Jour. Ind. Soc. Ag. Statistics
56(3), 2003 : 211-221

The Class of Unbiased Dual to Ratio Estimators

Virendra P. Singh
National Council of Educational Research and Training, New Delhi-110016
(Received : April, 1994)

SUMMARY

This paper proposes the class of unbiased dual to ratio estimators of the population mean in case of interpenetrating sub-samples design, which includes Srivenkataramana [7] estimator. The case of simple random sampling without replacement (SRSWOR) is also studied where similar class of unbiased dual to ratio estimators is developed which includes Srivenkataramana's estimator as a special case. Exact expression for the variance formula of the proposed class of estimators in SRSWOR is derived. The results are illustrated by means of a numerical example.

Key words : Dual to ratio estimators, Interpenetrating sub-sample design, SRSWOR.

1. Introduction

Consider a finite population of size N consisting of units $(U_1, U_2, \ldots, U_N)$. Let y be the characteristic of interest taking value Y_i on the unit U_i, $(i = 1, 2, \ldots, N)$. Information on an auxiliary characteristic x, related to y, taking value X_i on the unit U_i, is available in most of the sample survey situations. We are interested in estimating the parameters such as population total $Y = \sum_{i=1}^{N} Y_i$ or the population mean $\overline{Y} = \frac{Y}{N}$. The method of ratio estimation for estimating the population mean $\overline{Y}$ (or equivalently the total Y) of the study variate y, consists in getting an estimator R of the population ratio $R = Y/X = \sum_{i=1}^{N} Y_i \Big/ \sum_{i=1}^{N} X_i$ and multiplying this estimator by the known population mean $\overline{X} = X/N$ of the x-variate (or by X when estimating the total Y).

Let $\bar{y} = n^{-1}\sum_{i=1}^{n} y_i$ and $\bar{x} = n^{-1}\sum_{i=1}^{n} x_i$ be the unbiased estimators of population means $\bar{Y}$ and $\bar{X}$ corresponding to the variates y and x respectively. It is assumed that $\bar{X}$ is known. For simplicity, we assume all measurements to be nonnegative; and $\bar{X}$ and $\bar{x}$ to be positive. Let the correlation coefficient between y and x be positive. Then, the traditional ratio estimator is defined by

$$\hat{\bar{Y}}_R = (\bar{y}/\bar{x})\bar{X} \tag{1.1}$$

to estimate $\bar{Y}$.

Here, $n < N$ is sample size. Then clearly

$$\bar{x}^* = \frac{N\bar{X} - n\bar{x}}{N - n} \tag{1.2}$$

is also unbiased for $\bar{X}$ and correlation coefficient between $\bar{y}$ and $\bar{x}^*$ is negative.

It is well known that when the correlation coefficient between response and subsidiary variate is negative, the product method of estimation is quite effective to estimate $\bar{Y}$. Keeping this in view, Srivenkataramana [7] suggested a dual to ratio estimator

$$\hat{\bar{Y}}_d = \frac{\bar{y}\,\bar{x}^*}{\bar{X}} \tag{1.3}$$

to estimate $\bar{Y}$. The advantage of this estimator over ratio estimator is that the expression of its bias and mean square error can be derived exactly while it is not with ratio estimator.

It is known that the estimator $\hat{\bar{Y}}_d$ in (1.3) is biased. The bias of $\hat{\bar{Y}}_d$ is likely to be small ([see Srivenkataramana [7]). However, an unusual situation, which has a large coefficient of variation (CV) of x may exist vis-à-vis possibility of large bias arises. In such case, use of exactly unbiased estimators may be of great advantage (Rao [4]). The technique to make the estimator unbiased is to modify the sampling procedure such that the same estimator becomes unbiased, and other is to modify the form of estimator by correcting it for the bias (Hartley and Ross [1], Robson [5]). Murthy and Nanjamma [3] have extensively studied the problem of construction of unbiased ratio estimators for any sample design using the technique of interpenetrating sub-samples.

In this paper, we shall first consider the case of independent interpenetrating sub-samples and obtain a new class of unbiased dual to ratio estimators. An optimum estimator is obtained in this class and the results are modified for SRSWOR case.

2. The Class of Unbiased Dual to Ratio Estimators

First, we discuss the interpenetrating sub-sample model considered by Murthy and Nanjamma [3]. Let (y_i, x_i) be unbiased estimates of the population totals Y and X respectively from of the i^{th} independent sub-samples, (i = 1, 2, ..., n).

Let, $\hat{\bar{Y}}_d = \frac{\bar{y}\,\bar{x}^*}{\bar{X}}$ and $\hat{\bar{Y}}_d^* = n^{-1}\left(\sum_{i=1}^{n} y_i\left(\frac{x_i^*}{\bar{X}}\right)\right)$

where $\quad x_i^* = \frac{N\bar{X} - nx_i}{N - n}$

We, now consider the following weighted estimator for $\bar{Y}$ as

$$\hat{\bar{Y}} = W_0\bar{y} + W_1\hat{\bar{Y}}_d + W_2\hat{\bar{Y}}_d^* \tag{2.1}$$

where W_i's, (i = 0, 1, 2) are suitably chosen weights attached to different estimates of $\bar{Y}$ such that their sum is unity, i.e.

$$W_0 + W_1 + W_2 = 1 \tag{2.2}$$

$\hat{\bar{Y}}$ is unbiased for $\bar{Y}$ if

$$E(\hat{\bar{Y}}) = \bar{Y}$$

or if $\quad E\left[W_1(\hat{\bar{Y}}_d - \bar{Y}) + W_2(\hat{\bar{Y}}_d^* - \bar{Y})\right] = 0$

or if $\quad W_1B(\hat{\bar{Y}}_d) + W_2B(\hat{\bar{Y}}_d^*) = 0 \quad (2.3)$

$$\text{Now, } B(\hat{\bar{Y}}_d) = E\left[\bar{y}\left\{\frac{N\bar{X} - n\bar{x}}{N - n}\bar{X}\right\} - \bar{Y}\right]$$

$$= E\left[\frac{N}{(N-n)}(\bar{y} - \bar{Y}) - \frac{n}{(N-n)}\left(\bar{y}\left(\frac{\bar{x}}{\bar{\bar{X}}}\right)\right) - \bar{Y}\right]$$

$$= -\left[\left\{\frac{n}{(N-n)}\right\}\frac{1}{\bar{\bar{X}}}\{E(\bar{y}\,\bar{x} - \bar{Y}\,\bar{X})\}\right]$$

$$= -\left[\left\{\frac{1}{(N-n)}\right\}\left\{\frac{\mu_{11}(y, x)}{\bar{\bar{X}}}\right\}\right] \tag{2.4}$$

where, $\quad \mu_{11}(y, x) = N^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})$. Notice that for the interpenetrating sub-sample model, we have

$$B(\hat{\bar{Y}}_d^*) = E\left[\frac{1}{n\bar{X}}\sum_{i=1}^{n} y_i x_i^* - \bar{Y}\right]$$

$$= \left[\left(\frac{1}{\bar{X}}\right)E\,(y_i x_i^* - \bar{Y}\bar{X})\right]$$

$$= -\left[\left(\frac{n}{N-n}\right)\left(\frac{\mu_{11}(y, x)}{\bar{X}}\right)\right] \tag{2.5}$$

From (2.4) and (2.5), we have

$$B(\hat{\bar{Y}}_d^*) = nB(\hat{\bar{Y}}_d) \tag{2.6}$$

Now, letting $W_1 = W$, a constant, say, then from (2.3) and (2.6), we have $W_2 = W/n$. Putting, $W_1 = W$ and $W_2 = -n^{-1}W$ in (2.2), we obtain $W_0 = (1 - WC)$, where $C = \{(n-1)/n\}$.

Thus, substituting $W_0 = (1 - WC)$, $W_1 = W$ and $W_2 = -n^{-1}W$ in (2.1), we obtain a class of exactly unbiased dual to ratio estimators for $\bar{Y}$ as

$$\hat{\bar{Y}}_u = [(1 - WC)\bar{y} + W\{\hat{\bar{Y}}_d - (1 - C)\hat{\bar{Y}}_d^*\}] \tag{2.7}$$

Remark 2.1 : Notice that $W = 0$ gives the conventional unbiased estimator $\hat{\bar{Y}} = \bar{y}$. With $W = 1$, we have the estimator given by

$$\hat{\bar{Y}}(1) = \left\{\hat{\bar{Y}}_d + \frac{1}{n}(\bar{y} - \hat{\bar{Y}}_d^*)\right\}$$

While, $W = C^{-1} = \left(\frac{n}{n-1}\right)$, gives the estimator

$$\hat{\bar{Y}}(2) = \frac{n(\hat{\bar{Y}}_d - \hat{\bar{Y}}_d^*)}{n-1}$$

which is the estimator due to Srivenkataramana [7]. Several other estimators can be had from (2.7) just by putting the suitable values of W.

Remark 2.2 : In a similar fashion, the class of almost unbiased dual to product estimators for $\bar{Y}$ can be obtained as

$$\hat{\bar{Y}}_u^{(p)} = [(1 - W^*C)\bar{y} + W^*(\hat{\bar{Y}}_{pd} - (1 - C)\hat{\bar{Y}}_{pd}^*)]$$

where, $\hat{\bar{Y}}_{pd} = \bar{y}(\bar{X}/\bar{x}^*)$, $\hat{\bar{Y}}_{pd}^* = n^{-1}\sum_{i=1}^{n} y_i \frac{\bar{X}}{x_i^*}$, $C = n^{-1}(n-1)$ and W^* is a suitably chosen scalar.

3. Choice of an Optimum Estimator in the Class of $\hat{\bar{Y}}_u$ *in (2.7)*

From (2.7) we have

$$V(\hat{\bar{Y}}_u) = W^2 [C^2 V(\bar{y}) + V(\hat{\bar{Y}}_d) + (1-C)^2 V(\hat{\bar{Y}}_d^{\,*}) - 2C\,\mathrm{Cov}(\bar{y}, \hat{\bar{Y}}_d)$$
$$- 2(1-C)\,\mathrm{Cov}(\hat{\bar{Y}}_d, \hat{\bar{Y}}_d^{*}) + 2C(1-C)\,\mathrm{Cov}(\bar{y}, \hat{\bar{Y}}_d^{\,*})]$$
$$- 2W[CV(\bar{y}) - \mathrm{Cov}(\bar{y}, \hat{\bar{Y}}_d) + (1-C)\,\mathrm{Cov}(\bar{y}, \hat{\bar{Y}}_d^{*})] + V(\bar{y}) \quad (3.1)$$

Minimization of this leads to the optimum of W, which is given by

$$W_{(opt)} = A / B \quad (3.2)$$

where $A = CV(\bar{y}) - \mathrm{Cov}(\hat{\bar{y}}, \hat{\bar{Y}}_d) + (1-C)\,\mathrm{Cov}(\bar{y}, \hat{\bar{Y}}_d^{*})$, and

$$B = [C^2 V(\bar{y}) + V(\hat{\bar{Y}}_d) + (1-C)^2 V(\hat{\bar{Y}}_d^{*}) - 2C\,\mathrm{Cov}(\bar{y}, \hat{\bar{Y}}_d)$$
$$- 2(1-C)\,\mathrm{Cov}(\hat{\bar{Y}}_d, \hat{\bar{Y}}_d^{*}) + 2C(1-C)\,\mathrm{Cov}(\bar{y}, \hat{\bar{Y}}_d^{*})]$$

After some simplification, it can be shown that $W_{(opt)}$ can alternatively be written as

$$W_{(opt)} = \frac{\mathrm{Cov}\,(\bar{y}, \hat{\bar{Y}}')}{V(\hat{\bar{Y}}')} \quad (3.3)$$

where $\hat{\bar{Y}}' = \left[C\bar{y} - \hat{\bar{Y}}^{*}\right]$ and $\hat{\bar{Y}}^{*} = \left[\hat{\bar{Y}}_d - (1-C)\hat{\bar{Y}}_d^{\,*}\right]$

This gives, $V(\hat{\bar{Y}}_u^{(opt)}) = V(\bar{y})\,(1-\rho^2)$ (3.4)

where ρ is the correlation coefficient between $\bar{y}$ and $\hat{\bar{Y}}'$.

Remark 3.1 : From (3.4), it is immediate that $V(\hat{\bar{Y}}_u^{(opt)}) < V(\bar{y})$. Furthermore, it may be recalled that

$MSE(\hat{\bar{Y}}_d) = V(\bar{y}) + 2R\mathrm{Cov}(\bar{y}, \bar{x}^*) + R^2 V(\bar{x}^*)$, which leads to the condition that $\hat{\bar{Y}}_u^{(opt)}$ is better than the dual to ratio estimator, if

$$V(\hat{\bar{Y}}_u^{opt}) < MSE(\hat{\bar{Y}}_d) \quad (3.5)$$

4. The Class of Unbiased Dual to Ratio Estimators for SRSWOR Design

In case of SRSWOR, let y_i and x_i denote respectively, the y and x values of the i^{th} sampled unit, (i = 1, 2, ..., n). We have

$$\hat{\bar{Y}}_d = \bar{y}\frac{\bar{x}^*}{\bar{\bar{X}}} \text{ and } \hat{\bar{Y}}_d^* = n^{-1}\left(\sum_{i=1}^{n} y_i \frac{x_i^*}{\bar{\bar{X}}}\right) \tag{4.1}$$

where, $x_i^* = \dfrac{N\bar{\bar{X}} - nx_i}{N-n}$

It can easily be shown that under SRSWOR, the biases of $\hat{\bar{Y}}_d$ and $\hat{\bar{Y}}_d^*$ are given by

$$B(\hat{\bar{Y}}_d) = -\left\{\frac{1}{N}\frac{S_{yx}}{\bar{\bar{X}}}\right\} \tag{4.2}$$

$$B(\hat{\bar{Y}}_d^*) = -\left\{\frac{n}{N-n}\frac{N-1}{N}\frac{S_{yx}}{\bar{\bar{X}}}\right\} \tag{4.3}$$

where $\quad S_{yx} = \left\{\dfrac{N}{N-1}\mu_{11}(y, x)\right\}$

From (4.2) and (4.3), it follows that

$$\frac{B(\hat{\bar{Y}}_d)}{B(\hat{\bar{Y}}_d^*)} = \frac{N-n}{n(N-1)} \tag{4.4}$$

As before, we have the "class of unbiased dual to ratio estimators" for $\bar{Y}$ given by

$$\hat{\bar{Y}}_u^* = [(1 - WC^*)\bar{y} + W\{\hat{\bar{Y}}_d - (1 - C^*)\hat{\bar{Y}}_d^*\}] \tag{4.5}$$

where W is a suitably chosen scalar, and $C^* = \dfrac{N(n-1)}{n(N-1)}$.

Remark 4.1 : Notice that $W = 0$ gives the usual unbiased estimator $\hat{\bar{Y}} = \bar{y}$, and $W = \dfrac{1}{C^*}$ gives the estimator

$$\hat{\bar{Y}}_s^* = \left\{\frac{n(N-1)}{N(n-1)}\right\}\left\{\frac{\bar{y}\,\bar{x}^*}{\bar{\bar{X}}}\right\} - \left\{\frac{N-n}{N(n-1)}\right\}\left\{n^{-1}\sum_{i=1}^{n} y_i\left(\frac{x_i^*}{\bar{\bar{X}}}\right)\right\}$$

$$= \left[\left\{\frac{\bar{y}\,\bar{x}^*}{\bar{\bar{X}}}\right\} + \left\{\frac{1}{N}\frac{s_{yx}}{\bar{\bar{X}}}\right\}\right] \tag{4.6}$$

which is due to Srivenkataramana [7] where

$$s_{yx} = (n-1)^{-1} \sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})$$

Remark 4.2 : For $W = \frac{-q}{C^*}$, q being a constant, the estimator $\hat{\bar{Y}}_u^*$ can be expressed as

$$\hat{\bar{Y}}_s^{**}(q<0) = \left[(q+1)\bar{y} - q \left\{ \frac{\bar{y}\,\bar{x}^*}{\bar{X}} \right\} + \frac{1}{N} \frac{s_{yx}}{\bar{X}} \right] \tag{4.7}$$

This is analogous to Sastry [6] estimator, which is in fact a generalization of Srivenkatarmana [7] estimator (q = –1 gives Srivenkataramana's estimator).

5. *Variance of Estimator* $\hat{\bar{Y}}_u^*$ *in (4.5)*

From (4.5), we have

$$\begin{aligned} V(\hat{\bar{Y}}_u^*) = {} & W^2[C^{*2}V(\bar{y}) + V(\hat{\bar{Y}}_d) + (1-C^*)^2 V(\hat{\bar{Y}}_d^*) - 2C^*\mathrm{Cov}(\bar{y}, \hat{\bar{Y}}_d) \\ & + 2C^*(1-C^*)\mathrm{Cov}(\bar{y}, \hat{\bar{Y}}_d^*) - 2(1-C^*)\mathrm{Cov}(\bar{y}, \hat{\bar{Y}}_d^*)] \\ & - 2W[C^*V(\bar{y}) - \mathrm{Cov}(\bar{y}, \hat{\bar{Y}}_d) + (1-C^*)\mathrm{Cov}\,(\bar{y}, \hat{\bar{Y}}_d^*)] + V(\bar{y}) \end{aligned} \tag{5.1}$$

For determining the values of variances and covariances involved in (5.1), let

$$e_i = y_i - \bar{Y}, \qquad t_i = x_i - \bar{X}$$

and $\qquad \bar{e} = \bar{y} - \bar{Y}, \qquad \bar{t} = \bar{x} - \bar{X}$

where $\bar{e}$ and $\bar{t}$ denote the sample means of e_i's and t_i's.

We used the method of getting expectations of symmetric functions of sample observations in samples drawn by simple random sampling from a finite population, (see Sukhatme and Sukhatme [9]). The exact values of the variances and covariances involved in (5.1) can be obtained vis-à-vis hence the exact variance of the proposed estimator $\hat{\bar{Y}}_u^*$.

As remarked by Murthy [2] that the values of

$$E\left[\bar{e}\left(n^{-1}\sum_{i=1}^{n} e_i t_i\right)\right],\ E(\bar{e}^2\bar{t}),\ E\left[\bar{t}\left(n^{-1}\sum_{i=1}^{n} e_i t_i\right)\right]$$

$$E\left(n^{-1}\sum_{i=1}^{n} e_i t_i\right)^2, \quad E(\bar{t}^2\bar{e}), \quad E\left[\bar{e}\,\bar{t}\left(n^{-1}\sum_{i=1}^{n} e_i t_i\right)\right]$$

and $E\left(\bar{e}^2\,\bar{t}^2\right)$ are generally small even for moderate sample sizes, hence the terms involving these can be neglected. To get tangible idea about the variance of class of estimators $\hat{\bar{Y}}_u^*$, we take into consideration the variance for $\hat{\bar{Y}}_u^*$ upto the terms of $0(n^{-1})$ (or alternatively to the first degree of approximation). For this we have

$$V(\hat{\bar{Y}}_d) = V(\hat{\bar{Y}}_d^*) = \text{Cov}(\hat{\bar{Y}}_d, \hat{\bar{Y}}_d^*)$$
$$= \frac{f}{n}\bar{Y}^2\left[C_y^2 + g^*\,C_x^2(g^* - 2k)\right]$$

where $g^* = \left[\dfrac{n}{N-n}\right]$

$$\text{Cov}(\bar{y}, \hat{\bar{Y}}_d) = \text{Cov}(\bar{y}, \hat{\bar{Y}}_d^*)$$
$$= \frac{f}{n}\left[\mu_{20} - R\left(\frac{n}{N-n}\right)\mu_{11}\right]$$

and $V(\bar{y}) = \dfrac{f}{n}\mu_{20}$

where $\mu_{rs} = \dfrac{1}{N}\sum_{i=1}^{N} e_i^r t_i^s$ and $f = \left\{\dfrac{N-n}{N-1}\right\}$

Substituting these expressions in (5.1), we get the variance of $\hat{\bar{Y}}_u^*$, to terms of order $0(n^{-1})$

$$V(\hat{\bar{Y}}_u^*) = \bar{Y}^2\frac{f}{n}\left[C_y^2 + WgC_x^2(Wg - 2k)\right] \tag{5.2}$$

where $g = \left[N(n-1)/(N-1)(N-n)\right]$ and $k = \rho(C_y/C_x)\rho$ is the correlation coefficient of Y and X, and C_y and C_x are the CVs of Y and X, respectively.

The variance of $\hat{\bar{Y}}_u^*$ in (5.2) is minimized for

$$W = k/g = W_{opt}\,(\text{say}) \tag{5.3}$$

Thus, the minimum variance of $\hat{\bar{Y}}_u^*$ is given by

$$V_{min}(\hat{\bar{Y}}_u^*) = \bar{Y}^2 \frac{f}{n}(1-\rho^2)C_y^2 \qquad (5.4)$$

which is equivalent to the approximate variance of usual linear regression estimator.

6. Theoretical Comparisons

To compare $\hat{\bar{Y}}_u^*$ with $\bar{y}$, we note that the estimator $\hat{\bar{Y}}_u^*$ is better than $\bar{y}$ if

either $0 < W < (2k/g)$

or $2k/g < W < 0$ (6.1)

The proposed estimator $\hat{\bar{Y}}_u^*$ will dominate over Srivenkataramana's estimator $\hat{\bar{Y}}_d$ if

$$\text{either} \quad (1 - C^{*-1}) < W < \left\{\frac{2k - g^*}{g}\right\}$$

$$\text{or} \quad \left\{\frac{2k - g^*}{g}\right\} < W < (1 - C^{*-1}) \qquad (6.2)$$

Further, to compare $V(\hat{\bar{Y}}_u^*)$ with the usual ratio estimator, we first write the mean squared error of $\bar{y}_R$ to the first degree of approximation.

$$MSE(\bar{y}_R) = \frac{f}{n}\bar{Y}^2 [C_y^2 + C_x^2 (1-2k)] \qquad (6.3)$$

Thus it follows from (5.2) and (6.3) that the estimator $\hat{\bar{Y}}_u^*$ will be more efficient than $\bar{y}_R$ if

$$\text{either} \quad \frac{1}{g} < W < \frac{2k-1}{g}$$

$$\text{or} \quad \frac{2k-1}{g} < W < \frac{1}{g} \qquad (6.4)$$

7. Numerical Illustration

In this section, we study the performance of various estimators of population mean $\bar{Y}$ by considering the population studied by Srivenkataramana and Tracy [8]. The population consists of

y	:	12	22	38	15	18	31	15	20	10	25
x	:	14	25	37	18	20	30	15	21	12	28
y	:	11	17	12	22	14	26	08	16	13	19
x	:	14	19	12	23	16	28	09	15	15	20

Here, sample size $n = 5$, $\overline{Y} = 18.2$, $\overline{X} = 19.55$, $C_y = 0.40136$, $C_x = 0.35530$, $\rho = 0.9842$ and $k = 1.1118$.

Table 7.1 gives the relative variance of $\hat{\overline{Y}}_u^*$ for various choices of W, $\bar{y}$, $\bar{y}_R$, $\hat{\overline{Y}}_d$, $\hat{\overline{Y}}_s^*$ and per cent relative efficiencies (PRE's) of $\hat{\overline{Y}}_u^*$ for different choices of W, $\bar{y}_R$, $\hat{\overline{Y}}_d$, $\hat{\overline{Y}}_s^*$ with respect to $\bar{y}$. It shows that the performance of dual to ratio estimator $\hat{\overline{Y}}_d$ is poor than usual ratio estimator $\bar{y}_R$. Thus, one can recommend the use of $\bar{y}_R$ over $\hat{\overline{Y}}_d$ for such type of data. However, it is seen from the table that for a wide range of W, the performance of the estimator $\hat{\overline{Y}}_u^*$ is better than the ratio estimator $\bar{y}_R$. The range of the values of W are given below where $\hat{\overline{Y}}_u^*$ is respectively better than $\bar{y}$, $\hat{\overline{Y}}_d$ and $\bar{y}_R$

(i) $0 < W < 7.97$

(ii) $1.19 < W < 6.78$

(iii) $3.56 < W < 4.40$

Notice that the relative efficiencies corresponding to $\hat{\overline{Y}}_u^*$ for the optimum choice of W for 3.96 is maximum among the estimators considered.

Table 7.1. Exhibiting the relative variances (RVs) and percent relative efficiencies (PREs) of different estimators of $\overline{Y}$ with respect to $\bar{y}$

W	$RV(\hat{\overline{Y}}_u^*)$	PRE $(\hat{\overline{Y}}_u^*, \bar{y})$
0.0	0.16109	100.00
1.0	0.09225	174.00
1.5	0.06528	256.75
2.0	0.04392	372.08
3.0	0.01423	1131.72
3.25	0.01008	1598.49
3.50	7.1644×10^{-3}	2248.49
3.75	5.4945×10^{-3}	2931.89
$W_{(opt)} = 3.96$	5.0525×10^{-3}	3188.34
4.00	5.0678×10^{-3}	3178.72
4.25	5.8844×10^{-3}	2737.56
$\hat{\overline{Y}}_s^*$	0.08155	197.53
$\hat{\overline{Y}}_d$	0.08155	197.53
$\bar{y}_R$	6.630×10^{-3}	2429.74

ACKNOWLEDGEMENT

The author is grateful to Dr. H.P. Singh, Professor of Statistics, School of Studies in Statistics, Vikram University, Ujjain, M.P. 456010 for his valuable guidance and suggestions.

REFERENCES

[1] Hartley, H.O. and Ross, A. (1954). Unbiased ratio estimators. *Nature*, **174,** 270-271.

[2] Murthy, M.N. (1967). *Sampling Theory and Methods.* Calcutta Statistical Publishing Society, Calcutta.

[3] Murthy, M.N. and Nanjamma, N.S. (1959). Almost unbiased ratio estimates based on interpenetrating sub-sample estimate. *Sankhya,* **21,** 381-392.

[4] Rao, P.S.R.S. (1969). Comparison of four ratio-type estimators under a model. *J. Amer. Statist. Assoc.,* **64,** 574-580.

[5] Robson, D.S. (1957). Application of multivariate polykays to the theory of unbiased ratio-type estimation. *J. Amer. Statist. Assoc.,* **52,** 511-522.

[6] Sastry, N.S. (1974). *On a general class of unbiased ratio, product, ratio-cum-product and regression estimators.* Ph. D. Thesis submitted to the Indian Statistical Institute, Calcutta.

[7] Srivenkataramana, T. (1980). A dual to ratio estimator in sample surveys. *Biometrika,* **67,** 194-204.

[8] Srivenkataramana, T. and Tracy, D.S. (1981). Extending product methods of estimation to positive correlation case in surveys. *Austr. J. Statist.,* **23(1),** 95-100.

[9] Sukhatme, P.V. and Sukhatme, B.V. (1970). *Sampling Theory of Surveys with Applications.* Asia Publishing House, New Delhi.