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Improved Ratio-type Estimator for Variance Using Auxiliary Information

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SUMMARY

For estimating the population variance using information on several auxiliary variables, an improved ratio-type estimator is defined. It is shown that proposed estimator is more efficient than the usual unbiased estimator and the Isaki [5] estimator. Double sampling version of the suggested estimator is also discussed.

Key words : Ratio-type estimator, Multi-auxiliary information, Multivariate normal population.

1. Introduction

Suppose we have a population of N identifiable units on which (k + 1) characteristics $y, x_1, x_2, ..., x_k$ are defined. Here y is the characteristic of interest and x_i, (i = 1, 2, ..., k) are auxiliary characteristics whose population variances σ_i^2, (i = 1, 2, ..., k) are assumed to be known. Let (y_h, x_{ih}) denote the values of y and x_i on the unit h. Assume that a simple random sample of size n is drawn with replacement, (y_h, x_{ih}), (h = 1, 2, ..., n) are observed. Now assume that the problem is to estimate the population variance

$$S_y^2 = \left(\frac{1}{N}\right)\sum_{h=1}^{N} (y_h - \overline{Y})^2 = \theta \sum_{h \neq j}^{N} (y_h - y_j)^2, \theta = [N(N-1)]^{-1}$$

$$\overline{Y} = \left(\frac{1}{N}\right)\sum_{h=1}^{N} y_h$$

when S_x^2, the population variance of auxiliary character x is known, Isaki [5] considered the problem of estimating S_y^2 and suggested univariate, multivariate ratio and regression estimators with their properties. In this paper, we have proposed a multivariate ratio-type estimator for population variance of y and its properties are discussed in single phase as well as in two phase sampling.

Let $z = (y, \underset{\sim}{x})$ denote a $1 \times (k + 1)$ vector where the population variances of each component of the vector $\underset{\sim}{x}$ are assumed to be known. Let

$$\mu = (\mu_0, \mu_1, \mu_2, \ldots, \mu_k)' \text{ and } \Omega = (\sigma_{ij})$$

denote the mean and covariance of z where

$$\left.\begin{aligned} \sigma_{ij} &= \sigma_i^2 \quad \text{if } i = j, (i, j) = 0, 1, 2, \ldots, k \\ &= \rho_{ij}\, \sigma_i\, \sigma_j \text{ otherwise} \end{aligned}\right\} \tag{1.1}$$

In particular $\rho_{ij} = \rho, (i, j) = 0, 1, 2, \ldots, k$ the model (1.1) reduces to

$$\left.\begin{aligned} \sigma_{ij} &= \sigma_i^2 \quad \text{if } i = j, (i, j) = 0, 1, 2, \ldots., k \\ &= \rho\sigma_i\sigma_j \text{ otherwise} \end{aligned}\right\} \tag{1.2}$$

and $-k^{-1} < \rho < 1$. Assume that z possesses the same moments as a $1 \times (k + 1)$ multivariate normal variable up to the fourth order. Let

$$s_{ij} = (n - 1)^{-1} \sum_{h=1}^{n} (z_{ih} - \bar{z}_{i.}) z_{jh}, i, j = 0, 1, 2, \ldots, k \tag{1.3}$$

Then, following Olkin [10], Isaki [5] constructed the multivariate ratio estimator for σ_0^2 as

$$\hat{S}_{ym1}^2 = \sum_{i=1}^{k} w_i \hat{r}_i\, \sigma_i^2 \tag{1.4}$$

where $\hat{r}_i = s_{ii}^{-1}\, s_{00} = \left(\frac{s_0^2}{s_i^2}\right), i = 1, 2, \ldots, k; 0 < w_i < 1;$ $\sum_{i=1}^{k} w_i = 1$ and it is assumed that the σ_i^2, $i = 1, 2, \ldots, k$; are known while ρ and σ_0^2 are unknown.

Motivated by Shukla [14] and John [6] and Mohanty and Pattanaik [8], one may define the following alternative multivariate ratio-type estimators for σ_0^2 as

$$\hat{S}_{ym2}^2 = s_0^2 \left[\frac{\left(\sum_{i=1}^{k} w_i\, \sigma_i^2\right)}{\left(\sum_{i=1}^{k} w_i s_i^2\right)}\right] \tag{1.5}$$

$$\hat{S}^2_{ym3} = \prod_{i=1}^{k} \left(\hat{r}_i \, \sigma_i^2\right)^{w_i} \tag{1.6}$$

and

$$\hat{S}^2_{ym4} = \left(\sum_{i=1}^{k} \frac{w_i}{\hat{r}_i \, \sigma_i^2} \right)^{-1} \tag{1.7}$$

where w_i's (i = 1, 2, ..., k) are the weights such that $\sum_{i=1}^{k} w_i = 1$.

We note from Agrawal and Panda [1] that with $w_i' = \dfrac{\left(w_i \prod_{j \neq i}^{k} \sigma_j^2\right)}{\sum_{i=1}^{k} \left(w_i \prod_{j \neq i}^{k} \sigma_j^2\right)}$ and $\sum_{i=1}^{k} w_i' = 1$, the estimator S^2_{ym4} takes the form

$$\hat{S}'^2_{ym4} = s_0^2 \left[\frac{\left(\sum_{i=1}^{k} w_i' \, \sigma_i^2 \right)}{\left(\sum_{i=1}^{k} w_i' \, s_i^2 \right)} \right]$$

which is similar to Shukla [14] and John [6] type estimator $\hat{S}^2_{ym2}$.

Under the set up (1.2) the common minimum variance of the estimators $\hat{S}^2_{ymj}$ $(j = 1 \text{ to } 4)$ to the first degree of approximation is given by

$$\underset{j=1 \text{ to } 4}{\min.\,\mathrm{Var}}\left(\hat{S}^2_{ymj}\right) = \left[k(n-1)\right]^{-1} \left[2(k+1)\left(1-\rho^2\right)\sigma_0^4\right] \tag{1.8}$$

With the assumption that z possesses the same moments as in the above, the variance of s_0^2 is

$$\mathrm{Var}\left(s_0^2\right) = (n-1)^{-1} \, 2\sigma_0^4 \tag{1.9}$$

If follows from (1.8) and (1.9) that

$$\min_{j=1 \text{ to } 4} . \mathrm{Var}\left(\hat{S}_{ymj}^2\right) \le \mathrm{Var}\left(s_0^2\right) \text{iff } |\rho| \ge (k+1)^{-1/2} \tag{1.10}$$

The other relevant references related with the present investigation are Das and Tripathi [4], Srivastava and Jhajj [16], Prasad and Singh ([11] and [12]), Biradar and Singh [3], Singh *et al.* [15], Sthapit [17], Arcos Ceberian and Rueda Gracia [2], Rueda Gracia and Arcos Ceberain [13] and Upadhyaya and Singh [20] etc.

2. The Suggested Estimator and its Variance

We suggest the estimator for σ_0^2 as

$$\hat{S}_{ym}^{*2} = \sum_{i=1}^{k} w_i \, \hat{r}_i \, \sigma_i^2 + w_{k+1} \, s_0^2 \tag{2.1}$$

where $\theta < w_i < 1, \sum_{i=1}^{k+1} w_i = 1$

It is noticed from (2.1) that $\hat{S}_{ym}^{*2}$ reduces to the usual unbiased estimator s_0^2 if $w_i = 0$ for $i = 1, 2, ..., k$ and it reduces to Isaki [5] estimator $\hat{S}_{ym1}^2$ for $w_{k+1} = 0$. Under the set up (1.1) the variance of $\hat{S}_{ym}^{*2}$ to the first degree of approximation is given by

$$\mathrm{Var}\left(\hat{S}_{ym}^{*2}\right) = \left(\frac{2\sigma_0^4}{n-1}\right)[1 - 2\,\underset{\sim}{d}'\underset{\sim}{w} + \underset{\sim}{w}'\underset{\sim}{D}\,\underset{\sim}{w}] \tag{2.2}$$

where $\underset{\sim}{D} = \begin{bmatrix} 1 & \rho_{12}^2 & \cdots & \rho_{1(k-1)}^2 & \rho_{1k}^2 \\ \rho_{21}^2 & 1 & \cdots & \rho_{2(k-1)}^2 & \rho_{2k}^2 \\ \cdots & \cdots & \cdots\ \cdots & & \cdots \\ \rho_{(k-1)1}^2 & \rho_{(k-1)2}^2 & \cdots & 1 & \rho_{(k-1)k}^2 \\ \rho_{k1}^2 & \rho_{k2}^2 & \cdots & \rho_{k(k-1)}^2 & 1 \end{bmatrix}_{k \times k}$ and

$$\underset{\sim}{d}' = \begin{bmatrix} \rho_{01}^2 & \rho_{02}^2 & \rho_{03}^2 & \cdots & \rho_{0k}^2 \end{bmatrix}_{1 \times k}$$

Minimization of (2.2) yields

$$\underset{\sim}{w} = D^{-1}\,\underset{\sim}{d} \text{ and } w_{k+1} = (1 - \underset{\sim}{D}^{-1}\,\underset{\sim}{d}) \tag{2.3}$$

Substitution of (2.3) in (2.2) gives the minimum variance of $\hat{S}^{*2}_{ym}$ as

$$\min.\mathrm{Var}\left(\hat{S}^{*2}_{ym}\right)=\frac{2\sigma_0^4}{(n-1)}(1-\underset{\sim}{d}'\underset{\sim}{D}\underset{\sim}{d}) \tag{2.4}$$

which is smaller than that of the usual unbiased estimator s_0^2.

Under the set up (1.2), the variance of $\hat{S}^{*2}_{ym}$ to the first degree of approximation is given by

$$\mathrm{Var}\left(\hat{S}^{*2}_{ym}\right)=\left(\frac{2\sigma_0^4}{n-1}\right)[1-2\,\underset{\sim}{d}^{*'}\underset{\sim}{w}+\underset{\sim}{w}'\underset{\sim}{D}^{*}\underset{\sim}{w}] \tag{2.5}$$

which is minimized for

$$\underset{\sim}{w}=\underset{\sim}{D}^{*-1}\underset{\sim}{d}^{*} \text{ and } w_{k+1}=(1-\underset{\sim}{D}^{*-1}\underset{\sim}{d}^{*}) \tag{2.6}$$

with

$$\underset{\sim}{d}^{*}=\begin{bmatrix}\rho^2\\ \rho^2\\ \rho^2\\ \vdots\\ \rho^2\end{bmatrix}_{k\times 1} \quad \text{and } \underset{\sim}{D}^{*}=\begin{bmatrix}1 & \rho^2 & \dots & \rho^2\\ \rho^2 & 1 & \dots & \rho^2\\ \dots & \dots & \dots & \dots\\ \rho^2 & \rho^2 & \dots & \rho^2\\ \rho^2 & \rho^2 & \dots & 1\end{bmatrix}_{k\times k}$$

Substitution of (2.6) in (2.5) yields the minimum variance of $\hat{S}^{*2}_{ym}$ is given by

$$\mathrm{miv.Var}\left(\hat{S}^{*2}_{ym}\right)=\frac{2\sigma_0^4}{(n-1)}(1-\underset{\sim}{d}^{*'}\underset{\sim}{D}^{*}\underset{\sim}{d}^{*})$$

$$=\frac{2\sigma_0^4}{(n-1)}\left[1-\frac{k\rho^4}{\left(1+(k-1)\rho^2\right)}\right] \tag{2.7}$$

Thus to a first order of approximation, the estimator $\hat{S}^{*2}_{ym}$ is as efficient as a multivariate regression estimator. We mention in passing that the optimal choice of the weights w_i involves population quantity (ρ) which is unknown. However, this can be either assessed quite accurately from the past experience or estimated in a reasonable way from the sample at hand, for instance, see Murthy [9] and Tankou and Dharmadhikari [19].

We have from (1.10) and (2.7) that

$$\min.\operatorname{Var}\left(\hat{S}^{*2}_{ym}\right) - \min_{j=1\text{ to }4}.\operatorname{Var}\left(\hat{S}^{*2}_{mj}\right) = \frac{2\sigma_0^4}{(n-1)}\,\frac{\left(1-\rho^2\right)^2}{k\left\{1+(k-1)\rho^2\right\}} \geq 0 \quad (2.8)$$

which implies that

$$\min.\operatorname{Var}\left(\hat{S}^{*2}_{ym}\right) \leq \min.\operatorname{Var}\left(\hat{S}^{2}_{ym}\right) \quad (2.9)$$

It follows from (1.12) and (2.9) that

$$\min.\operatorname{Var}\left(\hat{S}^{*2}_{ym}\right) \leq \min_{j=1\text{ to }4}.\operatorname{Var}\left(\hat{S}^{2}_{ymj}\right) \leq \operatorname{Var}\left(s_0^2\right) \text{iff } |\rho| \geq (k+1)^{-1/2} \quad (2.10)$$

Further, we have from (1.11) and (2.7) that

$$\operatorname{Var}\left(s_0^2\right) - \min.\operatorname{Var}\left(\hat{S}^{*2}_{ym}\right) = \frac{2\sigma_0^4}{(n-1)}\,\frac{k\rho^4}{\left[1+(k-1)\rho^2\right]} \geq 0 \quad (2.11)$$

Thus it follows from (2.8) and (2.11) that the suggested estimator $\hat{S}^{*}_{ym}$ is more efficient than the usual unbiased estimator s_0^2 and Isaki [5] estimator $\hat{S}^{2}_{ym1}$.

3. Two Phase Sampling

If the required auxiliary information is not readily available for the population before sampling, it might pay to collect such information for a large preliminary sample and then collect more precise information for the variable of interest on a second phase sample. This procedure is known as two phase sampling (or double sampling), is very much use in the practice. The second phase sample may be either (i) a subsample of the large first-phase sample (designated as Case I) or (ii) it may be selected independently (designated as Case II).

Let the first and second phase sample sizes be n_1 and n respectively and

$$s^{*}_{ij} = \left(n_1 - 1\right)^{-1} \sum_{h=1}^{n_1} \left(z_{ih} - \bar{z}^{*}_{i.}\right) z_{jh}$$

and

$$s_{ij} = (n-1)^{-1} \sum_{h=1}^{n} \left(z_{ih} - \bar{z}_{i.}\right) z_{jh},\ (i, j) = 0, 1, 2, \ldots, k$$

represents the corresponding covariances of the auxiliary variables

$$\bar{z}^*_{i.} = \sum_{h=1}^{n_1} \frac{z_{ih}}{n_1} \text{ and } \bar{z}_{i}. = \sum_{h=1}^{n} \frac{z_{ih}}{n}$$

It is to be noted that the samples have been drawn by simple random sampling with replacement (SRSWR) scheme at both the phases. We simplify the notation by setting $s_{ii} = s_i^2, s^*_{ii} = s_i^{*2}$ and $s_{00} = s_0^2$. Now, we define the multivariate ratio estimator for σ_0^2 as

$$\hat{S}^2_{yd} = \sum_{i=1}^{k} w_i \hat{r}_i s_i^{*2} + w_{k+1} s_0^2; \sum_{i=1}^{k+1} w_i = 1 \tag{3.1}$$

For $w_{k+1} = 0, \hat{S}^2_{yd}$ reduces to

$$\hat{S}^2_{yd1} = \sum_{i=1}^{k} w_i \hat{r}_i s_i^{*2}, \sum_{i=1}^{k} w_i = 1 \tag{3.2}$$

which is double sampling version of Isaki [5] estimator $\hat{S}^2_{ym1}$ in (1.4). If we set $w_{k+1} = 0, \hat{S}^2_{yd}$ boils down to the usual unbiased estimator s_0^2.

Replacing σ_i^2 by s_i^{*2} in (1.5), (1.6) and (1.7) we get double sampling versions of $\hat{S}^2_{ym2}, \hat{S}^2_{ym3}$ and $\hat{S}^2_{ym4}$ respectively as

$$\hat{S}^2_{yd2} = s_0^2 \left[\frac{\left(\sum_{i=1}^{k} w_i s_i^{*2} \right)}{\left(\sum_{i=1}^{k} w_i s_i^2 \right)} \right] \tag{3.3}$$

$$\hat{S}^2_{yd3} = \prod_{i=1}^{k} \left(\hat{r}_i s_i^{*2} \right)^{w_i} \tag{3.4}$$

and

$$\hat{S}^2_{yd4} = \left(\sum_{i=1}^{k} \frac{w_i}{\hat{r}_i s_i^{*2}} \right)^{-1} \tag{3.5}$$

where w_i's $(i = 1, 2, ..., k)$ are suitably chosen constants such that $\sum_{i=1}^{k} w_i = 1$.

Now, we write the variances of $\hat{S}^2_{yd}$ in two cases

Case I: When the second phase sample is a subsample of first, then the variance of $\hat{S}^2_{yd}$ under set up (1.2), to the first degree of approximation, is given by

$$\operatorname{Var}\left(\hat{S}^2_{yd}\right)_I = \frac{2\sigma_0^4}{(n-1)}\left[1 + \frac{(n_1 - n)}{(n_1 - 1)}(\underset{\sim}{w}' \underset{\sim}{D}^* \underset{\sim}{w} - 2\underset{\sim}{w}' \underset{\sim}{d}^*)\right] \tag{3.6}$$

which is minimized for

$$\underset{\sim}{w} = \underset{\sim}{D}^{*-1} \underset{\sim}{d}^*, \; w_{k+1} = (1 - \underset{\sim}{D}^{*-1} \underset{\sim}{d}^*) \tag{3.7}$$

or

$$\left.\begin{aligned} w_i &= \frac{\rho^2}{\left[1 + (k-1)\rho^2\right]}; i = 1, 2, \ldots, k \\ w_{k+1} &= \frac{\left(1 - \rho^2\right)}{\left[1 + (k-1)\rho^2\right]} \end{aligned}\right\} \tag{3.8}$$

Thus the minimum variance of $\hat{S}^2_{yd}$ is given by

$$\min.\operatorname{Var}\left(\hat{S}^2_{yd}\right)_I = \frac{2\sigma_0^4}{(n-1)}\left[1 - \frac{(n_1 - n)}{(n_1 - 1)} \frac{k\,\rho^4}{\left\{1 + (k-1)\rho^2\right\}}\right] \tag{3.9}$$

which is equal to the approximate variance of the usual double sampling multivariate regression estimator

$$\hat{S}^2_{yrd} = s_0^2 + \sum_{i=1}^{k} \hat{\beta}_i \left(s_i^{*2} - s_i^2\right) \tag{3.10}$$

where $\hat{\beta}_i = \frac{s_{0i}}{s_i^2}, i = 1, 2, \ldots, k$

Case II: When the second phase sample is independent of the first phase, then the variance of $\hat{S}^2_{yd}$ under set up (1.2), to the first degree of approximation, is given by

$$\operatorname{Var}\left(\hat{S}^2_{yd}\right)_{II} = \frac{2\sigma_0^4}{(n-1)}\left[1 - 2\,\underset{\sim}{w}' \underset{\sim}{d}^* + \frac{(n_1 + n - 2)}{(n_1 - 1)}(\underset{\sim}{w}' \underset{\sim}{D}^* \underset{\sim}{w})\right] \tag{3.11}$$

which is minimized for

$$w = \frac{(n_1 - 1)}{(n_1 + n - 2)} \underset{\sim}{D}^{*-1} \underset{\sim}{d}^{*}, \; w_{k+1} = 1 - \frac{(n_1 - 1)}{(n_1 + n - 2)} \underset{\sim}{D}^{*-1} \underset{\sim}{d}^{*} \tag{3.12}$$

or

$$\left.\begin{aligned} w_i &= \frac{(n_1 - 1)}{(n_1 + n - 2)} \frac{\rho^2}{\left[1 + (k-1)\rho^2\right]}, \; i = 1, 2, \ldots, k \\ w_{k+1} &= \left[1 - \frac{(n_1 - 1)}{(n_1 + n - 2)} \frac{\rho^2}{\left[1 + (k-1)\rho^2\right]}\right] \end{aligned}\right\} \tag{3.13}$$

Hence the resulting (minimum) variance of $\hat{S}^2_{yd}$ is given by

$$\min.\text{Var}\left(\hat{S}^2_{yd}\right)_{II} = \frac{2\sigma_0^4}{(n-1)}\left[1 - \frac{(n_1 - 1)}{(n_1 + n - 2)} \frac{k\rho^4}{\left\{1 + (k-1)\rho^2\right\}}\right] \tag{3.14}$$

It follows from (3.9) and (3.14) that

$$\min.\text{Var}\left(\hat{S}^2_{yd}\right)_I - \min.\text{Var}\left(\hat{S}^2_{yd}\right)_{II} = \frac{2(n-1)\sigma_0^4}{(n_1 - 1)(n_1 + n - 2)} \frac{k\,\rho^4}{\left[1 + (k-1)\rho^2\right]} > 0 \tag{3.15}$$

which shows that the estimator $\hat{S}^2_{yd}$ in case II is always more efficient than in case I.

Further, in cases I and II the common minimum variance of $\hat{S}^2_{ydj}$ $(j = 1 \text{ to } 4)$ for the set up (1.2) are respectively given by

$$\underset{j=1 \text{ to } 4}{\min.\text{Var}}\left(\hat{S}^2_{ydj}\right)_I = \frac{2\sigma_0^4}{k(n-1)}\left[(k+1)(1-\rho^2) + \left(\frac{n-1}{n_1 - 1}\right)\left\{(k+1)\rho^2 - 1\right\}\right] \tag{3.16}$$

and

$$\underset{j=1 \text{ to } 4}{\min.\text{Var}}\left(\hat{S}^2_{ydj}\right)_{II} = \frac{2\sigma_0^4}{k(n-1)}\left[(k+1)(1-\rho^2) + \left(\frac{n-1}{n_1 - 1}\right)\left\{(k-1)\rho^2 + 1\right\}\right] \tag{3.17}$$

From (3.16) and (3.17), we have

$$\underset{j=1 \text{ to } 4}{\min.\text{Var}}\left(\hat{S}^2_{ydj}\right)_{II} - \underset{j=1 \text{ to } 4}{\min.\text{Var}}\left(\hat{S}^2_{ydj}\right)_I = \frac{4(1-\rho^2)\sigma_0^4}{k(n_1 - 1)} > 0 \tag{3.18}$$

which follows that multivariate ratio-type estimator $\hat{S}^2_{ydj}$ (j = 1 to 4) has smaller minimum variance in case I than that in Case II.

From (1.9), (3.9) and (3.16) we have

$$\operatorname{Var}\left(s_0^2\right)-\underset{j=1\text{ to }4}{\min.}\operatorname{Var}\left(\hat{S}^2_{ydj}\right)_I = 2\left[\frac{1}{n-1}-\frac{1}{n_1-1}\right]\frac{\left\{(k+1)\rho^2-1\right\}\sigma_0^4}{k}$$

$$\geq 0 \text{ iff } |\rho| \geq (k+1)^{-1} \tag{3.19}$$

and

$$\underset{j=1\text{ to }4}{\min.}\operatorname{Var}\left(\hat{S}^2_{ydj}\right)_I - \min.\operatorname{Var}\left(\hat{S}^2_{yd}\right)_I = 2\sigma_0^4\left(\frac{1}{n-1}-\frac{1}{n_1-1}\right)\frac{\left(1-\rho^2\right)^2}{k\left\{1+(k-1)\rho^2\right\}} \geq 0 \tag{3.20}$$

Thus we have the following inequality

$$\min.\operatorname{Var}\left(\hat{S}^2_{yd}\right)_I \leq \min.\operatorname{Var}\left(\hat{S}^2_{ydj}\right)_I \leq \min.\operatorname{Var}\left(\hat{S}^2_{yd}\right)_I \text{ if } |\rho| \geq (k+1)^{-1} \tag{3.21}$$

Further from (1.9), (3.14) and (3.17) we have

$$\operatorname{Var}\left(s_0^2\right)-\underset{j=1\text{ to }4}{\min}\operatorname{Var}\left(\hat{S}^2_{ydj}\right)=\frac{2\sigma_0^4}{k}\left(\frac{1}{n-1}+\frac{1}{n_1-1}\right)\left[\rho^2\left\{1+\frac{(n_1-n)}{(n_1+n-2)}k\right\}-1\right]$$

$$\geq 0 \text{ iff } |\rho| \geq \left[\frac{(n_1-n)k}{(n_1+n-2)}+1\right]^{-1/2} \tag{3.22}$$

and

$$\underset{j=1\text{ to }4}{\min.}\operatorname{Var}\left(\hat{S}^2_{ydj}\right)-\min.\operatorname{Var}\left(\hat{S}^2_{yd}\right)_{II}$$

$$=\frac{2\sigma_0^4}{k}\frac{\left[(n_1-1)\left(1-\rho^2\right)+(n-1)\left\{1+(k-1)\rho^2\right\}\right]^2}{(n_1-1)(n_1+n-2)\left\{1+(k-1)\rho^2\right\}} \geq 0 \tag{3.23}$$

Thus from (3.22) and (3.23) we have the following inequality

$$\min.\operatorname{Var}\left(\hat{S}^2_{yd}\right)_{II} \leq \underset{j=1\text{ to }4}{\min.}\operatorname{Var}\left(\hat{S}^2_{ydj}\right) \leq \operatorname{Var}\left(s_0^2\right) \text{if } |\rho| \geq \left[\frac{(n_1-n)k}{(n_1+n-2)}+1\right]^{-1/2} \tag{3.24}$$

Finally, we conclude that proposed estimator $\hat{S}^2_{yd}$ is better than s_0^2 and $\hat{S}^2_{ydj}$ $(j = 1 \text{ to } 4)$ in both the cases I and II.

Remark 3.1 : The efficiencies of the estimators discussed in this paper can be compared for fixed cost, following the procedure given in Sukhatme *et al.* [18] and Khan and Tripathi [7].

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