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On Non-response in Sampling on Two Occasions

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SUMMARY

The problem of estimation of finite population mean in mail surveys for the current occasion in the context of sampling on two occasions is attempted when there is non-response on both the occasions. Estimators for the current occasion are derived as a particular case when there is non-response on first occasion and second occasion only. The results obtained are demonstrated with the help of an empirical study.

Key words : Repeat surveys, Mail surveys, Non-response.

1. Introduction

Jessen (1942), Tikkiwal (1953), Yates (1949), Patterson (1950), Eckler (1955) and Raj (1979), contributed towards the development of the theory of unbiased estimation of mean of characteristics in successive sampling. Hansen and Hurwitz (1994) suggested a technique of handling non-response in mail surveys. These surveys have the advantage that the data can be collected relatively inexpensively. However, non-response is a common problem with mail surveys. Cochran (1977) and more recently Fabian and Hyunshik (2000) extended the Hansen and Hurwitz technique to the case when besides the information on character under study information is also available on auxiliary character. In this article the theory for use of Hansen and Hurwitz technique for estimation of population mean for current occasion in the context of sampling on two occasions has been developed. The results obtained are demonstrated with the help of an empirical study.

2. Estimation of Population Mean for Current Occasion in the Presence of Non-response on Both the Occasions

Consider a finite population $\Omega = (U_1, U_2, \cdots, U_N)$ of 'N' identifiable units. Let $(x_i, y_i;\ i = 1, 2, 3, \cdots, N)$ be the values of the characteristic on first and second occasion respectively. We assume that the population can be divided into two classes, those who will respond at the first attempt and those who will not. Let the sizes of these two classes be N_1 and N_2 respectively. Let on the first occasion, schedules through mail are sent to 'n' units selected by simple random sampling. On the second occasion, a simple random sample of $m = n\lambda$ units is retained while an independent sample of $u = n\mu = n - m$ units is selected (unmatched with the first occasion). We assume that in the unmatched portion of the sample on the two occasions u_1 units respond and u_2 units do not. Similarly, in the matched portion m_1 units respond and m_2 units do not. Let m_{h_2} denote the size of the sub sample drawn from the non-response class from the matched portion of the sample on the two occasions for collecting information through personal interview. Similarly, denote by u_{h_2} the size of the sub sample drawn from the non-response class in the unmatched portion of the sample on two occasions. Also, let σ^2 and σ_2^2 respectively denote population variance and the population variance pertaining to the non-response class. Similarly, ρ and ρ_2 denote correlation between units belonging to the matched portion and the correlation between non-respondents belonging to the matched portion.

Let, $\bar{x}_m^*$ and $\bar{x}_u^*$ denote the Hansen and Hurwitz estimator for matched and unmatched portion of the sample on the first occasion. Let the corresponding estimator for the second occasion be denoted by $\bar{y}_m^*$ and $\bar{y}_u^*$. Thus, we have the following setup

1st occasion	$\bar{x}_u^*$	$\bar{x}_m^*$	
2nd occasion		$\bar{y}_m^*$	$\bar{y}_u^*$

where $$\bar{y}_m^* = \frac{m_1\bar{y}_{m_1} + m_2\bar{y}_{m_{h_2}}}{m}, \quad \bar{y}_u^* = \frac{u_1\bar{y}_{u_1} + u_2\bar{y}_{u_{h_2}}}{u}$$

$$\bar{x}_m^* = \frac{m_1\bar{x}_{m_1} + m_2\bar{x}_{m_{h_2}}}{m}, \quad \bar{x}_u^* = \frac{u_1\bar{x}_{u_1} + u_2\bar{x}_{u_{h_2}}}{u}$$

Consider the following estimator

$$\hat{T}_{12} = a\bar{x}_u^* + b\bar{x}_m^* + c\bar{y}_m^* + d\bar{y}_u^*$$

Unbiasedness of $\hat{T}_{12}$ implies

$$a + b = 0 \text{ and } c + d = 1$$

Substituting the value of a and d we obtain

$$\hat{T}_{12} = a\left(\bar{x}_u^* - \bar{x}_m^*\right) + c\bar{y}_m^* + (1-c)\bar{y}_u^*$$

If $\hat{T}_{12}$ is a minimum variance linear unbiased estimator (MVLUE), it must be uncorrelated with every zero function, Rao (1952). Imposing this condition on $\hat{T}_{12}$ we get

$$\mathrm{Cov}(\hat{T}_{12}, \bar{x}^*_m) = \mathrm{Cov}(\hat{T}_{12}, \bar{x}^*_u)$$

$$\mathrm{Cov}(\hat{T}_{12}, \bar{y}^*_m) = \mathrm{Cov}(\hat{T}_{12}, \bar{y}^*_u)$$

It can be easily seen that

$$\mathrm{Cov}\left(\bar{x}^*_m, \bar{x}^*_u\right) = \mathrm{Cov}\left(\bar{x}^*_m, \bar{y}^*_u\right) = \mathrm{Cov}\left(\bar{y}^*_m, \bar{x}^*_u\right)$$

$$= \mathrm{Cov}\left(\bar{y}^*_m, \bar{y}^*_u\right) = \mathrm{Cov}\left(\bar{y}^*_u, \bar{x}^*_u\right) = 0$$

$$\mathrm{Cov}\left(\bar{x}^*_m, \bar{x}^*_m\right) = \mathrm{Var}\left(\bar{x}^*_m\right) = \left(\frac{\sigma^2}{m} + \frac{fN_2\sigma_2^2}{Nm}\right)$$

$$\mathrm{Cov}\left(\bar{x}^*_u, \bar{x}^*_u\right) = \mathrm{Var}\left(\bar{x}^*_u\right) = \left(\frac{\sigma^2}{u} + \frac{fN_2\sigma_2^2}{Nu}\right)$$

$$\mathrm{Cov}\left(\bar{y}^*_u, \bar{y}^*_u\right) = \mathrm{Var}\left(\bar{y}^*_u\right) = \left(\frac{\sigma^2}{u} + \frac{fN_2\sigma_2^2}{Nu}\right)$$

$$\mathrm{Cov}\left(\bar{y}^*_m, \bar{y}^*_m\right) = \mathrm{Var}\left(\bar{y}^*_m\right) = \left(\frac{\sigma^2}{m} + \frac{fN_2\sigma_2^2}{Nm}\right)$$

$$\mathrm{Cov}\left(\bar{y}^*_m, \bar{x}^*_m\right) = \left(\frac{\rho\sigma^2}{m} + \frac{\rho_2 fN_2\sigma_2^2}{Nm}\right)$$

Solving for ‘a’ and ‘c’ gives

$$a = \frac{c\mu(\rho\sigma^2 + \rho_2 A)}{(\sigma^2 + A)} \text{ and } c = \frac{1}{\left[\left(\sigma^2 + A\right)^2 - \mu^2\left(\rho\sigma^2 + \rho_2 A\right)^2\right]}\left[\left(\sigma^2 + A\right)^2\lambda\right]$$

where, $A = \dfrac{fN_2\sigma_2^2}{N}$ and $f = \dfrac{m_2}{m_{h_2}} = \dfrac{u_2}{u_{h_2}}$

Substituting the values of ‘a’ and ‘c’ the minimum variance linear unbiased estimator for the population mean on the second occasion is given by

$$\hat{T}^*_{12} = \frac{\lambda\left(\sigma^2 + A\right)}{\left[\left(\sigma^2 + A\right)^2 - \mu^2\left(\rho\sigma^2 + \rho_2 A\right)^2\right]}\left[\frac{\mu\left(\rho\sigma^2 + \rho_2 A\right)}{(\sigma^2 + A)}\left(\bar{x}^*_u - \bar{x}^*_m\right) + \bar{y}^*_m\right]$$

$$+ \left[\frac{\left(\sigma^2 + A\right)^2\mu - \mu^2\left(\rho\sigma^2 + \rho_2 A\right)^2}{\left(\sigma^2 + A\right)^2 - \mu^2\left(\rho\sigma^2 + \rho_2 A\right)^2}\right]\bar{y}^*_u \quad (2.1)$$

The variance of $\hat{T}_{12}^{*}$ is obtained as

$$V\left(\hat{T}_{12}^{*}\right) = Cov\left(\hat{T}_{12}^{*}, \bar{y}_u^{*}\right)$$

$$= \left[\frac{\left(\sigma^2 + A\right)^2 \mu - \mu^2\left(\rho\sigma^2 + \rho_2 A\right)^2}{\left(\sigma^2 + A\right)^2 - \mu^2\left(\rho\sigma^2 + \rho_2 A\right)^2}\right]\frac{\left(\sigma^2 + A\right)}{n\mu} \quad (2.2)$$

In case $\rho = \rho_2$, $V\left(\hat{T}_{12}^{*}\right)$ reduces to

$$V\left(\hat{T}_{12}^{*}\right) = \left[\frac{1 - \mu\rho^2}{1 - \mu^2\rho^2}\right]\frac{\left(\sigma^2 + A\right)}{n}$$

while if A = 0, i.e. there is no-response then $V\left(\hat{T}_{12}^{*}\right)$ reduces to

$$V\left(\hat{T}_0^{*}\right) = \left[\frac{1 - \mu\rho^2}{1 - \mu^2\rho^2}\right]\frac{\sigma^2}{n}$$

where, $\hat{T}_0^{*}$ is the usual estimator of population mean for the current occasion in the context of sampling on two occasions when there is no non-response.

Minimizing the variance of $\hat{T}_{12}^{*}$ gives the optimum fraction of unmatched and matched portion of the sample as

$$\mu_{opt} = \frac{\left(\sigma^2 + A\right)}{\left(\sigma^2 + A\right) + \sqrt{\left(\sigma^2 + A\right)^2 - \left(\rho\sigma^2 + \rho_2 A\right)^2}} \quad (2.3)$$

$$\lambda_{opt} = \frac{\sqrt{\left(\sigma^2 + A\right)^2 - \left(\rho\sigma^2 + \rho_2 A\right)^2}}{\left(\sigma^2 + A\right) + \sqrt{\left(\sigma^2 + A\right)^2 - \left(\rho\sigma^2 + \rho_2 A\right)^2}} \quad (2.4)$$

Remark 1: When there is non-response only on first occasion then, MVLUE for the population mean on current occasion can be obtained as follows

Define

$$\hat{T}_1 = a(\bar{x}_u^{*} - \bar{x}_m^{*}) + c\bar{y}_m + (1 - c)\bar{y}_u$$

where, $\bar{y}_m = \frac{1}{m}\sum_{i=1}^{m} y_i$ and $\bar{y}_u = \frac{1}{u}\sum_{i=1}^{u} y_i$

Imposing the unbiasedness and the minimum variance unbiased conditions the optimum values of constants 'a' and 'c' are given by

$$a = \frac{\mu\lambda\rho\sigma^2}{\left[(\sigma^2 + A) - \mu^2\rho^2\sigma^2\right]},\quad c = \frac{(\sigma^2 + A)\lambda}{\left[(\sigma^2 + A) - \mu^2\rho^2\sigma^2\right]}$$

Thus, the *MVLUE* for the current occasion in this case is given by

$$\hat{T}_1^* = \frac{1}{\left[(\sigma^2 + A) - \mu^2\rho^2\sigma^2\right]}\left[\begin{array}{r}\mu\lambda\rho\sigma^2\left(\bar{x}_u^* - \bar{x}_m^*\right) + (\sigma^2 + A)\lambda\bar{y}_m \\ + \mu\left((\sigma^2 + A) - \mu\rho^2\sigma^2\right)\bar{y}_u\end{array}\right] \tag{2.5}$$

and the corresponding minimum variance by

$$V(\hat{T}_1^*) = \left[\frac{(\sigma^2 + A) - \mu\rho^2\sigma^2}{(\sigma^2 + A) - \mu^2\rho^2\sigma^2}\right]\left(\frac{\sigma^2}{n}\right) \tag{2.6}$$

The optimum fraction to be unmatched is given by

$$\mu_{opt} = \frac{\sqrt{(\sigma^2 + A)^2 - \rho^2\sigma^2(\sigma^2 + A)}}{\left[(\sigma^2 + A) + \sqrt{(\sigma^2 + A)^2 - \rho^2\sigma^2(\sigma^2 + A)}\right]} \tag{2.7}$$

while, $\lambda_{opt} = 1 - \mu_{opt}$

Remark 2: When there is non-response only on the second occasion, the MVLUE for population mean on the current occasion can be obtained as follows

Define

$$\hat{T}_2 = a\left(\bar{x}_u - \bar{x}_m\right) + c\bar{y}_m^* + (1 - c)\bar{y}_u^*$$

where, $\bar{x}_u = \frac{1}{u}\sum_{i=1}^{u} x_i$ and $\bar{x}_m = \frac{1}{m}\sum_{i=1}^{m} x_i$

The optimum values of constant 'a' and 'c' are given by

$$c = \frac{\lambda(\sigma^2 + A)}{[(\sigma^2 + A) - \mu^2\rho^2\sigma^2]},\quad a = \frac{\lambda\mu\rho}{[(\sigma^2 + A) - \mu^2\rho^2\sigma^2]}(\sigma^2 + A)$$

Thus, the MVLUE in this case is given by

$$\hat{T}_2^* = \frac{1}{\left(\sigma^2 + A - \mu^2\rho^2\sigma^2\right)}\left[\begin{array}{r}\lambda(\sigma^2 + A)\mu\rho(\bar{x}_u - \bar{x}_m) + \lambda(\sigma^2 + A)\bar{y}_m^* \\ + \{(\sigma^2 + A)\mu - \mu^2\rho^2\sigma^2\}\bar{y}_u^*\end{array}\right] \quad (2.8)$$

and the corresponding minimum variance by

$$V(\hat{T}_2^*) = \left[\frac{(\sigma^2 + A) - \mu\rho^2\sigma^2}{(\sigma^2 + A) - \mu^2\rho^2\sigma^2}\right]\left[\sigma^2 + A\right]\frac{1}{n} \quad (2.9)$$

The optimum fraction to be unmatched is given by

$$\mu_{opt} = \frac{\sqrt{(\sigma^2 + A)^2 - \rho^2\sigma^2(\sigma^2 + A)}}{\left[(\sigma^2 + A) + \sqrt{(\sigma^2 + A)^2 - \rho^2\sigma^2(\sigma^2 + A)}\right]} \quad (2.10)$$

while, $\lambda_{opt} = 1 - \mu_{opt}$

3. Comparison between Variances of the Estimator

(*i*) Comparison Between Variance of $\hat{T}_{12}^*$ and $\hat{T}_2^*$

It can be seen that

$V(\hat{T}_{12}^*) > V(\hat{T}_2^*)$

$$\text{if, } A < \frac{\rho^2\sigma^2 - 2\rho_2\rho\sigma^2}{\rho_2^2} \quad (3.1)$$

(*ii*) Comparison Between Variance of $\hat{T}_2^*$ and $\hat{T}_1^*$

It can be seen that

$V(\hat{T}_2^*) > V(\hat{T}_1^*)$

if, $(\sigma^2 + A) > \sigma^2$

Since 'A' is a positive constant $V(\hat{T}_2^*)$ is always greater than $V(\hat{T}_1^*)$.

Thus, if (3.1) holds then

$V(\hat{T}_{12}^*) > V(\hat{T}_2^*) > V(\hat{T}_1^*)$

The percentage loss in precision of $\hat{T}_{12}^*$ over $\hat{T}_0^*$ is given by

$$L_{12}=\left[\frac{V(\hat{T}_{12}^*)}{V(\hat{T}_0^*)}-1\right]\times 100 \tag{3.2}$$

The percentage loss in precision of $\hat{T}_2^*$ and $\hat{T}_1^*$ over $\hat{T}_0^*$ can be obtained by replacing $V(\hat{T}_{12}^*)$ with $V(\hat{T}_2^*)$ and $V(\hat{T}_1^*)$ respectively in (3.2). We denote these by L_2 and L_1 respectively. The percentage loss in precision of $\hat{T}_{12}^*$, $\hat{T}_2^*$ and $\hat{T}_1^*$ over $\hat{T}_0^*$ for different values of σ^2, σ_2^2, ρ, ρ_2, W_2, f and μ is indicated in Table 1.1 and Table 1.2. It is assumed that N = 300 and n = 50.

Table 1.1. Percentage loss in precision of $\hat{T}_{12}^*$, $\hat{T}_2^*$ and $\hat{T}_1^*$ over $\hat{T}_0^*$ for different values of ρ, ρ_2, σ^2 and σ_2^2

ρ	ρ_2	μ	f	W_2	σ_2^2	σ^2	L_{12}	L_2	L_1
						$\sigma^2 < \sigma_2^2$			
0.7	0.2	0.7	2.5	0.8	0.4	0.3	313.427	311.359	12.189
0.7	0.2	0.7	2.5	0.8	0.6	0.3	466.876	465.809	13.162
0.7	0.2	0.7	2.5	0.8	0.9	0.3	696.639	697.310	13.901
						$\sigma^2 > \sigma_2^2$			
0.6	0.2	0.3	1.5	0.6	0.2	0.3	66.043	65.190	3.244
0.6	0.2	0.3	1.5	0.6	0.2	0.7	28.489	27.951	1.779
0.6	0.2	0.3	1.5	0.6	0.2	0.9	22.191	21.742	1.452
						$\sigma^2 = \sigma_2^2$			
0.8	0.3	0.7	2.0	0.7	0.1	0.1	179.893	179.210	16.338
0.8	0.3	0.7	2.0	0.7	0.3	0.3	179.893	179.210	16.338
0.8	0.3	0.7	2.0	0.7	0.8	0.8	179.893	179.210	16.338
						$\rho < \rho_2$			
0.1	0.7	0.6	2.5	0.5	0.5	0.4	142.356	156.628	0.147
0.3	0.7	0.6	2.5	0.5	0.5	0.4	141.274	159.863	1.410
0.6	0.7	0.6	2.5	0.5	0.5	0.4	149.093	174.387	7.078
						$\rho > \rho_2$			
0.8	0.1	0.3	2.0	0.5	0.5	0.4	152.968	146.338	9.484
0.8	0.4	0.3	2.0	0.5	0.5	0.4	143.459	146.338	9.484
0.8	0.9	0.3	2.0	0.5	0.5	0.4	119.242	146.338	9.484
						$\rho = \rho_2$			
0.2	0.2	0.8	1.5	0.5	0.5	0.5	75.000	75.503	0.288
0.5	0.5	0.8	1.5	0.5	0.5	0.5	75.000	79.127	2.358
0.9	0.9	0.8	1.5	0.5	0.5	0.5	75.000	114.237	22.421

A close perusal of Table 1.1 reveals that for all the cases i.e. $\sigma^2(<,=,>)\sigma_2^2$ and $\rho(<,=,>)\rho_2$ the percentage loss in precision over $\hat{T}_0^*$ is maximum in $\hat{T}_{12}^*$ while it is least in $\hat{T}_1^*$. It can be seen from Table 1.1 that in some cases the percentage loss in precision of $\hat{T}_{12}^*$ over $\hat{T}_0^*$ is less than that of $\hat{T}_2^*$ over $\hat{T}_0^*$. These are the cases for which condition (3.1) violated. For the case $\sigma^2 < \sigma_2^2$ percentage loss in precision of all the estimators over $\hat{T}_0^*$ increases with increase in the values of σ_2^2 while it decreases with increase in the values of σ^2 when $\sigma^2 > \sigma_2^2$ However, percentage loss in precision for all the estimators over $\hat{T}_0^*$ remain constant when $\sigma^2 = \sigma_2^2$. Also, for the case $\rho < \rho_2$ the percentage loss in precision of $\hat{T}_{12}^*$ over $\hat{T}_0^*$ first decreases and then increases with increase in the values of ρ while it increases for $\hat{T}_2^*$ and $\hat{T}_1^*$. When $\rho > \rho_2$ the percentage loss in precision of $\hat{T}_{12}^*$ over $\hat{T}_0^*$ decreases with increase in the values of ρ_2 while for $\hat{T}_2^*$ and $\hat{T}_1^*$ percentage loss remains constant. For the case $\rho = \rho_2$ the percentage loss in precision of $\hat{T}_{12}^*$ over $\hat{T}_0^*$ remains constant while it increases for both $\hat{T}_2^*$ and $\hat{T}_1^*$ with increase in the values of ρ and ρ_2.

Table 1.2. Percentage loss in precision of $\hat{T}_{12}^*$, $\hat{T}_2^*$ and $\hat{T}_1^*$ over $\hat{T}_0^*$ for different values of W_2, f, and μ

ρ	ρ_2	μ	f	W_2	σ_2^2	σ^2	L_{12}	L_2	L_1
						W_2			
0.7	0.2	0.6	2.5	0.1	0.4	0.6	21.054	19.937	2.803
0.7	0.2	0.6	2.5	0.3	0.4	0.6	61.677	59.439	6.292
0.7	0.2	0.6	2.5	0.6	0.4	0.6	121.084	118.269	9.134
						f			
0.8	0.3	0.4	1.0	0.5	0.4	0.6	41.887	40.788	5.591
0.8	0.3	0.4	3.0	0.5	0.4	0.6	122.882	121.759	10.880
0.8	0.3	0.4	3.5	0.5	0.4	0.6	142.890	141.947	11.668
						μ			
0.8	0.2	0.1	1.5	0.4	0.7	0.5	90.444	89.187	2.819
0.8	0.2	0.5	1.5	0.4	0.7	0.5	110.398	105.647	11.765
0.8	0.2	0.7	1.5	0.4	0.7	0.5	**113.415**	**108.654**	**13.399**
0.8	0.2	0.9	1.5	0.4	0.7	0.5	102.286	99.887	8.634

It can be seen from Table 1.2 that the percentage loss in precision of all the estimators over $\hat{T}_0^*$ increases with increase in the values of both W_2 and 'f'. The percentage loss in precision for all the estimators over $\hat{T}_0^*$ first increases and then deceases with increase in the values of μ.

To get an idea about saving in cost through mail surveys in the context of successive sampling on two occasions for different assumed values of σ^2, σ_2^2, ρ, ρ_2, W_2, f and μ is indicated in Table 2.1 and Table 2.2.

Also, let

$N = 300$ and $n = 50$

$c_1/c_0 = 4$, $c_2 = 45$

where

c_0 = Cost per unit for mailing a questionnaire (Rs 1.00)

c_1 = Cost per unit of processing the results from the first attempt respondents (Rs 4.00)

c_2 = Cost per unit for collecting data through personal interview (Rs 45.00)

We denote by

C_{00} = Total cost incurred in collecting the data by personal interview from the whole sample i.e. when there is no non-response. The cost functon in this case is given by (assuming that the cost incured on data collection for the matched and unmatched portion of the sample are same and also cost incured on data collection on both the occasions is same)

$$C_{00} = 2.n.c_2 \tag{3.3}$$

Substituting the values of n and c_2 in (3.3) the total cost work out to be Rs 4500.00.

Further, Let n_1 denote number of units which respond at the first attempt and n_2 denote number of units which do not respond.

(i) The cost function for the case when there is non-response on both occasions is given by

$$C_{12} = 2\left[c_0 n + c_1 n_1 + \frac{c_2 n_2}{f}\right]$$

The expected cost is given by

$$E(C_{12}) = 2n[c_0 + c_1 W_1 + c_2(W_2/f)]$$

where, $W_1 = \frac{N_1}{N}$ and $W_2 = \frac{N_2}{N}$, such that $W_1 + W_2 = 1$

(ii) The cost function for the case when there is non-response on second occasion only is given by

$$C_2 = 2c_0n + c_1n + \left[c_1n_1 + \frac{c_2n_2}{f}\right]$$

and the expected cost is given by

$$E(C_2) = n[2c_0 + c_1(W_1 + 1) + c_2(W_2 / f)]$$

(iii) The cost function for the case when there is non-response on first occasion only is given by

$$C_1 = \left[c_1n_1 + \frac{c_2n_2}{f}\right] + 2c_0n + c_1n$$

and the expected cost is given by

$$E(C_1) = n[2c_0 + c_1(W_1 + 1) + c_2(W_2 / f)]$$

By equating the variances of $V(\hat{T}_{12}^*)$, $V(\hat{T}_2^*)$ and $V(\hat{T}_1^*)$ respectively to $V(\hat{T}_0^*)$ and using the assumed values of different parameters, values of sample size for the three cases and corresponding expected cost of survey were determined in respect of $\hat{T}_{12}^*$, $\hat{T}_2^*$ and $\hat{T}_1^*$. Let the sample sizes so obtained for the three cases be denoted by n'_{12}, n'_2 and n'_1. The results of this exercise are presented in Table 2.1and Table 2.2.

As expected, sample sizes associated with the three estimators, which give equal precision to that of the estimator $\hat{T}_0^*$ is maximum for $\hat{T}_{12}^*$ while it is least for $\hat{T}_1^*$. It may be seen that for some cases sample size for $\hat{T}_2^*$ is greater than that of $\hat{T}_{12}^*$. These are the cases for which condition (3.1) violated. Also, saving in cost over $\hat{T}_0^*$ is maximum for $\hat{T}_1^*$ while it is least for $\hat{T}_{12}^*$.

A close perusal of Table 2.1 reveals that saving in cost for all the estimators decreases with increase in the values of σ_2^2 while it increases with increase in the values of σ^2. However, it remains constant with increase in the values of $\sigma^2 = \sigma_2^2$. Sample sizes for the three estimators which give equal precision to $\hat{T}_0^*$ increases with increase in the values of σ_2^2 while it decreases with increase in the values of σ^2. However, it remains constant with increase in the values of $\sigma^2 = \sigma_2^2$.

Saving in cost for all the estimators decreases with increase in the values of ρ. On the other hand saving in cost increases for $\hat{T}_{12}^*$ and remains constant for both $\hat{T}_2^*$ and $\hat{T}_1^*$ with increase in the values of ρ_2. When $\rho = \rho_2$ the saving in cost remains constant for $\hat{T}_{12}^*$ while it decreases for $\hat{T}_2^*$ and $\hat{T}_1^*$ with increase in the values of $\rho=\rho_2$. Sample sizes associated with the three estimators which provides equal precision to the estimator $\hat{T}_0^*$ increase with increase in the values of ρ while they decrease with increasing values of ρ_2 in case of $\hat{T}_{12}^*$ and remain constant for both $\hat{T}_2^*$ and $\hat{T}_1^*$. When $\rho=\rho_2$ the sample sizes remain constant for $\hat{T}_{12}^*$ and increase for the estimators $\hat{T}_2^*$ and $\hat{T}_1^*$ with increase in the values of $\rho=\rho_2$.

Table 2.1. Sample sizes and corresponding expected cost of survey, which give equal precision of $\hat{T}_{12}^*, \hat{T}_2^*$ and $\hat{T}_1^*$ over $\hat{T}_0^*$ for different values of $\sigma^2, \sigma_2^2, \rho$ and ρ_2

ρ	ρ_2	μ	f	W_2	σ_2^2	σ^2	n'_{12}	n'_2	n'_1	C_{12}	C_2	C_1
						$\sigma^2<\sigma_2^2$						
0.7	0.2	0.5	2.5	0.4	0.4	0.3	130	128	55	2745.76	1998.09	856.32
0.7	0.2	0.5	2.5	0.4	0.7	0.3	188	186	56	3976.88	2906.56	871.97
0.7	0.2	0.5	2.5	0.4	0.8	0.3	207	206	56	4385.79	3209.15	875.22
						$\sigma^2>\sigma_2^2$						
0.6	0.2	0.3	1.5	0.3	0.2	0.3	67	66	51	1705.01	1180.21	907.85
0.6	0.2	0.3	1.5	0.3	0.2	0.6	58	58	51	1493.33	1035.14	900.12
0.6	0.2	0.3	1.5	0.3	0.2	0.9	56	55	50	1422.44	986.77	897.07
						$\sigma^2=\sigma_2^2$						
0.8	0.3	0.7	2.0	0.5	0.2	0.2	115	114	57	3279.61	2202.93	1101.46
0.8	0.3	0.7	2.0	0.5	0.6	0.6	115	114	57	3279.61	2202.93	1101.46
0.8	0.3	0.7	2.0	0.5	0.9	0.9	115	114	57	3279.61	2202.93	1101.46
						$\rho<\rho_2$						
0.1	0.7	0.6	2.5	0.5	0.4	0.6	89	92	50	2127.91	1560.05	850.93
0.5	0.7	0.6	2.5	0.5	0.4	0.6	89	95	52	2129.54	1610.91	878.68
0.8	0.7	0.6	2.5	0.5	0.4	0.6	95	104	56	2276.18	1760.34	960.19
						$\rho>\rho_2$						
0.8	0.2	0.3	2.0	0.4	0.5	0.3	130	128	55	3223.67	2227.83	954.79
0.8	0.6	0.3	2.0	0.4	0.5	0.3	122	128	55	3026.66	2227.83	954.79
0.8	0.9	0.3	2.0	0.4	0.5	0.3	114	128	55	2817.08	2227.83	954.79
						$\rho=\rho_2$						
0.3	0.3	0.8	1.5	0.3	0.6	0.4	84	84	50	2144.00	1500.40	895.76
0.5	0.5	0.8	1.5	0.3	0.6	0.4	84	86	51	2144.00	1523.96	909.83
0.8	0.8	0.8	1.5	0.3	0.6	0.4	84	93	56	2144.00	1657.61	989.62

Table 2.2. Sample sizes and corresponding cost of survey, which give equal precision of $\hat{T}_{12}^*, \hat{T}_2^*$ and $\hat{T}_1^*$ over $\hat{T}_0^*$ for different values of W_2, f, and μ

ρ	ρ_2	μ	f	W_2	σ_2^2	σ^2	n'_{12}	n'_2	n'_1	C_{12}	C_2	C_1
							W_2					
0.7	0.2	0.6	2.5	0.2	0.4	0.6	71	70	52	1103.96	894.29	670.72
0.7	0.2	0.6	2.5	0.6	0.4	0.6	111	109	55	2962.52	2008.07	1004.04
0.7	0.2	0.6	2.5	0.8	0.4	0.6	130	129	55	4216.04	2728.01	1169.15
							f					
0.8	0.3	0.4	1.0	0.5	0.4	0.6	71	70	53	3618.12	2147.01	1610.26
0.8	0.3	0.4	1.5	0.5	0.4	0.6	81	81	54	2922.33	1852.41	1234.94
0.8	0.3	0.4	3.0	0.5	0.4	0.6	111	111	55	2340.26	1718.63	859.32
							μ					
0.8	0.2	0.2	1.5	0.4	0.7	0.5	98	97	53	3024.57	1978.83	1075.45
0.8	0.2	0.7	1.5	0.4	0.7	0.5	107	104	57	3286.60	2128.27	1156.67
0.8	0.2	0.9	1.5	0.4	0.7	0.5	101	100	54	3115.20	2038.85	1108.07

It may be seen from Table 2.2 that sample sizes which give equal precision to $\hat{T}_0^*$ increases with increase in the values of W_2 as well as increase in the values of f while, they first increase and then decrease with increases in the values of μ. Saving in cost decreases with increase in the values of W_2 while it increases with increase in the values of 'f'. It first decreases and then increases with increase in the values of μ.

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