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Analysis of Intercropping Experiments using Experiments with Mixtures Methodology

J.S. Dhekale, Rajender Parsad and V.K. Gupta
Indian Agricultural Statistics Research Institute, New Delhi-110012
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SUMMARY

An attempt has been made to establish an analogy between replacement series intercropping experiments and experiments with mixtures. The analytical procedure of obtaining optimum proportion of the area allocated to different crops and $100(1-\alpha)\%$ confidence intervals for the predicted response at the optimum proportions of the area allocated has been obtained. The procedure developed has been illustrated with the help of an example.

Key words : Experiments with mixtures, intercropping experiments, replacement series, optimum response.

1. Introduction

Intercropping is an important feature of dryland farming and it has proved very useful for survival of small and marginal farmers in tropics and sub tropics. Depending upon the row arrangements, the intercropping experiments are classified into two types *viz.* replacement series and additive series. In replacement series the component crop is introduced by replacing a part of the main crop and in additive series the component crop is introduced without reducing the plant population of the main crop. Intercropping experiments with replacement series form the subject of the present investigation. The treatments in these experiments are the different row ratios. One of the objectives of replacement series intercropping experiments is identification of row ratio that maximizes response (gross returns, net returns, total calories or nutrients, *etc.*) from the crops involved in the experiment.

For this purpose replacement series intercropping experiments are being conducted with sole crop treatments and there is no straightforward method of analysis of the data generated from experiments consisting of sole crop treatments. The quantitative evaluation of the treatments in these experiments is carried out using indices. These indices compare the treatments and do not test significance, as in most of the cases indices do not follow normal distribution.

Such experiments are being conducted in the National Agricultural Research System (NARS) using randomized complete block (RCB) designs. Different proportions of two crops are selected arbitrarily and the data on monetary returns, nutrients, etc. obtained from these crops is being analyzed by usual analysis of variance (ANOVA) method. This procedure helps in identifying the best treatment (proportion of area allocated to the component crops) among the treatments tried in the experiment. However, in these experiments, agronomist may be interested in predicting or interpolating the response at points within the experimental region that have not been tried in the experiment. This requires that a relationship may be built up between response and the proportion of crops and analysis may be carried out to get the proportion of area allocated to component crops, which maximizes net returns, even though this proportion is not included in the experiment.

A critical look at the treatment structure of these experiments reveals that in such experiments the total land resources *i.e.* area under each experimental unit is constant and the response varies only due to the different proportions of the crops. Therefore, these experiments are analogous to the experiments with mixtures with two components *viz.* two crops. In these experiments the variation in response is due to the varying proportions of the area allotted to the crops. The sole crop treatments are pure blends and different proportions (treatments) are treated as mixtures. Hence, the above questions can be answered by fitting second order polynomial of Scheffe ([5], [6]). In Section 2, we describe the procedure of fitting a second order canonical polynomial using the experiments with mixtures methodology. The procedure of obtaining the optimum proportion of the area allocated to different crops is also given in Section 2. The results obtained have been illustrated with the help of an example in Section 3.

2. *Experiments with Mixtures Methodology for Replacement Series Intercropping Experiments*

Let there be N design points and v distinct intercropping treatment combinations. In this situation N = vr as each treatment is replicated r times. Let the proportion of the area allocated to i^{th} crop in u^{th} experimental unit be x_{iu}, (i = 1, 2; u =1, 2, ..., N). A proportion of area allocated in u^{th} experimental unit is represented by x_{1u}, x_{2u}. The response y_u is assumed to have functional relationship with proportion x_{1u}, x_{2u} allocated to the crops in u^{th} experimental unit. This relationship can be explained by second order canonical polynomial of Scheffe ([5], [6])

$$y_u = \beta_1 x_{1u} + \beta_2 x_{2u} + \beta_{12} x_{1u} x_{2u} + e_u \qquad (2.1)$$

where β_1, β_2 and β_{12} are the usual regression coefficients. Here β_i, $i = 1, 2$ may be taken as effect due to proportion of area sown in crop A and crop B respectively and β_{12} may be taken as joint effect of crops A and B. If β_{12} is positive and significantly different from zero one may conclude that the crops in the mixture help each other. On the other hand if β_{12} has negative sign and is significantly different from zero, then one may conclude that the component crops have an adverse impact on each other. For more details, one may refer to Murthy and Das [3], Nigam [4], Cornell [2] and Batra *et al.* [1] *etc.* e_u's are random error terms associated with y_u and have mean zero and variance σ^2. Also

$$x_{1u} + x_{2u} = 1, \; 0 \le x_{1u}, x_{2u} \le 1; \; u = 1, 2, ..., N \tag{2.2}$$

The parameters β_1, β_2 and β_{12} can be estimated by using ordinary least squares. For simplicity, model (2.1) can be written in matrix notation as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e} \tag{2.3}$$

where $\mathbf{Y}$ is an $N \times 1$ vector of observations, $\mathbf{X}$ is an $N \times 3$ design matrix whose elements are x_{1u}, x_{2u} and $x_{1u}x_{2u}$, $\boldsymbol{\beta}$ is a 3×1 vector of unknown parameters β_1, β_2 and β_{12}, $\mathbf{e}$ is an $N \times 1$ vector of random errors with mean vector $\mathbf{0}$ and variance covariance matrix as $\sigma^2 \mathbf{I}_n$. Usual ANOVA for no intercept linear model can be performed with caution that regression sum of squares and total sum of squares must be corrected for general mean. The outline of the analysis is given as follows. The normal equations for estimating $\boldsymbol{\beta}$ are

$$\mathbf{X'X}\boldsymbol{\beta} = \mathbf{X'Y} \tag{2.4}$$

The ordinary least square estimator of $\boldsymbol{\beta}$ is

$$\hat{\boldsymbol{\beta}} = (\mathbf{X'X})^{-1}\mathbf{X'Y} \tag{2.5}$$

The variance–covariance matrix of estimates of parameters is given by

$$D(\hat{\boldsymbol{\beta}}) = \sigma^2 (\mathbf{X'X})^{-1}$$

The error sum of squares is given by

$$SSE = \mathbf{Y'Y} - \hat{\boldsymbol{\beta}}'\mathbf{X'Y} \tag{2.6}$$

The ANOVA for the above fitting model is given below

ANOVA for Experiments with Mixtures

Source of Variation	Degrees of freedom	Sum of Squares	Mean Square	F-Cal
Regression (fitted model)	2	$SSR = \hat{\boldsymbol{\beta}}'\mathbf{X'Y} - \frac{(\mathbf{1'Y})^2}{N}$	$MSR = \frac{SSR}{2}$	$\frac{MSR}{MSE}$
Error	N-3	$SSE = \mathbf{Y'Y} - \hat{\boldsymbol{\beta}}'\mathbf{X'Y}$	$MSE = \frac{SSE}{(N-3)}$	
Total	$N-1$	$SST = \mathbf{Y'Y} - \frac{(\mathbf{1'Y})^2}{N}$		

Let the fitted equation be

$$\hat{y} = \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_{12} x_1 x_2 \tag{2.7}$$

Now substituting (2.2) in (2.7) we get

$$\begin{aligned}\hat{y} &= \hat{\beta}_1 x_1 + \hat{\beta}_2 (1 - x_1) + \hat{\beta}_{12} x_1 (1 - x_1) \\ &= \hat{\beta}_2 + (\hat{\beta}_1 - \hat{\beta}_2 + \hat{\beta}_{12}) x_1 - \hat{\beta}_{12} x_1^2 \\ &= \hat{\beta}'_0 + \hat{\beta}'_1 x_1 - \hat{\beta}'_2 x_1^2 \end{aligned} \tag{2.8}$$

It is a quadratic equation in x_1, which can be used for obtaining optimum proportion of area allocated to different crops for maximizing the gross returns

$$x_{1opt} = \hat{\beta}'_1 / \left(2\hat{\beta}'_2\right) \tag{2.9}$$

where $\hat{\beta}'_0 = \hat{\beta}_2$; $\hat{\beta}'_1 = \hat{\beta}_1 - \hat{\beta}_2 + \hat{\beta}_{12}$ and $\hat{\beta}'_2 = \hat{\beta}_{12}$

Now using (2.2) we get

$$x_{2opt} = 1 - x_{1opt}$$

Using this, the predicted optimum response is given by

$$\hat{y}_{opt} = \hat{\beta}_1 x_{1opt} + \hat{\beta}_2 x_{2opt} + \hat{\beta}_{12} x_{1opt} x_{2opt}$$

The 100(1-α)% confidence interval for optimum predicted response obtained from intercropping is given by $\hat{y}_{opt} \pm t_{(N-3),\alpha/2} \sqrt{Var(\hat{y}_{opt})}$. This procedure has been explained with the help of an example in the next section.

3. Illustration

Example 3.1: Consider an intercropping experiment conducted at Dryland Research Station, Dr. Panjabrao Deshmukh Krishi Vidyapeeth, Akola, on Redgram and Safflower during 1980-81. The objective of the experiment was to find the optimum proportion of area allocated to redgram and safflower that maximizes gross returns. The experiment consisted of five treatments with five replications in Randomized Complete Block (RCB) Design. Treatment details are as given below

T_1: Sole Redgram

T_2: Redgram + Safflower in the row ratio of 2:1

T_3: Redgram + Safflower in the row ratio of 1:2

T_4: Redgram + Safflower in the row ratio of 1:1

T_5: Sole Safflower

In this experiment the five treatments are the proportions of area allocated to crops redgram and safflower *viz.* 1: 0, 2/3:1/3, 1/3:2/3, 1/2:1/2, 0:1. One can see that all these treatments satisfy conditions of experiments with mixtures *i.e.* the sum of components (area allocated) is one. Therefore, this can be analyzed by using experiment with mixtures methodology and results are compared with the results of usual ANOVA method.

For analysis purpose gross returns are calculated for fourteen different relative price ratios. In selecting these price ratios, we have kept in mind both small and large variations in prices of two component crops. The data on gross returns obtained from different treatments were analyzed by usual analytical procedure of RCB design. The treatment differences were found to be significant at 5% level of significance at all the price ratios and block differences were not significant. The mean gross returns for different price ratios along with critical differences are presented in Table 1 and maximum response and its 95% confidence intervals are presented in Table 2.

Table 1. Mean gross returns (Rs./plot) obtained from intercropping treatments and critical differences for different price ratios obtained by ANOVA

Relative Price Ratio	Treatment means					CD at 5%
	T_1	T_2	T_3	T_4	T_5	
1:0.75 (P_1)	2.85	3.13	3.70	3.27	3.32	0.51
1:0.80 (P_2)	2.85	3.25	3.90	3.42	3.54	0.53
1:0.85 (P_3)	2.85	3.37	4.10	3.57	3.76	0.55
1:0.90 (P_4)	2.85	3.50	4.31	3.73	3.98	0.57
1:0.95 (P_5)	2.85	3.62	4.51	3.88	4.20	0.60
1:1.00 (P_6)	2.85	3.74	4.72	4.03	4.42	0.62
1:1.05 (P_7)	2.85	3.87	4.92	4.18	4.65	0.65
1:1.10 (P_8)	2.85	3.99	5.12	4.33	4.87	0.67
1:1.15 (P_9)	2.85	4.11	5.33	4.49	5.09	0.70
1:1.20 (P_{10})	2.85	4.24	5.53	4.64	5.31	0.73
1:1.25 (P_{11})	2.85	4.36	5.74	4.79	5.53	0.75
1:1.30 (P_{12})	2.85	4.49	5.94	4.94	5.75	0.78
1:1.50 (P_{13})	2.85	4.98	6.76	5.55	6.64	0.89
1:1.75 (P_{14})	2.85	5.60	7.78	6.31	7.74	1.03

It can be observed from Table 1 that for all the relative price ratios, the gross returns obtained from T_3 (row ratio of red gram to safflower 1:2) were maximum and significantly more than T_1 and T_2. Also the gross returns obtained were significantly more than the gross returns obtained from T_4 (except P_1 and P_2) but at par with T_5, indicating that for maximum gross returns one third area should be allocated to red gram and two third area to safflower.

The data for different price ratios were also analyzed as per experiments with mixtures methodology. The observations unadjusted for block effects were considered, as the block differences were not significant. The estimates of parameters for different price ratios, optimum area to be allocated to red gram by experiments with mixtures, optimum response and its 95% confidence intervals are presented in Table 2.

Table 2. Parameters of fitted equation, optimum area to be allotted to red gram, gross returns with confidence limits for different price ratios

Price Ratio	By ANOVA		By Experiments with Mixture				
	x_{1Max}	95% CI for estimated yield at x_{1max}	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_{12}$	x_{1opt}	95% CI for $\hat{y}_{opt}$
P_1	33.3	3.70 ± 0.36	2.795** (0.170)	3.387** (0.170)	1.160 ^{ns} (0.687)	24.49	3.46 ± 0.21
P_2	33.3	3.90 ± 0.37	2.795** (0.178)	3.610** (0.178)	1.368 ^{ns} (0.718)	20.20	3.67 ± 0.23
P_3	33.3	4.10 ± 0.39	2.794** (0.186)	3.833** (0.186)	1.576 * (0.750)	17.05	3.88 ± 0.25
P_4	33.3	4.31 ± 0.41	2.794** (0.194)	4.056** (0.194)	1.783* (0.783)	14.62	4.09 ± 0.27
P_5	33.3	4.51 ± 0.42	2.794** (0.203)	4.279 ** (0.203)	1.991 * (0.818)	12.71	4.31 ± 0.29
P_6	33.3	4.72 ± 0.44	2.793** (0.211)	4.502** (0.211)	2.199* (0.853)	11.15	4.53 ± 0.32
P_7	33.3	4.92 ± 0.46	2.793** (0.220)	4.724** (0.220)	2.406* (0.889)	9.86	4.75 ± 0.34
P_8	33.3	5.12 ± 0.48	2.793** (0.230)	4.947** (0.230)	2.614** (0.926)	8.78	4.97 ± 0.36
P_9	33.3	5.33 ± 0.49	2.792** (0.239)	5.170** (0.239)	2.822** (0.964)	7.86	5.19 ± 0.39
P_{10}	33.3	5.53 ± 0.51	2.792** (0.248)	5.393** (0.248)	3.029** (1.002)	7.06	5.41 ± 0.41
P_{11}	33.3	5.74 ± 0.53	2.792** (0.258)	5.616** (0.258)	3.236** (1.041)	6.37	5.63± 0.44
P_{12}	33.3	5.94 ± 0.55	2.791** (0.268)	5.839** (0.268)	3.444** (1.081)	5.76	5.85 ± 0.46
P_{13}	33.3	6.76 ± 0.63	2.791** (0.308)	6.731** (0.308)	4.275** (1.242)	3.91	6.74 ± 0.56
P_{14}	33.3	7.78 ± 0.73	2.790** (0.359)	7.845** (0.359)	5.313** (1.449)	2.41	7.85 ± 0.69

Figures in parenthesis are standard errors
* and ** significant at 5% and 1% level of significance respectively, and
ns: indicate non-significant
CI : Confidence interval
x_{1max} : denotes the percentage of the area allocated to redgram that gives maximum gross returns through RCB design analysis
x_{1opt} : denotes the percentage of the area allocated to redgram that gives maximum gross returns through experiments with mixtures methodology

It was observed from Tables 1 and 2 that if we use analytical procedure of RCB design then for all price ratios, the row ratio is constant. But if we analyze the same data by experiment with mixture methodology then this area will not remain constant. The area to be allocated to red gram decreased with decrease in the price ratio of red gram: safflower *i.e.* as price of safflower increases, area to be allocated to red gram decreases. The area to be allocated to red gram decreased gradually from 24.49 percent to 2.41 percent. The minimum area to be allocated to red gram was 2.41 percent for the price ratios 1:1.75 and it was maximum (24.49 percent) for price ratio 1:0.75. This indicated that for price ratio 1:0.75 nearly 25 percent (*i.e.* row ratio of red gram: safflower should be 1:3) area should be allocated to red gram so as to get maximum gross returns (Rs.3.46 ± 0.21 per plot), whereas if price ratio is 1:1.75, area allocated to red gram is 2.41% *i.e.* it would be better to have sole crop of safflower. This type of results cannot be obtained from ANOVA method.

It can also be seen that the 95% confidence intervals of optimum predicted response obtained by using experiment with mixtures method were smaller than the 95% confidence intervals obtained by ANOVA method. Thus it indicates that experiments with mixtures can usefully be employed for analysis of replacement series intercropping experiments with sole crop treatments.

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REFERENCES

[1] Batra, P.K., Parsad, R., Gupta, V.K., Khanduri, O.P. (1999). Strategy for analysis of experiments involving split application of fertilizers. *Statistics and Application*, **1(2)**, 175-187.

[2] Cornell, J.A. (1981). *Experiments with Mixtures: Designs, Models and Analysis of Mixture Data.* John Wiley & Sons, New York.

[3] Murthy, J.S. and Das, M.N. (1968). Design and analysis of experiments with mixtures. *Ann. Math. Statist.*, **39**, 1517-1539.

[4] Nigam, A.K. (1969). *Contribution to design and analysis of experiments with mixtures*. Unpublished Ph. D. thesis, B. H. U. , Banaras.

[5] Scheffe, H. (1958). Experiments with mixture. *J. Roy. Statist. Soc.*, **B20**, 344-360.

[6] Scheffe, H. (1963). The simplex centroid design for experiments with mixtures. *J. Roy. Statist. Soc.*, **B25**, 235-263.