J. Ind. Soc. Agril. Statist.
58(3), 2004 : 257-268

CATANOVA for Analysis of Nominal Data from Repeated Measures Design

B. Singh
Indian Veterinary Research Institute, Izatnagar–243 122
(Received : May, 2001)

SUMMARY

The existing CATANOVA method has been examined for its suitability for analysis of nominal data from repeated measures design. The modified tests are developed for single and multi-group repeated measures designs, separately. The computed results reveal that in single group repeated measures design the actual size of existing CATANOVA test is more for negative correlation and less for positive correlation than the stated size. This implies that the existing test may yield the non-existing effects as significant for negative correlation and may not even detect the real effects for positive correlation among repeated observations. Similar results are obtained for repeated factor and interaction effects and just reverse results for group effect in multi-group repeated measures design. These results show that the existing CATANOVA tests are not valid for repeated measures designs and hence the modified tests should be used for analysis of nominal data from such designs.

Key words : Actual size, CATANOVA, Modified tests, Repeated measures design, Stated size.

1. Introduction

The term repeated is used to describe measurements which are made on some characteristic of the same individual but on more than one occasion. The repeated measurements may be continuous or categorical. A general methodology for analysis of categorical data from repeated measures designs has been discussed by Koch *et al.* (1977) by using Grizzle *et al.* (1969) linear model approach. Their approach is problem specific and not applicable to general situations of repeated measurements.

The CATANOVA method is analogous to ANOVA method for quantitative data. This method has been studied by Light and Margolin (1971), Onukogu (1985 a, b) and Singh (1993, 1996), among others for one-way and two-way classified nominal data. Here, we examine the validity of existing CATANOVA method to analyze the nominal data from single and multi-group repeated measures designs. The modified CATANOVA tests for such designs are also developed.

2. Single Group Repeated Measures Design

2.1 Model

Let n_{ijk} be the response of j^{th} individual at k^{th} time point in i^{th} response category. Here n_{ijk} assumes value 1 if response is in i^{th} category otherwise it takes value zero. The statistical model for repeated measurements on categorical data from single group repeated measure design is defined as

$$E(n_{ijk}) = \Pi_i + t_{ik} \tag{2.1}$$

$$(i = 1, 2, \ldots, I;\ j = 1, 2, \ldots, J;\ k = 1, 2, \ldots, K)$$

where Π_i and t_{ik} represent the constant and k^{th} time point effect in i^{th} response category, respectively. The response variables n_{ijk} are assumed to have the variances and co-variances as follows

$$V(n_{ijk}) = \pi_i(1-\pi_i),\ Cov(n_{ijk}, n_{i'jk}) = -\pi_i\pi_{i'},\ i' \neq i$$

$$Cov(n_{ijk}, n_{ijk'}) = \rho\pi_i(1-\pi_i),\ k' \neq k \text{ and}$$

$$Cov(n_{ijk}, n_{i'jk'}) = -\rho\pi_i\pi_{j'},\ i' \neq i,\ k' \neq k$$

where ρ is correlation among repeated measurements and π_i is the probability of getting the response in i^{th} category.

The null hypothesis of interest is

$$H_0 : t_{ik} = 0,\ \forall\ k,\ i = 1, 2, \ldots, I \tag{2.2}$$

2.2 Sums of Squares

The components sum of squares for CATANOVA under model (2.1) are defined as under

$$\text{TSS(Total SS)} = \sum_{ijk} n_{.jk}\{(n_{ijk}/n_{.jk}) - (n_i/n)\}^2$$

$$\text{BST (Time Point SS)} = \sum_{ik} n_{..k}\{(n_{i.k}/n_{..k}) - (n_i/n)\}^2$$

$$\text{BSA (Individual SS)} = \sum_{ij} n_{.j.}\{(n_{ij.}/n_{.j.}) - (n_i/n)\}^2$$

$$\text{Int. SS} = \text{TSS} - \text{BST} - \text{BSA} \tag{2.3}$$

$$n_i = \sum_{jk} n_{ijk},\ n_{ij.} = \sum_{k} n_{ijk},\ n_{i.k} = \sum_{j} n_{ijk},\ n_{.jk} = \sum_{i} n_{ijk} = I$$

$$n_{..k} = \sum_{j} n_{.jk} = J,\ n_{.j.} = \sum_{k} n_{.jk} = K \text{ and } n = \sum_{jk} n_{.jk} = JK$$

The expressions in (2.3) may be expressed for computational purpose as under

$$TSS = n - \sum_i (n_i^2/n)$$

$$BST = \sum_{ik} (n_{i.k}^2/n_{..k}) - \sum_i (n_i^2/n)$$

$$BSA = \sum_{ij} (n_{ij.}^2/n_{.j}) - \sum_i (n_i^2/n)$$

2.3 Expected Values

The expected values of sums of squares (2.3) are obtained under the model (2.1) and conditions

$$\sum_k t_{ik} = 0, i = 1, 2, \ldots, I$$

as follows

$$V(n_i) = n\pi_i(1-\pi_i)[1+(K-1)\rho], \quad V(n_{i.k}) = n_{..k}\,\pi_i(1-\pi_i)$$

$$V(n_{ij}) = n_{.j}\pi_i(1-\pi_i)\,[1+(K-1)\rho]$$

$$E(TSS) = \sum_{ik} n_{..k}t_{ik}^2 + (n-1)\,[1-\{(K-1)/(n-1)\}\rho]\left(1-\sum_i \pi_i^2\right)$$

$$E(BST) = \sum_{ik} n_{..k}t_{ik}^2 + (K-1)(1-\rho)\left(1-\sum_i \pi_i^2\right)$$

$$E(BSA) = (J-1)[1+(K-1)\rho]\left(1-\sum_i \pi_i^2\right) \text{ and}$$

$$E(Int.SS) = (J-1)(K-1)(1-\rho)\left(1-\sum_i \pi_i^2\right) \qquad (2.4)$$

Under null hypothesis (2.2), the expressions (2.4) reduce to

$$E(TSS) = (n-1)[1-\{(K-1)/(n-1)\}\rho]\left(1-\sum_i \pi_i^2\right)$$

$$E(BST) = (K-1)(1-\rho)\left(1-\sum_i \pi_i^2\right)$$

$$E(BSA) = (J-1)\,[1+(K-1)\rho]\left(1-\sum_i \pi_i^2\right) \text{ and}$$

$$E(\text{Int. SS}) = (J-1)(K-1)(1-\rho)\left(1-\sum_i \pi_i^2\right) \tag{2.5}$$

2.4 Testing Procedure

Existing CATANOVA Test

The CATANOVA test statistic (see Light and Margolin (1971)) is defined by

$$C = (I-1)(n-1)\ \text{BST}/\text{TSS} \tag{2.6}$$

The asymptotic null distribution of C is given by

$$[(1-\rho)/\{1-\{(K-1)/(n-1)\}\rho]\ \chi^2_{(I-1)(K-1)}$$

The actual size of CATANOVA test under (2.1) is expressed as

$$\Pr(C \geq C_{0\alpha}) = 1 - I_w[(I-1)(K-1)/2] \tag{2.7}$$

where $w = (C_{0\alpha}/2)[1-\{(K-1)/(n-1)\}\rho]/(1-\rho)$, $I_w(.)$ is the value of incomplete gamma and $C_{0\alpha}$ is the upper α (stated size) percent tail point for chi-square distribution on $(I-1)(K-1)$ degrees of freedom.

Modified CATANOVA Test

The expressions (2.4) and (2.5) for expected values of sums of squares reveal that the existing CATANOVA statistic does not provide unbiased testing. An unbiased testing is provided by the following test statistic

$$C_m = [\{\text{BST}/(K-1)\}/\{\text{Int.SS}/(J-1)(K-1)\}] \tag{2.8}$$

with asymptotic null distribution as F on $[(I-1)(K-1), (I-1)(J-1)(K-1)]$ degrees of freedom. The actual size of this test will be approximately the same as the stated size.

3. Multi Group Repeated Measures Design

3.1 Model

Let n_{ijkl} is the response of j^{th} individual in l^{th} group measured at k^{th} time point in i^{th} response category. Here n_{ijkl} assumes value 1 if response is in i^{th} response category otherwise it takes value zero. The statistical model for repeated measurements on categorical data from multi-group repeated measures design is given by

$$E(n_{ijkl}) = \Pi_i + g_{il} + t_{ik} + \Delta_{ikl} \tag{3.1}$$

$$(i = 1, 2, \ldots, I;\ j = 1, 2, \ldots, J;\ k = 1, 2, \ldots, K;\ l = 1, 2, \ldots, L)$$

where Π_i, t_{ik}, g_{il} and Δ_{ikl} represent the constant, k^{th} time point effect, l^{th} group effect and their interaction effect for i^{th} response, respectively. The response variables n_{ijk} are assumed to have the variances and co-variances as follows

$$V(n_{ijkl}) = \pi_i(1-\pi_i),\ Cov(n_{ijkl}, n_{i'jkl}) = -\pi_i\pi_{i'},\ i' \neq i$$

$$Cov(n_{ijkl}, n_{ijk'l}) = \rho\,\pi_i(1-\pi_i),\ k' \neq k$$

$$Cov\ (n_{ijkl}, n_{i'jk'l}) = -\rho\pi_i\pi_{i'},\ i' \neq i,\ k' \neq k$$

where ρ is correlation among repeated measurements and π_i is the probability of getting the response in i^{th} category.

The null hypotheses of interest are

$$H_{01} : g_{il} = 0\ \forall\ l,\ H_{02} : t_{ik} = 0\ \forall\ k \text{ and } H_{03} : \Delta_{ikl} = 0\ \forall\ k, l \tag{3.2}$$

3.2 Sums of Squares

The components sum of squares for CATANOVA under model (3.1) are defined as under

$$TSS = \sum_{ijkl} n_{.jkl}\{(n_{ijkl}/n_{.jkl}) - (n_i/n)\}^2$$

$$BSG = \sum_{il} n_{...l}\{(n_{i..l}/n_{...l}) - (n_i/n)\}^2$$

$$SSE_1 = \sum_{ijl} n_{.j.l}\{(n_{ij.l}/n_{.j.l}) - (n_{i..l}/n_{...l})\}^2$$

$$BST(Time) = \sum_{ik} n_{..k}\{(n_{i.k}/n_{..k}) - (n_i/n)\}^2$$

$$Bet\ cell\ SS = \sum_{ikl} n_{..kl}\{(n_{i.kl}/n_{..kl})\} - (n_i/n)\}^2$$

$$Int.SS = Bet\ cell\ SS - BST - BSG$$

$$TSS(l) = \sum_{ijk} n_{.jkl}\{(n_{ijkl}/n_{.jkl}) - (n_{i..l}/n_{...l})\}^2$$

$$BST(l) = \sum_{ik} n_{..kl}\{(n_{i.kl}/n_{..kl}) - (n_{i..l}/n_{...l})\}^2$$

$$BSA(l) = \sum_{ij} n_{.j.l}\{(n_{ij.l}/n_{.j.l}) - (n_{i..l}/n_{...l})\}^2$$

$$SSE_2 = \sum_{l}[TSS(l) - BSA(l) - BST(l)] \tag{3.3}$$

$$n_{i.kl} = \sum_{j} n_{ijkl},\ n_{ij.l} = \sum_{k} n_{ijkl},\ n_{i..l} = \sum_{jk} n_{ijkl},\ n_{i.k.} = \sum_{jl} n_{ijkl}$$

$$n_{i} = \sum_{jkl} n_{ijkl},\ n_{.jkl} = \sum_{i} n_{ijkl} = I,\ n_{..kl} = \sum_{j} n_{.jkl} = J,\ n_{.j.l} = \sum_{k} n_{.jkl} = K$$

$$n_{.j} = \sum_{jk} n_{.jkl} = JK,\ n_{..k} = \sum_{jk} n_{.jkl} = JL,\ n = \sum_{jkl} n_{.jkl} = JKL$$

The expressions in (3.3) may be expressed for computational purpose as under

$$\text{Total SS} = n - \sum_{i}(n_i^2/n),\quad \text{BSG (Group)} = \sum_{il}(n_{i...l}^2/n_{...l}) - \sum_{i}(n_i^2/n)$$

$$SSE_1 = \sum_{il}\left[\sum_{l}(n_{ij.l}^2/n_{.j.l}) - (n_{i..l}^2/n_{...l})\right]$$

$$\text{BST(Time)} = \sum_{ik}(n_{i.k}^2/n_{..k}) - \sum_{i}(n_i^2/n)$$

$$\text{Bet cell SS} = \sum_{ikl}(n_{i.kl}^2/n_{..kl}) - \sum_{i}(n_i^2/n)$$

$$\text{TSS(l)} = n_{...l} - \sum_{i}(n_{i...l}^2/n_{...l})$$

$$\text{BST(l)} = \sum_{ik}(n_{i.kl}^2/n_{..kl}) - \sum_{i}(n_{i..l}^2/n_{...l})$$

$$\text{BSA(l)} = \sum_{ij}(n_{ij.l}^2/n_{.j.l}) - \sum_{i}(n_{i..l}^2/n_{..l})$$

3.3 Expected Values

Similar to Section 2.3, the expected values of sums of squares (3.3) under model (3.1) and conditions

$$\sum_{i} g_{il} = 0,\ \sum_{k} t_{ik} = 0 \text{ and } \sum_{l} A_{ikl} = \sum_{k} A_{ikl} = 0,\ \forall\, i$$

are derived as

$$V(n_i) = n\pi_i(1-\pi_i)[1+(K-1)\rho],\ V(n_{i..l}) = n_{...l}\pi_i(1-\pi_i)[1+(K-1)\rho]$$

$$V(n_{i.k}) = n_{..k}\pi_i(1-\pi_i),\ V(n_{i.kl}) = n_{..kl}\pi_i(1-\pi_i)$$

$$V(n_{ij.l}) = n_{.j.l}\pi_i(1-\pi_i)[1+(K-1)\rho]$$

$$E(TSS) = \sum_{il} n_{...l} g_{il}^2 + \sum_{ik} n_{..k} t_{ik}^2 + \sum_{ikl} n_{..kl} \Delta_{ikl}^2$$
$$+ (n-1)[1 - \{(K-1)\rho/(n-1)\}]\left(1 - \sum_i \pi_i^2\right)$$

$$E(BSG) = \sum_{il} n_{...l} g_{il}^2 + (L-1)[1+(K-1)\rho]\left(1 - \sum_i \pi_i^2\right)$$

$$E(BST) = \sum_{ik} n_{..k} t_{ik}^2 + (K-1)[1-\rho]\left[1 - \sum_i \pi_i^2\right]$$

$$E(Int.SS) = \sum_{ikl} n_{.kl} \Delta_{ikl}^2 + (K-1)(L-1)[1-\rho]\left(1 - \sum_i \pi_i^2\right)$$

$$E(SSE_1) = ml\,[1+(K-1)\rho](1 - \sum_i \pi_i^2),\ ml = L(J-1)$$

$$E(SSE_2) = m2\,[1-\rho]\left(1 - \sum_i \pi_i^2\right),\ m2 = n - KL - L(J-1) \tag{3.4}$$

Expressions in (3.4) under corresponding null hypothesis in (3.2) reduce to

$$E(BSG) = (L-1)[1+(K-1)\rho]\left(1 - \sum_i \pi_i^2\right)$$

$$E(BST) = (K-1)[1-\rho]\left(1 - \sum_i \pi_i^2\right)$$

$$E(Int\,SS) = (K-1)(L-1)[1-\rho]\left(1 - \sum_i \pi_i^2\right)$$

$$E(SSE_1) = m_1[1+(K-1)\rho]\left(1 - \sum_i \pi_i^2\right)$$

$$E(SSE_2) = m_2[1-\rho]\left(1 - \sum_i \pi_i^2\right) \tag{3.5}$$

3.4 Testing Procedure

Existing CATANOVA Tests

The CATANOVA test statistic for group effect is defined by

$$C_g = (I-1)(n-1)BSG/TSS \tag{3.6}$$

The asymptotic null distribution of C_g is given by

$$[\{1+(K-1)\rho\}/\{1-\rho(K-1)/(n-1)\}]\chi^2_{(I-1)(L-1)}$$

The actual size of CATANOVA test under model (3.1) is obtained as

$$\Pr(C_g \geq C_{0\alpha}) = 1 - I_{w1}\,[(I-1)(L-1)/2] \tag{3.7}$$

where $w1 = (C_{0\alpha}/2)[\{1-\rho(K-1)/(n-1)\}/\{1+(K-1)\rho\}]$, and $C_{0\alpha}$ is the α percent upper tail point for chi square distribution on $(I-1)(L-1)$ degrees of freedom.

The CATANOVA test statistic for repeated factor effect is defined by

$$C_r = (I-1)(n-1)\mathrm{BST}/\mathrm{TSS} \tag{3.8}$$

The asymptotic null distribution of C_r is given by

$$[\{1-\rho\}/\{1-\rho(K-1)/(n-1)\}]\chi^2_{(I-1)(K-1)} \tag{3.9}$$

The actual size of CATANOVA test under (3.1) is obtained as

$$\Pr(C_r \geq C_{0\alpha}) = 1 - I_{w2}[(I-1)(K-1)/2] \tag{3.10}$$

where $w2 = (C_{0\alpha}/2)[\{1-\rho(K-1)/(n-1)\}/\{1-\rho\}]$ and $C_{0\alpha}$ is the α percent upper tail point for chi square distribution on $(I-1)(K-1)$ degrees of freedom.

The CATANOVA test statistic for interaction effect is defined by

$$C_I = (I-1)(n-1)\,\mathrm{IntSS}/\mathrm{TSS} \tag{3.11}$$

The asymptotic distribution of C_I is given by

$$[\{1-\rho\}/\{1-\rho(K-1)/(n-1)\}]\chi^2_{(I-1)(K-1)(L-1)} \tag{3.12}$$

The actual size of CATANOVA test under (3.1) is obtained as

$$\Pr(C_I \geq C_{0\alpha}) = 1 - I_{w3}\,[(I-1)(K-1)(L-1)/2] \tag{3.13}$$

where $w3 = (C_{0\alpha}/2)[\{1-\rho(K-1)/(n-1)\}/\{1-\rho\}]$ and $C_{0\alpha}$ is the α percent upper tail point for chi square distribution on $(I-1)(K-1)(L-1)$ degrees of freedom.

Modified CATANOVA Tests

Expressions (3.4) and (3.5) for expected values of sums of squares reveal that the existing CATANOVA tests do not provide unbiased testing even for large n and depend upon (ρ), the correlation among repeated measurements. We here develop the following test statistics whose asymptotic null distributions and hence the testing of different effects under model (3.1) do not depend on the correlation among repeated measurements.

The modified test statistic for group effect is obtained as

$$F_g = [\{BSG/(L-1)\}/(SSE_1/m1)] \tag{3.14}$$

with asymptotic null distribution as standard F on $[(I-1)(L-1), (I-1)m1]$ degrees of freedom. The actual size of this test will be approximately the same as the stated size.

The modified test statistic for repeated factor effect is obtained as

$$F_r = [\{BST/(K-1)\}/(SSE_2/m2)] \tag{3.15}$$

with asymptotic null distribution as standard F on $[(I-1)(K-1), (I-1)m2]$ degrees of freedom. The actual size of this test will be approximately the same as the stated size.

The modified test statistic for interaction effect is obtained as

$$F_I = [\text{Int. SS.}/(L-1)(K-1)/(SS_2/m2)] \tag{3.16}$$

with asymptotic null distribution as standard F on $[(I-1)(K-1)(L-1), (I-1)\,m2]$ degrees of freedom. The actual size of this test will be approximately the same as the stated size.

4. Numerical Results

The computed values for actual size of CATANOVA test under single group model (2.1) are presented in Table 1 for following apriori values of parameters

Stated size (α) = 0.05; I (No. of response categories) = 2, 3, 5

J (No. of individuals) = 3, 5; K (No. of time points) = 3, 5

ρ (Correlation among repeated measurements) = –0.1(0.1)0.5

The corresponding values of actual size of tests for group, repeated factor and interaction effects under multi-group model (3.1) are given in Tables 2 to Table 4, respectively, for $\alpha = 0.05; I = 2, 5; J = 3, 5; K = 3, 5; L = 3, 5$ and $\rho = -0.1(0.1)0.5$.

4.1 Single Group Repeated Measures Design

The computed results (Table 1) for single group design reveal that the actual size of CATANOVA test is less than the stated size for negative correlation ($\rho = -0.1$) and more for positive correlation ($\rho > 0$). This implies that sometimes the real effect may not be detected by the existing CATANOVA test for positive correlation. Similarly, in case of negative correlation, the existing CATANOVA test may show significant effect for non-existing effects due to the repeated factor in single group repeated measures design. This effect increases with increase in number of time points.

Table 1. Actual size of CATANOVA test in single group repeated measurements of nominal data

			Correlation coefficient among repeated measurements						
I	J	K	–0.1	0	0.1	0.2	0.3	0.4	0.5
2	5	3	0.0644	0.05	0.0367	0.0249	0.0151	0.0078	0.0031
2	5	5	0.0692	0.05	0.0334	0.0202	0.0102	0.0041	0.0011
3	5	3	0.0701	0.05	0.0329	0.0193	0.0096	0.0037	0.0010
3	5	5	0.0774	0.05	0.0287	0.0141	0.0053	0.0014	0.0002
5	5	3	0.0783	0.05	0.0283	0.0134	0.0051	0.0012	0.0002
5	5	5	0.0898	0.05	0.0231	0.0082	0.0021	0.0003	12×10^{-6}
2	3	3	0.0636	0.05	0.0373	0.0258	0.0161	0.0086	0.0036
2	3	5	0.0676	0.05	0.0343	0.0212	0.0113	0.0048	0.0014
3	3	3	0.0693	0.05	0.0333	0.0199	0.0102	0.0041	0.0011
3	3	5	0.0761	0.05	0.0294	0.0147	0.0058	0.0016	0.0003
5	3	3	0.0777	0.05	0.0286	0.0138	0.0052	0.0013	0.0002
5	3	5	0.0887	0.05	0.0235	0.0086	0.0021	0.0003	15×10^{-6}

Table 2. Actual size of CATANOVA test for group effect in multi-group repeated measurements of nominal data

				Correlation coefficient among repeated measurements						
I	J	K	L	–0.1	0	0.1	0.2	0.3	0.4	0.5
2	3	3	3	0.0233	0.05	0.0832	0.1196	0.1517	0.1942	0.2301
2	3	5	3	0.0000	0.05	0.1166	0.1922	0.2011	0.3226	0.3766
5	3	3	3	0.0128	0.05	0.1152	0.1993	0.2006	0.3804	0.4638
5	3	5	3	0.0011	0.05	0.1985	0.3786	0.5333	0.6561	0.7444
2	5	3	3	0.0235	0.05	0.0829	0.1188	0.1577	0.1922	0.2274
5	5	3	3	0.0126	0.05	0.1149	0.1985	0.2893	0.3786	0.4614
2	5	5	3	0.0067	0.05	0.1184	0.1911	0.2591	0.3199	0.3734
5	5	5	3	0.0011	0.05	0.1981	0.3775	0.5338	0.6541	0.7424
2	3	3	5	0.0182	0.05	0.0857	0.1499	0.2075	0.2651	0.3205
5	3	3	5	0.0076	0.05	0.1467	0.2819	0.4261	0.5573	0.6662
2	3	5	5	0.0032	0.05	0.1492	0.2633	0.3697	0.4613	0.5379
5	3	5	5	0.0002	0.05	0.2812	0.5558	0.7499	0.8629	0.9246
2	5	3	5	0.0183	0.05	0.0955	0.1492	0.2063	0.2633	0.3181
5	5	3	5	0.0076	0.05	0.1464	0.2812	0.4248	0.5558	0.6645
2	5	5	5	0.0032	0.05	0.1488	0.2622	0.3679	0.4591	0.5351
5	5	5	5	0.0002	0.05	0.2807	0.5548	0.7488	0.8621	0.9238

Table 3. Actual size of CATANOVA test for repeated measurements of nominal data

				Correlation among repeated measurements						
I	J	K	L	–0.1	0	0.1	0.2	0.3	0.4	0.5
2	3	3	3	0.0659	0.05	0.0363	0.0243	0.0451	0.0073	0.0028
2	3	5	3	0.0701	0.05	0.0329	0.0193	0.0096	0.0037	0.0001
5	3	3	3	0.0787	0.05	0.0281	0.0132	0.0048	0.0012	0.0002
5	3	5	3	0.0906	0.05	0.0228	0.0078	0.0019	0.0002	1×10^{-5}
2	5	3	3	0.0653	0.05	0.0361	0.0241	0.0143	0.0171	0.0027
5	5	3	3	0.0789	0.05	0.0279	0.0131	0.0047	0.0012	0.0002
2	5	5	3	0.0705	0.05	0.0326	0.0189	0.0093	0.0035	0.0009
5	5	5	3	0.0911	0.05	0.0227	0.0079	0.0018	0.0002	1×10^{-5}
2	3	3	5	0.0653	0.05	0.0361	0.0241	0.0143	0.0071	0.0027
5	3	3	5	0.0789	0.05	0.0279	0.0131	0.0047	0.0012	0.0002
2	3	5	5	0.0705	0.05	0.0326	0.0189	0.0093	0.0035	0.0009
5	3	5	5	0.0911	0.05	0.0227	0.0079	0.0018	0.0002	1×10^{-5}
2	5	3	3	0.0654	0.05	0.0361	0.0239	0.0141	0.0070	0.0026
5	5	3	5	0.0791	0.05	0.0279	0.0131	0.0047	0.0011	0.0001
2	5	5	5	0.0708	0.05	0.0325	0.0187	0.0091	0.0034	0.0008
5	5	5	5	0.0912	0.05	0.0226	0.0078	0.0018	0.0002	94×10^{-6}

Table 4. Actual size of CATANOVA test for interaction effect in multi-group repeated measurements of nominal data

				Correlation among repeated measurements						
I	J	K	L	–0.1	0	0.1	0.2	0.3	0.4	0.5
2	3	3	3	0.0702	0.05	0.0328	0.0191	0.0095	0.0037	0.0009
2	3	5	3	0.0777	0.05	0.0288	0.0138	0.0052	0.0013	0.0002
5	3	3	3	0.0908	0.05	0.0228	0.0079	0.0018	0.0002	1×10^{-5}
5	3	5	3	0.1097	0.05	0.0169	0.0037	0.0004	2×10^{-5}	1×10^{-7}
2	5	3	3	0.0708	0.05	0.0325	0.0188	0.0092	0.0035	0.0009
5	5	3	3	0.0911	0.05	0.0227	0.0078	0.0018	0.0002	96×10^{-6}
2	5	5	3	0.0783	0.05	0.0283	0.0134	0.0050	0.0013	0.0002
5	5	5	5	0.1103	0.05	0.0167	0.0036	0.0004	16×10^{-6}	1×10^{-7}
2	3	3	5	0.0785	0.05	0.0282	0.0134	0.0049	0.0012	0.0002
5	3	3	5	0.1104	0.05	0.0167	0.0036	0.0004	15×10^{-6}	1×10^{-7}
2	3	5	5	0.0902	0.05	0.0231	0.0081	0.0019	0.0003	11×10^{-5}
5	3	5	5	0.1425	0.05	0.0106	0.0011	3×10^{-5}	1×10^{-10}	1×10^{-10}
2	5	3	5	0.0788	0.05	0.0281	0.0132	0.0048	0.0012	0.0002
5	5	3	5	0.1107	0.05	0.0166	0.0036	0.0004	15×10^{-6}	9×10^{-8}
2	5	5	5	0.0907	0.05	0.0228	0.0079	0.0019	0.0002	1×10^{-5}
5	5	5	5	0.1425	0.05	0.0106	0.0011	3×10^{-5}	1×10^{-7}	1×10^{-10}

4.2 Multi-Group Repeated Measures Design

The computed results for multi-group (L = 3, 5) design reveal that the actual size is more for negative correlation ($\rho = -0.1$) and less for positive correlation ($\rho > 0$) than the stated size of CATANOVA test for group effect (Table 2). This implies that the real group effect sometimes may not be detected by the existing CATANOVA test for negative correlation. Similarly, in case of positive correlation, the existing CATANOVA test can show significant effect for non-existing effects. This effect increases with increase in number of time points, response categories and number of groups. For repeated factor and interaction effects the results are of reverse nature (Table 3 & 4).

Thus, the existing CATANOVA tests are not found valid (robust) for analysis of categorical data from single and multi-group repeated measures designs. Hence, the modified CATANOVA tests should be applied for analysis of nominal data from such designs.

REFERENCES

Grizzle, J.E., Starmer, L.E. and Koch, C.G. (1969). Analysis of data by linear models. *Biometrics*, **25**, 489-504.

Koch, G.G., Landis, J.R., Freeman, J.L., Daniel, H., Freeman Jr. and Lehnen, R.G. (1977). A general methodology for the analysis of experiments with repeated measurement of categorical data. *Biometrics*, **33**, 133-158.

Light, R.J. and Margolin, B.H. (1971). Analysis of variance for categorical data. *J. Amer. Statist. Assoc.*, **66**, 534-544.

Onukogu, I.B. (1985a). An analysis of variance of nominal data. *J. Biom.*, **27(4)**, 375-384.

Onukogu, I.B. (1985b). Reasoning by analogy from ANOVA to CATANOVA. *J. Biom.*, **27(8)**, 839-849.

Singh, B. (1993). On the analysis of variance method for nominal data. *Sankhya*, **B55**, 40-47.

Singh, B. (1996). On CATANOVA method for analysis of two-way classified nominal data. *Sankhya*, **B58(3)**, 379-388.