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Estimation in Mail Surveys Under PSNR Sampling Scheme

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SUMMARY

In the set up of stratified scheme, when frames of each stratum are not available survey practitioners are advised to use post-stratification scheme for the estimation of population parameters. Suppose post-stratified units of the sample have a few respondents and a large number of non-respondents then estimation of population parameter is difficult. This is a most likely situation in mail surveys where questionnaires are mailed to respondents and expected to return after completion, earlier to a deadline. This paper presents a new "Post-stratified non-response (PSNR)" sampling scheme in order to handle a high rate of non-response likely to occur in mail surveys. The scheme has several steps to complete before the estimation of population mean. An unbiased estimator has been found and its precision is examined. The cost analysis is incorporated to estimate the optimal sample size required under prefix cost. Due to certain limitations, a new alternative approach is proposed and observed solution oriented for cost optimal sample size. All the derived results are numerically supported and the PSNR scheme is found useful and effective for the mean estimation.

Key words : Post-stratification, SRSWOR, Optimal, PSNR, Respondents (RS), Non-respondents (NRS).

1. Introduction

A mail survey is cheaper and faster in the present days of advanced electronic communication but, it fails to achieve the object of cent percent response. Suppose a population is stratified naturally, with known size of strata but, without the list of units of each stratum like a voter list of a city which cannot provide frames if strata are made as lower, middle and higher income groups. One could get information from other records that there are 30% lower, 60% middle and 10% higher-income group voters in the city and this needs to be utilized. It motivates to choose post-stratified sampling scheme where a random sample is drawn from the population by SRSWOR and stratified according to stratum formed by information of other records. If these post–stratified units of individuals (voters) are mailed questionnaires and are expected to return back till the deadline there may be a few respondents and a huge amount of non-response. To cope with this, a motivation is derived from Hansen and Hurwitz [2], Agrawal and Panda [1] and a post-stratified non-response (PSNR) scheme is developed.

2. *Notations*

Let N be the size of a population divided into k strata each of size N_i, $(i = 1, 2, 3, ..., k)$ such that $\sum_{i=1}^{k} W_i = \frac{N_i}{N} = 1$. Further, i^{th} strata contains N_i' respondents (say RS) and N_i'' non–respondents (say NRS) and these are suppose to be known like a stratum of literate people has 70% response while that of illiterate has only 30%. The variable Y is under study with population mean $\overline{Y}_i$ of i^{th} strata and $\overline{Y} = \sum_{i=1}^{k} W_i \overline{Y}_i$ alongwith population mean squares in usual notations, S^2 and S_i^2. Within the i^{th} strata, response group (R) has S_{1i}^2 and non-response group (NR) has S_{2i}^2 as their population mean squares. To note that $N_i = N_i' + N_i''$.

3. *A Post Stratified Non-response (PSNR) Sampling Scheme*

Step I : Select a sample of size n by SRSWOR from the population N and post-stratified into k strata, such that n_i units represent to $N_i \left(\sum_{i=1}^{k} n_i = n \right)$. An auxiliary source of information or prior guess may be used for this purpose.

Step II : Mail questionnaires to all the random n_i units for response over the variable Y under study and wait until a deadline. If there is a possible complete response, the $\overline{y}_i$ is sample mean from i^{th} strata and $\overline{y} = (n)^{-1} \sum_{i=1}^{k} n_i \overline{y}_i$.

Step III : Assume that non-response observed when the deadline of returning questionnaire is over, and there are n_i' respondents, n_i'' non-respondents in the i^{th} strata $(n_i' + n_i'' = n_i)$. The $\overline{y}_i'$ is the mean of responding n_i' units.

Step IV : From non-responding n_i'', select sub-samples of size n_i''' by SRSWOR, maintaining a prefixed fraction $f_i = \left(\frac{n_i''}{n_i'''}\right)$ over all the k strata.

Step V : Conduct a personal interview for n_i''' units and assume all these responded well during that period. The $\bar{y}_i'''$ is the mean based on n_i'''.

4. Estimation Under PSNR

Deriving a motivation from Hansen and Hurwitz [2], to estimate $\bar{Y}$, the proposed estimation strategy is

$$\bar{y}_{PSNR} = \sum_{i=1}^{k} W_i \left[\left(\frac{n_i'}{n_i}\right)\bar{y}_i' + \left(\frac{n_i''}{n_i}\right)\bar{y}_i'''\right] \tag{4.1}$$

Theorem 4.1: $\bar{y}_{PSNR}$ is unbiased for $\bar{Y}$.

Proof: The conditional expectation of $\bar{y}_{PSNR}$ given n_i and (n_i', n_i'') is denoted by $E[(.)/n_i, (n_i', n_i'')]$. Also, we know $E[(\bar{y}_i''')/n_i, (n_i', n_i'')] = \bar{y}_i''$ which is the mean of n_i'' units.

$$E(\bar{y}_{PSNR}) = E[E[E\{\bar{y}_{PSNR}\}/n_i, (n_i', n_i'')]]$$

$$E\left[E\left\{\sum_{i=1}^{k} W_i \left\{\frac{n_i'}{n_i}\right\}\bar{y}_i' + \left(\frac{n_i''}{n_i}\right)E(\bar{y}_i''')\right\}\Big/ n_i, (n_i', n''_i)\right]$$

$$E\left[E\left\{\sum_{i=1}^{k} W_i \left\{\left(\frac{n_i'}{n_i}\right)\bar{y}_i' + \left(\frac{n_i''}{n_i}\right)\bar{y}_i''\right\}\right\}\Big/ n_i\right]$$

$$E\left[\sum W_i \{E(\bar{y}_i)\}/n_i\right] = \sum W_i \bar{Y}_i = \bar{Y}$$

Theorem 4.2: The variance of $\bar{y}_{PSNR}$ is

$$V(\bar{y}_{PSNR}) = \sum_{i=1}^{k} W_i^2 \left[\left\{\frac{1}{nW_i} + \frac{(N-n)(1-W_i)}{n^2(N-1)W_i^2}\right\}\left\{A_i + S_i^2\right\}\right] - \left\{\frac{1}{N}\sum_{i=1}^{k} W_i S_i^2\right\} \tag{4.2.1}$$

where $A_i = \left[\frac{N_i''}{N_i}(f_i - 1)S_{2i}^2\right]$ (4.2.2)

Proof: We use a standard result

$$E\left(\frac{1}{n_i}\right)=\left[\frac{1}{nW_i}+\left\{\frac{(N-n)(1-W_i)}{n^2(N-1)W_i^2}\right\}\right] \qquad (4.2.3)$$

and $V\left(\bar{y}_i'''/n_i,(n_i',n_i'')\right)=\left[\left\{\frac{1}{n_i'''}-\frac{1}{n_i''}\right\}S_{2i}^2\right]$ using a double sampling set-up (4.2.4)

$$V\left[\bar{y}_{PSNR}\right]=E\left[E\left[V\left\{\bar{y}_{PSNR}\right\}/n_i,(n_i',n_i'')\right]\right]+E\left[V\left[E\left\{\bar{y}_{PSNR}\right\}/n_i,(n_i',n_i'')\right]\right] + V\left[E\left[V\left\{\bar{y}_{PSNR}\right\}/n_i,(n_i',n_i'')\right]\right] \qquad (4.2.5)$$

The conditional variance is a sum of three components. Consider the first one

$$E\left[E\left[V\left(\bar{y}_{PSNR}\right)/n_i,(n_i',n_i'')\right]\right]$$

$$=E\left[E\left[V\left\{\sum_{i=1}^{k}\left(\frac{W_i}{n_i}\right)(n_i'\bar{y}_i')+\sum_{i=1}^{k}\left(\frac{W_i}{n_i}\right)(n_i''\bar{y}_i''')\right\}/n_i,(n_i',n_i'')\right]\right]$$

$$=E\left[E\left[\left\{0+\sum_{i=1}^{k}\left(\frac{W_i n_i''}{n_i}\right)^2 V(\bar{y}_i''')\right\}/n_i,(n_i',n_i'')\right]\right]$$

$$=E\left[\left\{\sum\left(\frac{W_i n_i''}{n_i}\right)^2\left\{\left(\frac{1}{n_i'''}\right)-\left(\frac{1}{n_i''}\right)\right\}E\left(s_{2i}^2\right)\right\}/n_i,(n_i',n_i'')\right]$$

$$=E\left[\left\{\sum\left(\frac{W_i^2}{n_i}\right)\left(\frac{n_i''}{n_i}\right)(f_i-1)\left(S_{2i}^2\right)\right\}/n_i,(n_i',n_i'')\right]$$

$$=E\left[\left\{\sum\left(\frac{W_i^2}{n_i}\right)(f_i-1)\left(S_{2i}^2\right)\left(E\left(\frac{n_i''}{n_i}\right)\right)\right\}/n_i\right]$$

$$=\sum W_i^2\left\{E\left(\frac{1}{n_i}\right)\right\}\left(\frac{N_i''}{N_i}\right)(f_i-1)S_{2i}^2 \qquad (4.2.6)$$

$$=\sum W_i^2\left\{E\left(\frac{1}{n_i}\right)\right\}A_i$$

The term s_{2i}^2 is sample mean of the non-response group among n units and a result $E\left[\left\{\frac{n_i''}{n_i}\right\}/n_i\right]=\left(\frac{N_i''}{N_i}\right)$ is used above. The second component provides

$$E[V[E(\bar{y}_{PSNR})/n_i,(n_i',n_i'')]]=E\left[V\left[\sum_{i=1}^{k} W_i\left\{\left(\frac{n_i'}{n_i}\right)\bar{y}_i'+\left(\frac{n_i''}{n_i}\right)E(\bar{y}_i'')\right\}\Big/ n_i,(n_i',n_i'')\right]\right]$$

$$=E\left[V\left\{\sum_{i=1}^{k} W_i\bar{y}_i\right\}\Big/ n_i\right]=E\left[\left\{\sum W_i^2\left\{\left(\frac{1}{n_i}\right)-\left(\frac{1}{N_i}\right)\right\}S_i^2\right\}\Big/ n_i\right]$$

$$=\sum W_i^2\left\{E\left(\frac{1}{n_i}\right)\right\}S_i^2-\sum\left\{\frac{W_i^2S_i^2}{N_i}\right\} \tag{4.2.7}$$

The third component vanishes and the addition of (4.2.6) and (4.2.7) with the substitution of $E\left(\frac{1}{n_i}\right)$ provides the proof.

Remark 4.2.1: The variance expression be expressed in the form

$$V(\bar{y}_{PSNR})=\sum W_i^2\left\{E\left(\frac{1}{n_i}\right)\right\}\{A_i+S_i^2\}-\left(\frac{1}{N}\right)\sum W_iS_i^2 \tag{4.2.8}$$

5. Cost Analysis

Assume the following components of cost, involved in a mail survey

(i) C_0 : Cost of including sample units n_i of the i^{th} stratum in sample n

(ii) C_1 : Cost of collecting, editing and processing per n_i unit in the i^{th} strata of the response class

(iii) C_2 : Cost of personal interview and processing information per n_i''' unit in the non-response class

The cost function for i^{th} strata is

$$C_i=C_0n_i+C_1n_i'+C_2n_i''' \tag{5.1}$$

With total cost over k strata

$$T_c=\sum_{i=1}^{k} C_i=\sum_{i=1}^{k}[C_0n_i+C_1n_i'+C_2n_i''] \tag{5.2}$$

and the expected cost $$E(T_c)=\left(\frac{n}{N}\right)\sum_{i=1}^{k}\left\{C_0N_i+C_1N_i'+\frac{C_2N_i''}{f_i}\right\} \tag{5.3}$$

To get optimum f_i and n, define a function δ.

$\delta = E(T_c) + \lambda[V(\bar{y}_{PSNR}) - V_0]$ with V_0 a prefixed level of variance of $\bar{y}_{PSNR}$ and λ a lagrange multiplier. Differentiating with respect to f_i, n and λ and equating to zero, the three equations are

$$\lambda\left[Q_{1i} S_{2i}^2\right] = \left(\frac{n}{N}\frac{C_2 N_i''}{f_i^2}\right) \quad (5.4)$$

$$\lambda\{n^3(N-1)\}\sum_{i=1}^{k}\left[Q_{2i}\left\{\left(\frac{N_i''}{N_i}\right)(f_i - 1)S_{2i}^2 + S_i^2\right\}\right] = \left(\frac{1}{N}\right)\sum_{i=1}^{k}\left[C_0 N_i + C_1 N_i' + \frac{C_2 N_i''}{f_i}\right] \quad (5.5)$$

and

$$\sum_{i=1}^{k}\left[Q_{1i}\left(\frac{N_i''}{N_i}\right)(f_i - 1) S_{2i}^2 + Q_{1i} S_i^2 - \left(\frac{W_i^2 S_i^2}{N_i}\right)\right] = V_0 \quad (5.6)$$

where

$$Q_{1i} = \left[\left\{\left(\frac{W_i}{N}\right) + \left(\frac{(N-n)(1-W_i)}{n^2(N-1)}\right)\right\}\right], \; Q_{2i} = [(2N-n) + NW_i(n-2)]$$

Case I: The (5.4) generates k equations for k strata, therefore in total $(k+2)$ equations for $(k+2)$ unknowns $f_1, f_2, f_3, \ldots f_k$, n and λ. But, the powers of n and f_i are not necessary positive unity, so the assurance of finding solution is hard.

Case II: The limitation of case I improvise to put extra condition to determine λ. Let us consider

$$n = \sum_{i=1}^{k} n_i = \sum_{i=1}^{k} n_i' + \sum_{i=1}^{k} n_i'' \text{ or } \left(n - \sum_{i=1}^{k} n_i'\right) = \sum_{i=1}^{k} n_i''' f_i \quad (5.7)$$

Using (5.4), $$f_i = \sqrt{\left\{\frac{C_2 N_i''}{Q_{1i} S_{2i}^2}\right\}\left(\frac{1}{\lambda}\right)} \quad (5.8)$$

The (5.7) helps to eliminate λ and we get

$$f_i = \left(n - \sum_{i=1}^{k} n_i'\right) R_i \left[\sum_{i=1}^{k}\left[n_i''' R_i\right]\right]^{-1} \quad (5.9)$$

with $$R_i = N_i \left[\{nW_i (N-1)N_i + (N-n)(1-W_i)\}S_{2i}^2\right]^{-1} \tag{5.10}$$

The equation (5.9) provides the value of f_i. We can rewrite (5.6)

$$\left[V_0 + \sum_{i=1}^{k} \frac{W_i^2 S_i^2}{N_i}\right] = \sum_{i=1}^{k} Q_{1i} \left\{\left(\frac{N_i''}{N_i}\right)(f_i - 1)S_{2i}^2 + S_i^2\right\} \tag{5.11}$$

On substituting f_i the equation (5.11) provides solution for optimal n which is included in the term Q_{1i}.

Case III: Another approach for solution is as under

Let $\sum_{i=1}^{k} f_i = M$, where M is a prefixed constant. This provokes to keep the total of sub-sample fractions equal to M with no restriction on f_i and, it is used to eliminate λ. The solution is

$$f_i = \left[MR_i \Big/ \sum_{i=1}^{k} R_i\right] \tag{5.12}$$

Substituting f_i in (5.11), n could be obtained by solving the following

$$n^2(N-1)\sum_{i=1}^{k}\left[\sqrt{R_i}\left\{V_0 + \sum_{i=1}^{k}\left(\frac{W_i^2}{N_i}\right)S_i^2\right\}\right] - \sum_{i=1}^{k}\left[X_i\left\{\sum_{i=1}^{k}\sqrt{R_i}\left(S_i^2\right)\right\}\right]$$

$$+\sum_{i=1}^{k}\left[\sqrt{R_i}\left(X_i S_{2i}^2\right)\right] - \sum_{i=1}^{k}\left[X_i\left\{\sum_{i=1}^{k} f_i S_{2i}^2 \sqrt{R_i}\right\}\right] = 0 \tag{5.13}$$

where $X_i = [(N-n) + NW_i(n-1)]$

6. *An Alternative Cost Strategy*

In case I, the solution of f_i and n are not certain whereas case II and III do not ensure a concrete optimal value of n, therefore an alternative solution may think of for optimal choices. Consider i^{th} stratum-based approach by assuming cost C_{0i}, C_{1i} and C_{2i} varying over all k strata then the reset cost strategy is

(i) C_{0i} : cost of n_i units of the i^{th} stratum to be included into sample n

(ii) C_{1i} : cost of collecting, editing and processing per n'_i units in i^{th} strata of response class

(iii) C_{2i} : cost of personal interview and processing per n_i'' units in the i^{th} strata for non-response class

The i^{th} stratum has total cost

$$C_i = [C_{0i}n_i + C_{1i}n_i' + C_{2i}n_i''] \text{ and variance of } \bar{y}_{PSNR} \tag{6.1}$$

$$V(\bar{y}_{PSNR})_i = E\left(\frac{1}{n_i}\right)\left[\left(\frac{N_i''}{N_i}\right)(f_i - 1)S_{2i}^2 + S_i^2\right] - \left(\frac{1}{N_i}\right)S_i^2 \tag{6.2}$$

Define function δ_i', with lagrange multiplier λ_i and pre-fixed level of variance V_{0i}

$$\delta_i' = [\text{expected cost of } i^{th} \text{ strata}] + \lambda_i\left[V(\bar{y}_{PSNR})_i - V_{0i}\right] \tag{6.3}$$

$$\delta_i' = \left(\frac{n}{N}\right)\left[C_{0i}N_i + C_{1i}N_i' + C_{2i}\frac{N_i''}{f_i}\right]$$
$$+\lambda_i\left[\left\{E\left(\frac{1}{n_i}\right)\left\{\left(\frac{N_i''}{N_i}\right)(f_i - 1)S_{2i}^2 + S_i^2\right\} - \left(\frac{1}{N_i}\right)S_i^2\right\} - V_{0i}\right] \tag{6.4}$$

On differentiating with respect to f_i, λ and n, the three equations are

$$\left[\left(\frac{n}{N}\right)\frac{C_{2i}N_i''}{f_i^2} - \lambda_i\left\{E\left(\frac{1}{n_i}\right)\left(\frac{N_i''}{N_i}\right)S_{2i}^2\right\}\right] = 0 \tag{6.5}$$

$$\left[E\left(\frac{1}{n_i}\right)\left\{\left(\frac{N_i''}{N_i}\right)(f_i - 1)S_{2i}^2 + S_i^2\right\} - \left\{V_{0i} + \left(\frac{1}{N_i}\right)S_i^2\right\}\right] = 0 \tag{6.6}$$

$$\left[\left(\frac{1}{N}\right)\left\{C_{0i}N_i + C_{1i}N_i' + \frac{C_{2i}N_i''}{f_i}\right\}\right.$$
$$\left. - \left(\frac{\lambda_i}{n}\right)\left\{E\left(\frac{1}{n_i}\right) + \left\{\frac{N(1 - W_i)}{n^2(N - 1)W_i^2}\right\}\right\}\left\{\left(\frac{N_i''}{N_i}\right)(f_i - 1)S_{2i}^2 + S_i^2\right\}\right] = 0 \tag{6.7}$$

The equation (6.5) gives $\lambda_i = \{nC_{2i}N_i\}\left\{f_i^2\, NE\left(\frac{1}{n_i}\right)S_{2i}^2\right\}^{-1}$ (6.8)

which on substituting (6.7), provides

$$f_i = \left\{\frac{1}{2\delta_{1i}}\right\}\left[\delta_{2i} \pm \sqrt{\delta_{2i}^2 - 4\delta_{1i}\,\delta_{3i}}\right] \tag{6.9}$$

$$\delta_{1i} = \left(\frac{1}{N}\right)\{C_{0i}N_i + C_{1i}N_i'\},\ \delta_{2i} = C_{2i}N_i''\left[\left\{\frac{1-W_i}{n^2(N-1)W_i^2}\right\} - \left(\frac{(1-n)}{N}\right)\right]$$

$$\delta_{3i} = \left[\left\{\frac{N_i S_i^2}{N S_{2i}^2}\right\} - \left(\frac{N_i''}{N_i}\right) + \left\{\frac{(1-W_i)}{n^2(N-1)W_i^2}\right\}\left\{N_i'' - \left(\frac{S_i^2}{E(1/n_i)S_{2i}^2}\right)\right\}\right]$$

The positive value of f_i will occur if $\delta_{2i}^2 > (\delta_{1i}\ \delta_{3i})$

Remark 6.1: Suppose f_i is prefixed (say $f_i = 1.5$) for all i then (6.9) gives optimum selection of C_{0i}, C_{1i} and C_{2i} provided the ratio $\frac{S_i^2}{S_{2i}^2}$ is guessed well for all k strata.

Remark 6.2: Using (6.6), the optimum n is

$$(n_{opt})_i = \left[(NW_i - 1) \pm \sqrt{(NW_i - 1)^2 + 4\gamma_i (N-1) W_i^2\, N(1 - W_i)}\right]\left[2\gamma_i (N-1) W_i^2\right]^{-1} \tag{6.10}$$

where $$\gamma_i = \left[\left\{V_{0i} + \frac{S_i^2}{N_i}\right\} \Big/ \left\{\left(\frac{N_i''}{N_i}\right)(f_i - 1)S_{2i}^2 + S_i^2\right\}\right] \tag{6.10.1}$$

If N is sufficiently large then

(i) $(N-1) \approx N$, and

(ii) $(NW_i - 1) \approx NW_i$

and then (6.10) reduces into $(n_{opt})_i = N\left[1 \pm \sqrt{1 + 4\gamma_i\,(1 - W_i)}\right][2N_i\gamma_i]^{-1}$ (6.11)

This equation would provide the optimum $(n_{opt})_i$, when f_i from (6.9) is substituted along with a suitable choice of V_{0i}.

Remark 6.3: The (6.11) varies over i, therefore, provide k values of optimum sample size $(n_{opt})_i$ over different strata. One can choose any one of these for practical purpose. Another way may be to choose an average integer value of these $(n_{opt})_i$ or minimum/maximum of $(n_{opt})_i$ over all k strata.

Remark 6.3.1: The justification of remark 6.3 is based on unique selection of a value over availability of k different optimal n. A criteria is to be designed by the survey practitioner for single-valued choice like

(i) selection of any one value on the basis of experience

(ii) selection of an average integer value

(iii) selection of a minimum value, due to cost constraint

(iv) selection of a maximum value, due to objective of high precision

All these depends on the situation and circumstances to be faced.

Remark 6.4: In (6.11), the suitable $(n_{opt})_i$ exists only if choice of γ_i lies in a certain range. This helps to choose a prefixed level of variance V_{0i} for i^{th} strata using (6.10.1).

7. Numerical Illustrations

Suppose a population of 400 units, has four strata I, II, III, IV and each one is divided into two groups as response (R) and non-response (NR). The detail of these is in appendix A and population parameters are given below

$N = 400;\ \bar{Y} = 81.65;\ S^2 = 1857.025$

Table 1

Type	Divison	Strata			
		I	II	III	IV
Size of strata	Response Class (R)	$N_1' = 30$	$N_2' = 40$	$N_3' = 60$	$N_4' = 70$
	Non-Res. Class (NR)	$N_1'' = 30$	$N_2'' = 40$	$N_3'' = 60$	$N_4'' = 70$
	Total N_i	$N_1 = 60$	$N_2 = 80$	$N_3 = 120$	$N_4 = 140$
Mean	$\bar{Y}_i$	$\bar{Y}_1 = 13.48$	$\bar{Y}_2 = 48.47$	$\bar{Y}_3 = 85.50$	$\bar{Y}_4 = 126.88$
Mean Squares	Response Class (R)	$S_{11}^2 = 67.06$	$S_{12}^2 = 156.67$	$S_{13}^2 = 228.01$	$S_{14}^2 = 240.20$
	Non-Res. Class (NR)	$S_{21}^2 = 62.97$	$S_{22}^2 = 204.87$	$S_{23}^2 = 248.79$	$S_{24}^2 = 254.74$
	S_i^2	$S_1^2 = 64.38$	$S_2^2 = 180.46$	$S_3^2 = 225.63$	$S_4^2 = 245.74$
Weights	W_i	$W_1 = 0.15$	$W_2 = 0.20$	$W_3 = 0.30$	$W_4 = 0.35$

A sample of size n = 80 is drawn by SRSWOR and post-stratified into n_i units. When a deadline of receiving mailed questionnaire is over, n'_i and n''_i units are as below along with prefixed n'''_i and f_i.

Table 2

Type / Sample size	Strata I	II	III	IV	Total
n_i	16	18	22	24	80
n'_i	06	08	09	10	33
n''_i	10	10	13	14	47
n'''_i	03	04	04	05	16
$f_i = n''_i/n'''_i$	3.33	2.50	2.50	2.00	10.33

(i) We have $V(\bar{y}_{PSNR}) = 2.228$ which is quite small showing a high precision of $\bar{y}_{PSNR}$.

(ii) For calculation purpose alternative strategy is considered with cost C_{0i}, C_{1i} and C_{2i}. Moreover, the cost optimal f_i is calculated using (6.9) and shown below

Table 3

Strata I	Strata II	Strata III	Strata IV
$C_{01} = 01.0$	$C_{02} = 01.0$	$C_{03} = 01.0$	$C_{04} = 02.0$
$C_{11} = 03.0$	$C_{12} = 03.0$	$C_{13} = 03.0$	$C_{14} = 03.0$
$C_{21} = 0.50$	$C_{22} = 01.0$	$C_{23} = 01.0$	$C_{24} = 01.0$
$f_1 = 8.00$	$f_2 = 15.80$	$f_3 = 15.78$	$f_4 = 12.27$

Table 3 reveals that C_{0i} and C_{2i} are nearly same but C_{1i} is required much higher to these two.

(iii) The optimal sample size $(n_{opt})_i$ and values of V_{0i} are tabulated below

Table 4

	Strata I	Strata II	Strata III	Strata IV
V_{0i}	100	150	275	125
$(n_{opt})_i$	23	59	27	40

As per remark (6.3), an average integer value of cost optimal n is 37 taken over all the four strata.

(iv) The calculation of population mean $\bar{Y}$ using estimator $\bar{y}_{PSNR}$ is performed over four samples and displayed below.

Table 5

Strata		Sample 1	Sample 2	Sample 3	Sample 4
I	n'_1	6(9, 6, 11, 15, 19, 7)	6(4, 13, 14, 30, 7, 10)	6(9, 12, 14, 8, 1, 29)	6(2, 17, 22, 19, 7, 20)
	n''_1	10	10	10	10
	n'''_1	3(22, 4, 15)	3(26, 30, 15)	3(22, 2, 20)	3(20, 8, 15)
	n_1	16	16	16	16
	$\bar{y}'_1$	11.167	13.00	12.167	14.5
	$\bar{y}'''_1$	13.667	23.67	14.670	14.33
II	n'_2	8(59, 35, 47, 26, 34, 42, 60, 58)	8(35, 50, 62, 56, 43, 64, 47, 60)	8(49, 36, 42, 31, 64, 26, 58, 34)	8(38, 62, 42, 55, 43, 48, 39, 63)
	n''_2	10	10	10	10
	n'''_2	4(26, 70, 50, 69)	4(36, 66, 69, 58)	4(66, 28, 53, 44)	4(36, 70, 62, 53)
	n_2	18	18	18	18
	$\bar{y}'_2$	45.125	52.125	42.500	48.75
	$\bar{y}'''_2$	53.750	57.250	47.75	55.25
III	n'_3	9(93, 68, 62, 80, 84, 59, 66, 100, 70)	9(88, 71, 91, 86, 100, 106, 74, 68, 49)	9(84, 66, 92, 84, 59, 102, 70, 81, 47)	9(62, 91, 78, 66, 84, 96, 111, 105, 47)
	n''_3	13	13	13	13
	n'''_3	4(109, 67, 89, 95)`	4(107, 99, 65, 72)	4(109, 67, 89, 72)	4(109, 67, 83, 96)
	n_3	22	22	22	22
	$\bar{y}'_3$	75.780	81.440	76.110	82.220
	$\bar{y}'''_3$	90.000	85.750	84.250	88.750
IV	n'_4	10(112, 104, 149, 133, 120, 102, 131, 137, 121, 107)	10(101, 148, 133, 117, 124, 149, 105, 150, 137, 129)	10(146, 108, 126, 133, 136, 127, 144, 135, 106, 150)	10(127, 112, 124, 148, 136, 105, 121, 150, 128, 140)
	n''_4	14	14	14	14
	n'''_4	5(139, 146, 106, 111, 121)	5(127, 145, 113, 106, 132)	5(149, 127, 109, 132, 110)	5(109, 147, 113, 132, 127)
	n_4	24	24	24	24
	$\bar{y}'_4$	121.600	129.300	131.1	129.1
	$\bar{y}'''_4$	125.200	124.600	125.4	125.6
$\bar{y}_{PSNR}$		80.3608	83.3530	80.0590	79.3960

(v) Table 5 reveals that sample estimates $\bar{y}_{PSNR}$ are very close to the true value of the population mean $\bar{Y}$. Clearly, the variability $V(\bar{y}_{PSNR})$ would be very small.

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APPENDIX–A

Population (N = 400)

Strata – I

Response (R)

05	04	02	09	18	03	06	12	11	13
07	06	14	15	17	16	30	22	08	09
01	19	21	23	25	07	29	20	10	02

Non Response (NR)

04	22	14	26	08	30	20	15	06	08
02	04	16	08	10	22	14	06	18	20
01	13	25	07	19	21	15	15	27	09

Strata – II

Response (R)

25	67	59	41	53	35	47	49	50	30
26	38	36	62	34	56	68	42	51	43
31	42	43	64	55	26	47	38	48	58
60	39	58	27	66	55	34	63	52	41

Non-Response (NR)

41	26	36	64	66	66	70	48	49	30
50	62	25	46	28	40	62	34	63	62
25	28	47	24	55	40	69	47	50	53
55	58	61	66	53	49	67	44	42	70

Strata- III

Response (R)

60	88	67	93	84	85	60	71	68	79
62	66	65	77	92	80	91	62	72	100
89	98	84	86	78	90	59	74	96	70
66	100	84	102	78	76	70	96	106	88
80	84	74	86	68	81	111	113	105	68
96	71	62	83	94	90	96	47	98	49

Non-Response (NR)

61	103	105	107	109	70	63	84	98	95
90	99	98	67	96	85	74	83	72	101
84	65	67	89	70	106	98	79	105	87
89	91	87	96	72	75	97	80	81	102
108	92	63	90	104	107	88	95	109	120
120	83	62	105	74	107	66	98	109	112

Strata- IV

Response (R)

100	101	112	123	104	108	146	127	148	108
149	122	112	133	110	145	126	117	108	129
120	124	124	136	148	130	102	134	136	148
121	133	105	127	149	131	133	135	137	149
149	121	122	143	144	135	146	107	108	149
121	112	134	106	128	108	106	109	102	140
131	138	143	111	115	150	137	138	129	146

Non Response (NR)

139	148	137	130	149	138	127	116	145	124
109	150	147	106	145	144	113	112	141	110
100	104	106	108	140	132	124	146	108	128
121	103	105	127	149	111	118	125	147	109
130	149	108	147	106	125	124	125	102	121
119	107	145	104	143	121	132	108	146	144
132	134	126	148	110	112	134	146	128	150