle (Louis (1982)). The observed information matrix, I, can be computed from complete information matrix, I_c, and missing information matrix, I_m, with the relation:

$$I = I_c - I_m = E\left(-N \frac{\partial^2 \ell}{\partial \theta^2}\Big| \theta, Y \right)_{\hat{\theta}} - \mathrm{Var}\left(N \frac{\partial \ell}{\partial \theta}\Big| \theta, Y \right)_{\hat{\theta}} \qquad (8)$$

The formulae for computing I_c and I_m are given in Wong (1998). The dispersion matrix of estimates $\hat{\theta}$ is given by inverse of observed information matrix, I. Since there is no software package available for execution of EM algorithm and for estimating information matrices, relevant computer programs are developed in MATLAB, 5.3. To save space, salient parts of the same are appended as Annexure – I. However, entire listing can be obtained from the first author on request.

4. STABILITY OF FITTED MAR-ARCH MODEL AND FORECASTING IN HOLD-OUT-DATA

A number of tests related to breakpoint in structural change in the relationship examine whether parameters of best-fitted model based on BIC are stable across various subsamples of data. The idea of Chow breakpoint test is to fit the model separately for each subsample and to see whether there are significant differences in estimated equations. A significant difference indicates a structural change in the relationship. The F-statistic is based on comparison of restricted and unrestricted sum of squared residuals. In the simplest case involving a single breakpoint, F-statistic is computed as

$$F = \left[\left\{ \tilde{\mathbf{u}}'\tilde{\mathbf{u}} - (\mathbf{u}'_1\mathbf{u}_1 + \mathbf{u}'_2\mathbf{u}_2) \right\} \Big/ k \right] \Big/ \left[\left\{ \mathbf{u}'_1\mathbf{u}_1 + \mathbf{u}'_1\mathbf{u}_1 \right\} \Big/ (T-2k) \right] \qquad (9)$$

where $\tilde{\mathbf{u}}'\tilde{\mathbf{u}}$ is restricted sum of squared residuals, $\mathbf{u}'_i\mathbf{u}$ is sum of squared residuals from subsample i, T is total number of observations and k is the number of parameters in the model.

Before carrying out forecasts based on fitted MAR-ARCH model, Chow forecast test is employed by estimating parameters of the model for a subsample comprising first T_1 observations. This fitted model is then used to predict values of remaining T_2 hold-out data. The test for stability of model related to forecasting is carried out by F- statistic defined by

$$F = \left\{ (\tilde{\mathbf{u}}'\tilde{\mathbf{u}}' - \mathbf{u}'\mathbf{u}) \big/ T_2 \right\} \Big/ \left\{ \mathbf{u}'\mathbf{u} / (T_1 - k \right\} \qquad (10)$$

where $\tilde{\mathbf{u}}'\tilde{\mathbf{u}}$ is residual sum of squares when model is fitted to all T observations, $\mathbf{u}'\mathbf{u}$ is residual sum of squares when fitted to T_1 observations and k is number of estimated parameters.

5. AN ILLUSTRATION

Weekly wholesale onion price data of Nasik variety at Azadpur Mandi, New Delhi during the period from first week of April, 1998 to first week of November, 2001 collected from N.A.F.E.D., New Delhi is considered. It showed marked volatility by touching value of Rs. 4000 per quintal in October, 1998 and remained stable in the range of Rs. 450 to Rs. 700, depicting flat stretches with occasional bursts of large amplitude to the tune of Rs. 850 to Rs. 900, during October, 1999. In subsequent years, price remained on the average of Rs.350 in first half and above Rs. 500 for second half exhibiting another phase of flat stretches.

5.1 Fitting of Trend and Seasonal Airline Model

Formal test procedure for testing presence of stochastic trend in case of integrated process is based on the model

$$y_t = \mu + \beta t + \rho y_{t-1} + e_t^* \quad (11)$$

where e_t^* is a stationary process with mean zero and variance σ^2. The null hypothesis that $\rho = 1$ is based on the statistic analogous to the regression statistic $\hat{\tau}_\tau$ and is given by -1.84, which is not significant at 5% level. Substituting $\rho = 1$, in (11), the model to be considered reduces to

$$\Delta_1 y_t = \mu + \beta t + e_t^* \quad (12)$$

Regressing $\Delta_1 y_t$ on linear trend, fitted model is

$$\Delta_1 y_t = 6.06 - 0.04t + \hat{e}_t^* \quad (13)$$

Box-Jenkins approach to model seasonal time-series data is to apply various differencing filters Δ_k to y_t and investigating Estimated Autocorrelation Functions (EACF) of transformed series. Usually differencing is applied until EACF shows an easily interpretable pattern with only a few significant autocorrelations. We consider EACF of residual time-series $\{\hat{e}_t^*\}$ and compare it with

Table 1 : Estimated autocorrelation functions of weekly onion price data

Lags	e_t	$\Delta_1 e_t$	$(\Delta_1 e_t)^c$	$\Delta_1\Delta_{12}e_t$
1	1.00**	0.31**	0.78**	0.84**
2	0.74**	-0.38**	0.02	0.01
3	0.71**	-0.04	0.10	0.06
4	0.64**	-0.11	0.56**	0.54**
5	0.53**	0.56**	0.35**	-0.09
6	0.61**	-0.53**	0.64**	-0.06
7	-0.54**	0.04	-0.56**	-0.19
8	0.61**	-0.54**	0.74**	0.17
9	-0.53**	-0.58**	0.52**	0.13
10	-0.51**	0.64**	0.16	0.54**
11	-0.55**	0.25**	0.02	0.87**
12	-0.65**	0.51**	0.32**	0.79**
13	0.73**	0.59**	-0.24**	0.89**

** Significant at 5% level

autocorrelation function for seasonal ARMA type models. For seasonal time-series, however, we should also consider the correlation around s, 2s,... where s denotes number of seasons per year. Onion is grown in the country in four seasons, namely, July – September, October – December, January – March and April – June. Hence in our case s =4. The four relevant EACF's are presented in Table 1. The significant EACF values of $\Delta_1\Delta_{12}e_t$ at 1,11,12 and 13 suggests parsimonious model structures, known as 'Box-Jenkins airline model', given by

$$\Delta_t \Delta_s \hat{e}_t^* = (1+\theta_1 L)(1+\theta_s L^s)\varepsilon_t,\ t = s+2, s+3,\ldots \quad (14)$$

For given data, estimates of parameters θ_1 and θ_2 are obtained as -0.17 and 0.15 respectively.

5.2 Testing for ARCH

One method of testing for ARCH in $\hat{\varepsilon}_t$ in fitted model (14) is based on TR^2, where R^2 is obtained from fitting a regression of squared residuals, $\hat{\eta}_t$ (obtained after fitting $\hat{\varepsilon}_t$ on its conditional mean μ_t) on a constant and p of its lags. Assuming the conditional mean is correctly specified, Engle (1982) shows that TR^2 is asymptotically equivalent to a Lagrange multiplier (LM) test and is distributed asymptotically as a $\chi^2(p)$ random variable under null hypothesis. In our case, TR^2 and LM values are computed as 14.87 and 26.47 respectively, which are significant at 5% level. Lumsdaine and Ng (1999) suggested 'naive' approach which approximates unknown conditional mean in a better way by computing 'recursive residuals' containing true conditional mean not captured by regression function. The final model is

$$\hat{e}_t = Z_t' r + g(\hat{w}_{t-1}) + v_t \quad (15)$$

where Z_t is a vector of lagged values of $\hat{e}_t$, $g(\hat{w}_{t-1})$ is a (possibly nonlinear) function of recursive residuals $\hat{w}_{t-1}$. The quantity $\hat{v}_t^2$ is used for testing ARCH effects through TR^2 and LM statistics and their values are computed as 7.40 and 20.01 respectively, and are found to be significant at 5% level.

5.3 Fitting of MAR-ARCH Model

We consider two-component and three-component MAR-ARCH models for detrended and deseasonalised weekly onion price series. The order selection criterion followed here is Bayesian Information Criterion (BIC) as, unlike other criteria, viz. Akaike Information Criterion and Final Prediction Error, it leads to a consistent order selection (Fan and Yao (2003)). The best two-component MAR-ARCH model, defined by model (1) with $\phi_{ko} = 0, k = 1,2$ is found to be MAR-ARCH (2;0,1; 1,1) having BIC value of 307.98. The model is given by

$$F(\hat{e}_t \mid \hat{e}_{t-1}) = 0.75\Phi\left\{e_{1,t}/\sqrt{h_{1,t}}\right\} + 0.24\Phi\left\{e_{2,t}/\sqrt{h_{2,t}}\right\} \tag{16}$$

where $e_{1,t} = e_t, h_{1,t} = 0.14 + 0.38e^2_{1,t-1}, e_{2,t} = e_t + 0.84e_{t-1}$ and $h_{2,t} = 1.61 + 1.54\, e^2_{2,t-1}$. The standard errors for $(\hat{\alpha}_1, \hat{\alpha}_2, \hat{\phi}_{21}, \hat{\beta}_{10}, \hat{\beta}_{11}, \hat{\beta}_{20}, \hat{\beta}_{21})$ are (0.08, 0.03, 0.29, 0.04, 0.16, 0.61, 0.83) respectively. It is observed that occasional outliers in time-period July, 1998 and August, 1998, October, 1999 and November, 2001 are captured by large $h_{2,t}$ with small $\hat{\alpha}_2$ and bursts can be accommodated with a larger $\hat{\alpha}_1$ during October, 1998. Flat stretches during April, 1999 to June, 1999 are captured by low value of $\mathrm{Var}(y_t|y^{t-1})$. The best three-component model is a MAR-ARCH (3;1,1,0;2,0,1) with $\phi_{k0} = 0$, k = 1,2,3 having BIC value of 442.05. The parameter estimates and standard errors of $(\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\phi}_{11}, \hat{\phi}_{21}, \hat{\beta}_{10}, \hat{\beta}_{11}, \hat{\beta}_{12}, \hat{\beta}_{20}, \hat{\beta}_{30}, \hat{\beta}_{31})$ are (0.44, 0.48, 0.09, 0.41, -0.27, 0.13, 0.37, 1.35, 0.19, 5.54, 0.42) and (0.11, 0.13, 0.20, 0.21, 0.04, 0.09, 0.26, 0.91, 0.05, 3.56, 1.13) respectively. Wong and Li (2000) have pointed out that another related criterion, viz. BIC* may have better performance regarding selection criterion of number of components. According to this criterion also, best MAR-ARCH model is found to be MAR-ARCH (2;0,1;1,1). It is noticed that best MAR-ARCH(2;0,1;1,1) model satisfies first and second order stationary conditions, given in Wong and Li (2001). The fitted MAR-ARCH(2;0,1;1,1) model along with data points and residuals is depicted in Fig.1.

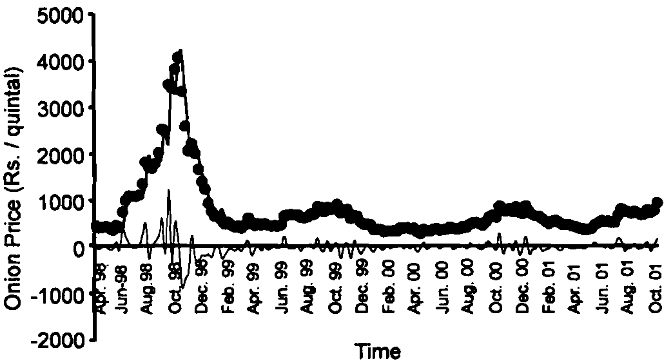

Fig. 1 : Fitted MAR-ARCH (2; 0, 1; 1,1) model along with actual data points and error series

As discussed in introduction, we study one-step and two-step predictive distributions from fitted two-component MAR-ARCH model. One-step predictive distributions, $F(\hat{\varepsilon}_t \mid \Im'_{t-1})$ for the series at time t = 31 and t = 108 for fitted MAR-ARCH model are shown in Figs. 2(a) and 3(a). Similarly two-step predictive distributions are obtained by 'naive' approach (Granger

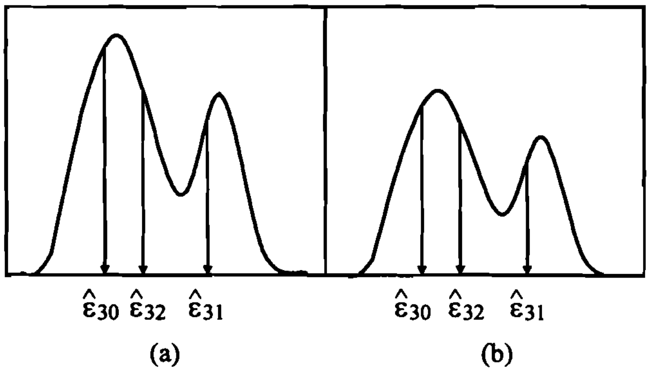

Fig. 2 : (a) One-step, and (b) two-step predictive distributions of $\{\hat{\varepsilon}_t\}$ series at t = 30 along with actual values of series at t – 1, t and t +1

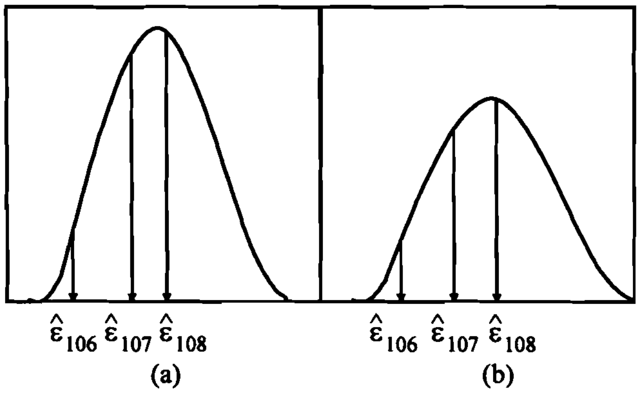

Fig. 3 : (a) One-step, and (b) two-step predictive distributions of $\{\hat{\varepsilon}_t\}$ series at t = 107 along with actual values of series at t – 1, t and t +1

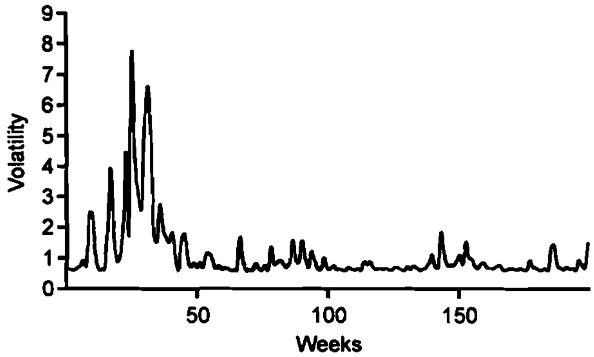

Fig. 4 : Volatility computed from fitted MAR-ARCH (2;0,1;1,1) model in seasonally adjusted onion price series

and Terasvirta (1993)) and exhibited in Figs. 2(b) and 3(b). For t = 31, distribution is bimodal while for t = 108, it is unimodal. This is justified by the fact that volatility function obtained from eq. (2) shows high volatility during time t = 31 whereas it is low during t = 108, as shown in Fig.4.

5.4 Test of ARCH in Post Model Fitting

McLeod and Li (1983) proposed a portmanteau test for linearity based on squared residuals after fitting the model. This approach is motivated by the result that if process $\{Z_t\}$ is a stationary Gaussian time-series, then

$$\rho_\tau(Z_t^2) = \{\rho_\tau(Z_t)\}^2,\ \rho_\tau(Z_t) = \mathrm{Corr}(Z_t, Z_{t+\tau}) \quad (17)$$

If residual series is having ARCH, where squared errors are correlated (even though errors are uncorrelated), McLeod and Li test statistic, analogous to Ljung-Box portmanteau test statistic, is

$$Q = N^*(N^*+2)\sum_{k=1}^{m} r_k^2/(N^*-k) \quad (18)$$

detects nonlinearity, where r_k is sample autocorrelation function of squared residuals, N* is the number of observations after fitting MAR-ARCH model. Comparative studies by Lukkonnen *et al.* (1988) suggest that McLeod and Li test is of value when testing linearity against ARCH-type alternatives. In our case, value of statistic comes out to be 21.30 for fitted MAR-ARCH model. Comparison with χ^2 tabulated value shows that null hypothesis of absence of ARCH is rejected at 5% level.

5.5 Cross-Validation of Fitted Model

The cross validation of fitted MAR-ARCH (2;0,1;1,1) model is carried out using eqs. (9) and (10). It is found that during third week of October, 1998 to second week of November, 1998, there are bursts in the data. The F-statistic based on eq. (9) reveals that null hypothesis that, price behaviour in pre- and post-November, 1998 period remain unaltered, can not be rejected at 5% level. So, fitted model is stable over entire time-period. Stability of the model for forecasting purposes is examined by predicting three hold-out-data during 3rd week of October, 2001 to 1st week of November, 2001 using eq. (16). Forecast values of wholesale onion price along with actual values in brackets for these weeks are respectively Rs. 805.42 (775.33), Rs. 749.83 (787.50) and Rs. 762.83 (750.00). Thus mean absolute percentage error came out as 2.32%. Formal tests based on *F*-statistic in eq. (10) which compares between residual sum of squares when equation is fitted to all observations and residual sum of squares when equation is fitted to first subsample reveals that forecast error is insignificant. In case of interval forecast, volatility values for these weeks are computed as Rs. 88.31, Rs. 83.06 and Rs. 122.06 respectively. This clearly shows that actual values for all three weeks lie within intervals of corresponding forecast values plus/minus respective volatility values.

To sum up, it is concluded that two-component MAR-ARCH model provides an excellent description of data under consideration.

6. CONCLUDING REMARKS

In this paper, we consider mixture nonlinear time-series models for analyzing onion price data. It is shown that two-component mixture autoregressive conditional heteroscedastic model performed better than other mixture models for modelling as well as forecasting purposes. Generally, in studying price phenomenon over time, standardization of prices prevailing during various time-epochs with respect to some standard price index is required. However, in the present instance, as per discussions with subject matter specialists it was decided that there was no need to do this for the present data set. It is hoped that other research workers, in future, would also apply these models to their data sets having sudden bursts, flat stretches and outliers. Work is in progress to investigate another extension of MAR nonlinear time-series models, called LMARX models, which involve logistic mixture autoregressive process with exogenous variables, like market availability, and shall be reported elsewhere in due course. The advantages of LMARX model over other nonlinear time-series models include a wider range of shape-changing predictive distributions, ability to handle cycles and conditional heteroscedasticity.

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ANNEXURE-I

PROGRAM TO ESTIMATE PARAMETERS, S.E., BIC OF MAR-ARCH MODEL

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fpo=fopen('d:\mararch\mar arch\(3;1,1,0;2,0,1).xls','w');
fpi=fopen('d:\mararch\data.txt','r');
n=172;k=3; ip(1)=1;ip(2)=1;ip(3)=0;iq(1)=2;iq(2)=0;iq(3)=1;
ipmax=ip(1);
for i=2:k
if (ip(i)>ipmax)
ipmax=ip(i);
end (2 times)
iqmax=iq(1);
for i=2:k
if (iq(i)>iqmax)
iqmax=iq(i);
end (2 times)
for i=1:k
icord(i)=0.0;
for j=1:i
icord(i)=icord(i)+ip(j)+iq(j)+1;
end (2 times)
for iter=1:10
ipmax = ip(1);
for i=2:k
if (ip(i) > ipmax)
ipmax = ip(i);
end (2 times)
iqmax = iq(1);
for i=2:k
if (iq(i) > iqmax)
iqmax = iq(i);
end (2 times)
```

(i) E-STEP

```
for i=ipmax+iqmax+1:n
for j=1:k
com = y(i);
for jj=1:ip(j)
com = com - phat(j,jj)*y(i-jj);
end
ht = shat(j,1);
for jj=1:iq(j)
error = y(i-jj) ;
for jjj=1:ip(j)
error = error - phat(j,jjj)*y(i-jj-jjj);
end
ht = ht + shat(j,jj+1)*error^2.0;
end
h(j) = 1.0/sqrt(ht)*exp(-com^2.0/ht/2.0);
end
denom = 0.0;
for j=1:k
denom = denom + ahat(j)*h(j);
end
for j=1:k
z(i,j) = ahat(j)*h(j)/denom; end; end
```

(ii) M-STEP

```
for j=1:k
asn(j) = 0.0;asumd = 0.0;
end
for i=ipmax+iqmax+1:n
for j=1:k
asn(j) = asn(j) + z(i,j);asumd = asumd + z(i,j);
end (2 times)
for j=1:k
ahat1(j) = asn(j)/asumd;
end
```

(iii) COMPUTING OF INFORMATION MATRIX

```
for i=1:k-1+icord(k)
for j=1:k-1+icord(k)
dim(i,j) = 0.0;
end (2 times)
for k1=1:k-1
for l=1:k-1
if (k1 = l)
dic(k1,l) = (n-ipmax-iqmax)* (1.0/ahat(k1) + 1.0/ahat(k));
else
dic(k1,l) = (n-ipmax-iqmax)/ahat(k);
end (3 times)
for it=ipmax+iqmax+1:n
for k1=1:k
for i=1:ip(k1)
for j=1:ip(k1)
i1=k-1+icord(k1)-ip(k1)-iq(k1)-1+i;
i2=k-1+icord(k1)-ip(k1)-iq(k1)-1+j;
dic(i1,i2)=dic(i1,i2)+(it,k1)*(1/2/hhh(k1,it)^2*fih(k1,it,i)*
    fih(k1,it,j)+1/hhh(k1,it)*fie(k1,it,i)*fie(k1,it,j));
end (2 times)
for i=1:iq(k1)+1
for j=1:iq(k1)+1
i1 = k-1+icord(k1)-iq(k1)-1+i; i2 = k-1+icord(k1)-iq(k1)-1+j;
dic(i1,i2)=dic(i1,i2)+z(it,k1)/2/
    hhh(k1,it)^2*bih(k1,it,i)*bih(k1,it,j)
end (4 times)
idem = k-1+icord(k); vtheta=inv(dim);dl = 0.0;dlik=0.0;
for it=ipmax+iqmax+1:n
dlt = 0.0;lik = 0.0;
for i=1:k
dlt = dlt + ahat(i)/sqrt(hhh(i,it))* exp(-eee(i,it)^2.0/hhh(i,it)/2.0);
lik = lik + z(it,i)*log(ahat(i))- (z(it,i)/2.0)*log(hhh(i,it))-
    z(it,i)*eee(i,it)*eee(i,it)/(2*hhh(i,it)^2);
end
dl = dl +log(dlt); dlik = dlik + lik;
end
bic=-2.0*dlik + log((n-ipmax-iqmax))*(k-1+icord(k));
```

(iv) STANDARD ERRORS

```
for i=1:k-1+icord(k)
vse(i)=sqrt(vtheta(i,i));
end
vseakt=0.0;
 for i=1:k-1
 for j=1:k-1
 vseakt=vseakt+vtheta(i,j);
 end (2 times)
vseakt=sqrt(vseakt);
st=fclose(fpo);
```