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On Small Area Estimation–An Empirical Study

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SUMMARY

The importance of small area statistics in agriculture are becoming more and more important in the context of Agro-Climatic Regional Planning process initiated by the Planning Commission, Govt. of India, in the early nineties to enforce the bottom–up planning in the county. In India, the crop production/productivity are being estimated at district level through crop-cutting experiments. For planning the development process at block level under the present system of Panchayat Raj, we need crop–statistics at block level. This necessitates the small area estimates at block level or at other small levels. Most of the methods of small area estimation are integral part of the sample survey in which direct or indirect (synthetic) estimators are developed for small areas. However, how to scale down the statistics available at large area to small area is still untouched in the context of small area estimation methods. In the present paper, an effort has been made to develop estimators for crop–production at block level using crop–production and other related information at district level. Exploring the relationship between crop–production (Y) and other related variables (X_j) at district level, weights for X_j at block level were worked out. Using these weights, three estimators at block level were developed. Their relative efficiencies were also worked out. An empirical study was carried out for rice crop in Faizabad district. The results of the empirical study were quite encouraging as it tallied very closely with block estimate of rice production reported by State Govt. during some of the years in the past.

Key words : Crop–cutting experiments, Block estimates, Synthetic estimator.

1. Introduction

The estimates of items at micro–level (small area) are now in great demand by public as well as private sectors in order to prepare policy formulation for research and development process specific to small areas. It is well known that in India the estimates of crop production based on crop–cutting experiments are being reported at district level and these estimates are aggregated at State and Country level. However, the demand for the estimate at block–level

(Community Development Block) have been growing considerably in general during recent years in the context of decentralised planning for agricultural development process on the pretext of Agro–Climatic Regional Planning (ACRP) approach initiated by the Planning Commission, Govt. of India, in 1988.

A first attempt, quite earlier, was made by Panse *et al.* [2] and Singh [3] to develop a methodology for estimating the crop yield at block level using double sampling approach but the method developed by them could not succeed due to certain physical constraints. Various small area estimation methods have been developed in recent past. Synthetic method of estimation is one of the most widely used method in small area estimation approaches. Stasny *et al.* [5] have also made an effort to develop small area estimation technique for estimating wheat production at county level in USA. They, in fast developed a prediction model using multiple regression technique with data obtained on sample of farms selected purposely from the different counties. Recently, Srivastava *et al.* [4] made an attempt to develop the synthetic estimators for crop production/productivity at block level within the frame of survey sampling. They also performed a simulation study to find out the performance of estimators. Most of the methods of small area estimation developed so far are integral part of sample surveys in which direct or indirect (synthetic) estimators are developed for small areas. Infact no efforts have been made in past as to how the estimates available at macro level can be scaled down to the smaller level, and this is what the main aim of the present paper.

Generally, the estimates of crop production/productivity are available at district level. There are other variables related to the crop production/productivity, which are available at district level as well as at block level. Such variables are area under the crop, irrigated area under the crop, fertilizer consumption, rainfall etc. Therefore, at first, an attempt has been made to build a suitable multiple regression model between the crop production and related variables (referred as predictor or independent variables) at district level. Then, the methodologies have been developed to scale down the district level crop-production estimates at block level using the information contained in the fitted regression model at district level in the sections 2 & 3. The relative efficiencies of the estimators developed are discussed in section 4. An empirical study has been carried out to illustrate the methodologies in section 5.

2. *The Model and Development of Estimator*

Consider a general regression model at district level

$$Y_i = f\left(X_{ij} \mid \beta\right) + \varepsilon_i \qquad (2.1)$$

where Y_i is crop production in the i^{th} year (i = 1, 2, ..., n), X_{ij} is value of the j^{th} predictor in the i^{th} year, (j = 1, 2, ..., p), β are unknown parameters and ε_i is error term to follow IIN (0, σ^2). The set of predictor variables may include area under the crop, irrigated area under the crop, fertilizer consumption, rainfall etc. Let the fitted model, using least square (l.s.) technique, be denoted as

$$\hat{Y}_i = f\left(X_{ij} \mid \hat{\beta}\right) \tag{2.2}$$

where $\hat{\beta}$ is the l.s. estimate of β and $\hat{Y}_i$ is the estimated value of Y_i for corresponding values of X_{ij}.

If the model (2.1) holds true at block level then simplest estimator for crop production at block level would be the predictor $\hat{Y}_i$. But, the model (2.1) may not generally hold true at block level as volume of some of X_{ij}'s would be quite small at block level as compared to district level. However, the relationship between Y_i and X_{ij}'s in (2.1) at district level could be explored in developing estimator for crop production at block level. In the analysis of variance of the model (2.1), let the sum of squares due to regression be denoted as SS_R. Following Montgomery *et al.* [1], the relative contribution of individual predictor X_j to Y in terms of sum of squares can be obtained by decomposing $SS_R\left(\beta_1, \beta_2, ..., \beta_p \mid \beta_0\right)$, i.e.

$$\begin{aligned} SS_R\left(\beta_1, \beta_2, ..., \beta_p \mid \beta_0\right) &= SS_R\left(\beta_1 \mid \beta_0\right) + SS_R\left(\beta_2 \mid \beta_0, \beta_1\right) + ... \\ &\quad + SS_R\left(\beta_p \mid \beta_0, \beta_1, \beta_2, ... \beta_{p-1}\right) \end{aligned} \tag{2.3}$$

The terms in R.H.S. of (2.3) give the relative contribution of the individual predictor. There is no definite hierarchical order of the terms in the R.H.S. of (2.3) as it holds true for any order of inclusion of first and other additional predictors in sequence in the model (2.1). For example,

$$\begin{aligned} SS_R\left(\beta_1, \beta_2, ..., \beta_p \mid \beta_0\right) &= SS_R\left(\beta_i \mid \beta_0\right) + SS_R\left(\beta_j \mid \beta_0, \beta_i\right)_{i \neq j} \\ &\quad + SS_R\left(\beta_k \mid \beta_0, \beta_i, \beta_j\right)_{i \neq j \neq k} + ... \end{aligned} \tag{2.4}$$

holds always true.

One of the criterion, being in practice for choosing the order of inclusion of predictors in the model, is the magnitude of correlation coefficient between Y and X_j. Therefore, in decomposing $SS_R\left(\beta_1, \beta_2, ..., \beta_p \mid \beta_0\right)$, the predictor having highest correlation with Y is chosen first, next highest at the second and so on. Once the relative contribution of individual predictor is obtained, a weight (ω) is assigned to each predictor to be used at block level. The ω is defined as

$$\omega_j = \frac{\text{SS due to jth predictor as mentioned in (2.3)}}{SS_R\left(\beta_1, \beta_2, ..., \beta_p \mid \beta_0\right)} \tag{2.5}$$

Using these weights, an estimator of crop production (Y_q) for the q^{th} block, (q = 1, 2, 3, ..., Q), is constructed as

$$\hat{Y}_q = \left[\sum_{j=1}^{P} \omega_j X_j\right] \hat{\bar{Y}} \tag{2.6}$$

where $\hat{\bar{Y}} = \frac{\hat{Y}}{A}$; $\hat{Y}$ is obtained through the fitted model (2.2). A is the area under the crop in a given year and X_j is the value of j^{th} predictor at block level in a given year.

The natural choice could be the $\bar{Y}$ in place of $\hat{\bar{Y}}$ in (2.6), where $\bar{Y}$ is the average yield of a crop at district level in a given year obtained through crop cutting experiments. However, since the standard error of $\bar{Y}$ is not being reported in the Statistical Bulletin published by State Govt./Union Govt., it will not be possible to estimate the standard error of $\hat{Y}_q$ if we use $\bar{Y}$ in place of $\hat{\bar{Y}}$. The estimator $\hat{Y}_q$ is naturally an unbiased estimator of Y_q as $\left[\sum_{j=1}^{P} \omega_j X_j\right]$ is a constant quantity for a given block and expected value of $\hat{\bar{Y}}$ is $\bar{Y}$ under assumption of the model (2.1).

The variance of $\hat{Y}_q$ is obtained as

$$V(\hat{Y}_q) = \left[\sum_{j=1}^{P} \omega_j X_j\right]^2 V\left(\hat{\bar{Y}}\right) = \left[\sum_{j=1}^{P} \omega_j X_j\right]^2 \frac{1}{A^2} V(\hat{Y}) \tag{2.7}$$

The variance of $\hat{Y}$ is easily available by fitting the model (2.1), which is equal to σ^2, the estimated error variance.

The estimator $\hat{Y}_q$ can also be referred to as synthetic estimator because $\hat{Y}_q$ also borrows the strength from $\hat{\bar{Y}}$ which is an estimate at the district level, whereas the coefficient of $\hat{\bar{Y}}$ in (2.6) is the weighted value of the predictors at the block level.

3. Scaling Block Estimates to Sum to District Total

It is obvious that in general $\sum_{q=1}^{Q} \hat{Y}_q \neq Y$, where Y is the actual crop production reported at district level through crop cutting experiments in a given year. Thus, a new estimator of Y_q is proposed as

$$\tilde{Y}_q = a_q \hat{Y}_q \tag{3.1}$$

where a_q are constants such that $\sum_{q=1}^{Q} \tilde{Y}_q = \sum_{q=1}^{Q} a_q \hat{Y}_q = Y$

The question arises that how to choose a_q. The two alternatives choice of a_q are suggested below

Choice I: The simplest choice of a_q is to take $a_q = a$, i.e., the constant for each block. It can easily be shown that

$$a = \frac{Y}{\sum_{q=1}^{Q} \hat{Y}_q} \tag{3.2}$$

Thus, a new estimator of Y_q is given by

$$\tilde{Y}_q^{(1)} = \hat{Y}_q \left[\frac{Y}{\sum_{q=1}^{Q} \hat{Y}_q} \right] \tag{3.3}$$

Choice II: Another choice of a_q could be to minimize the sum of squared differences between $\tilde{Y}_q$ and $\hat{Y}_q$. To do so, we minimize the sum of squared differences between $\tilde{Y}_q$ and $\hat{Y}_q$ subject to the condition $\sum_{q=1}^{Q} a_q \hat{Y}_q = Y$. Using a lagrange multiplier λ, to impose the desired constraint, we minimize the function

$$\phi = \sum_{q=1}^{Q} \left(a_q \hat{Y}_q - \hat{Y}_q\right)^2 + 2\lambda\left(Y - \sum_{q=1}^{Q} a_q \hat{Y}_q\right)$$

Differentiating ϕ w.r.t. a_q and equating it to zero, and solving for a_q, we get

$$a_q = 1 + \frac{\left(Y - \sum_{q=1}^{Q} \hat{Y}_q\right)}{Q\hat{Y}_q} \tag{3.4}$$

Thus, another new estimator of Y_q is given by

$$\tilde{Y}_q^{(2)} = \hat{Y}_q + \frac{\left(Y - \sum_{q=1}^{Q} \hat{Y}_q\right)}{Q} \tag{3.5}$$

Note that the scaled estimator $\tilde{Y}_q^{(2)}$ are obtained by adjusting the original estimator $\hat{Y}_q$ by adding a factor which is ratio of the difference between the actual crop production at district level, Y and the sum of the original block estimates, $\sum_{q=1}^{Q} \hat{Y}_q$, to number of blocks in the district.

4. Relative Performance of the Estimators

In order to find out the relative performance of the estimators, the variance of these estimators are derived.

The variance of $\hat{Y}_q$ is already given in (2.7). However, an alternative expression for $V(\hat{Y}_q)$ is given by

$$V(\hat{Y}_q) = \left(\frac{\delta_q^2}{A^2}\right) V(\hat{Y}) \quad \text{where } \delta_q = \sum_{j=1}^{P} \omega_j X_j \tag{4.1}$$

The variance of $\tilde{Y}_q^{(1)}$ and $\tilde{Y}_q^{(2)}$ are obtained as

$$V\left(\tilde{Y}_q^{(1)}\right) = a^2 \left(\frac{\delta_q^2}{A^2}\right) V(\hat{Y}) \tag{4.2}$$

$$V\left(\tilde{Y}_q^{(2)}\right) = \frac{Q-1}{Q} \left(\frac{\delta_q^2}{A^2}\right) V(\hat{Y}) \tag{4.3}$$

In order to study the relative efficiency of the estimators, the variances of these three estimators are compared.

The estimator $\tilde{Y}_q^{(1)}$ will be more efficient than the estimator $\hat{Y}_q$, if

$$V(\hat{Y}_q) - V\left(\tilde{Y}_q^{(1)}\right) > 0$$

i.e., $a < 1$

Since $a = \left(Y \Big/ \sum_{q=1}^{Q} \hat{Y}_q\right)$, it may be less than or greater than one. Therefore, the relative efficiency of $\tilde{Y}_q^{(1)}$ over $\hat{Y}_q$ depends on the value of a. Both the estimators will be equally efficient if a = 1.

The estimator $\tilde{Y}_q^2$ will be more efficient than the estimator $\hat{Y}_q$, if

$$V(\hat{Y}_q) - V\left(\tilde{Y}_q^{(2)}\right) > 0$$

i.e., $$\left(1-\frac{Q-1}{Q}\right)>0 \tag{4.4}$$

The L.H.S. of (4.4) will always be positive as $\frac{(Q-1)}{Q}$ is always less than one. It implies that $\tilde{Y}_q^{(2)}$ will always be more efficient than the estimator $\hat{Y}_q$.

The estimator $\tilde{Y}_q^2$ will be more efficient than the estimator $\tilde{Y}_q^{(1)}$, if

$$V\left(\tilde{Y}_q^{(1)}\right)-V\left(\tilde{Y}_q^{(2)}\right)>0$$

i.e., $$a^2>\frac{Q-1}{Q} \tag{4.5}$$

Since $\frac{(Q-1)}{Q}$ is always less than one but very close to one, the value of $a=\frac{Y}{\sum_{q=1}^{Q}\hat{Y}_q}$ must be greater than one or very close to one so as to hold the inequality (4.5). However, the estimator $\tilde{Y}_q^{(2)}$ will always be more efficient than the estimator $\tilde{Y}_q^{(1)}$, if $a>1$.

The relative efficiencies of $\tilde{Y}_q^{(1)}$, and $\tilde{Y}_q^{(2)}$, over $\hat{Y}_q$ are computed in an empirical illustration in next section using the following formula

$$E_i=\frac{V\left(\hat{Y}_q\right)}{V\left(\tilde{Y}_q^{(i)}\right)}\times 100;\ i=1,2 \tag{4.6}$$

5. *Empirical Illustration*

An empirical study has been carried out to illustrate the working of entire procedure. It is envisaged to estimate rice production at block level in Faizabad district of Uttar Pradesh (India).

Consider the model at district level

$$Y_i=\beta_0+\beta_1X_{i1}+\beta_2X_{i2}+\beta_3X_{i3}+\beta_4X_{i4}+\varepsilon_i \tag{5.1}$$

where

Y_i : Rice production (in quintal) of Faizabad district during i^{th} year

X_{i1} : Total area (in hectare) under rice crop during i^{th} year

X_{i2} : Fertilizer consumption (Kg/hectare) for rice crop during i^{th} year

X_{i3} : Irrigated area (in hectare) under rice crop during i^{th} year

X_{i4} : Rainfall index

β_j : Regression coefficient j = 0, 1, 2, 3, 4

and ε_i : Random error component assumed to follow independent normal distribution with mean 0 and variance σ^2

The rainfall index (X_{i4}) for the ith year was constructed as follows

$$X_{i4} = \sum_{j=1}^{K} r_j W_{ij}; \text{ K is the number of weeks} \tag{5.2}$$

where r_j is the correlation coefficient between rainfall data in j^{th} week and time series data on rice production (Y_i), and W_{ij} is the rainfall data in j^{th} week of i^{th} year.

The time series data on rice production, area under rice, irrigated area under rice and fertilizer consumption (N + P + K) pertaining to the period 1968-69 to 1995-96 for Faizabad district were obtained from the Bulletin of Agricultural Statistics, published by Directorate of Agricultural Statistics and Crop Insurance, Govt. of Uttar Pradesh, Lucknow (India). Since the fertilizer consumption (N + P + K) is not being reported crop wise, the total fertilizer consumption in a year was apportioned for the rice crop. Approximately one-third of the total fertilizer consumption in a year was considered to have been used for rice crop. The weekly rainfall data of Faizabad district starting from June to October and for the period 1968-69 to 1995-96 were obtained from the Department of Agricultural Meteorology, N.D.U.A. & T., Kumarganj, Faizabad, Uttar Pradesh (India).

The block-wise data on the variables X_1, X_2 and X_3 in Faizabad district were also obtained for some of the recent years from District Statistical Bulletins, published by Directorate of Economics and Statistics, Govt. of Uttar Pradesh (India). The block wise actual data of rice production based on crop cutting experiments in Faizabad district during the years 1981-82 and 1982-83 were also available in District Statistical Bulletins. The rainfall index developed at district level was also used for the block level in obtaining estimates of rice production at block level.

The multiple linear regression model (5.1) was fitted with the data mentioned above using least square technique.

The estimate of regression coefficients, their standard errors and value of coefficient of determination (R^2) etc. are presented in Table 1. The effects of total area under rice and fertilizer consumption have shown positive and significant effect on rice production in the district. The standard error of Y has also been presented in Table 1. The value of coefficient of determination (R^2) was found to be quite high, i.e. 92.43% which is indicative of the fact that these variables included in the model have been quite sufficient to explain the variability in the data of rice production at district level.

Table 1. The estimate of regression coefficients, their standard error and R^2

Variable	Regression coefficient	Standard error	R^2(%)
X_1	36.5542**	5.3679	92.43
X_2	7311.0099*	3295.3489	
X_3	0.3581	3.0085	
X_4	1486.7387	1480.6572	
Constant	–4295576.2561		

* $P < 0.05$ ** $P < 0.01$

Standard Error of Estimates $(\hat{Y}) = 324324.6080$

The analysis of variance for regression analysis of the aforesaid model is presented in Table 2. This also shows overall significance of the model fitted.

Table 2. Analysis of variance for regression analysis

Source	Sum of Squares	D.F.	Mean Square	F ratio	Prob.
Regression	2.8267×10^{13}	4	7.0668×10^{12}	67.183**	5.2×10^{-12}
Residual	0.2314×10^{13}	22	0.1052×10^{12}		
Total	3.0581×10^{13}	26			

Using the analysis of variance Table 2, the contribution of individual variables towards sum of squares due to regression are presented in Table 3. In order to find out the contribution of individual variable, the first variable namely total area under rice (X_1) was included in the model followed by fertilizer consumption in kg/ha (X_2), irrigated area under rice (X_3) and rainfall index (X_4) as Y had the highest correlation with X_1 (0.93) followed by X_2 (0.86), X_3 (0.78) and X_4 (0.58). On the basis of their contribution, the weights (ω_j) were calculated and are also presented in Table 3.

Utilizing the weights and block-wise data on X_1, X_2, X_3 and X_4 (rainfall index calculated at district level), the block estimates of rice production based on three estimators and their relative efficiencies were computed for the years 1981-82 and 1982-83 as crop-cutting estimates were also available for the blocks during these years, which would naturally make possible to compare the performance of the estimators.

Table 3. Contribution of individual variable towards sum of squares due to regression and the value of weights (ω_j)

S. No.	Variable	Contribution of variable		ω_j
1	X_1	$SS_R(\beta_1 \mid \beta_0)$	$= 2.6406 \times 10^{13}$	0.93416
2	X_2	$SS_R(\beta_2 \mid \beta_0, \beta_1)$	$= 0.1753 \times 10^{13}$	0.06202
3	X_3	$SS_R(\beta_3 \mid \beta_0, \beta_1, \beta_2)$	$= 0.0002 \times 10^{13}$	0.00007
4	X_4	$SS_R(\beta_4 \mid \beta_0, \beta_1, \beta_2, \beta_3)$	$= 0.0106 \times 10^{13}$	0.00375
	Total	$SS_R(\beta_4 \mid \beta_0, \beta_1, \beta_2, \beta_3)$	$= 2.8267 \times 10^{13}$	1.00000

The estimates and their relative efficiencies for the year 1981-82 and 1982-83 are presented in Table 4 and 5. Table 4 shows that the estimator $\tilde{Y}_q^{(2)}$ has been always efficient than the estimator $\hat{Y}_q$, which is on the expected line of theoretical results obtained in the previous section. The estimates based on $\tilde{Y}_q^{(1)}$ were less precise than that of $\hat{Y}_q$ as a > 1. However, the estimates based on $\tilde{Y}_q^{(1)}$ and $\tilde{Y}_q^{(2)}$ were found to be very close to crop cutting estimates as compared to that of $\hat{Y}_q$. The results of Table 4 has also been presented diagrammatically in Fig 1 to illustrate the closeness of estimates.

For the year 1982-83, the value of 'a' was less than one and, therefore, the estimators $\tilde{Y}_q^{(1)}$ and $\tilde{Y}_q^{(2)}$ were found to be more efficient than $\hat{Y}_q$ (Table 5). However, gain in precision was obtained to be more appreciable due to $\tilde{Y}_q^{(1)}$ as compared to $\tilde{Y}_q^{(2)}$. The results of Table 5 also indicate that estimates based on $\tilde{Y}_q^{(1)}$ and $\tilde{Y}_q^{(2)}$ were very close to crop-cutting estimates as against that of based on $\hat{Y}_q$. The results of Table 5 has also been depicted diagrammatically in Fig 2 to illustrate the closeness of the estimates. Note that the value of a is constant for a given year, and that is why the relative efficiency (E_i) of the estimators comes out to be same for all blocks in the given year. Similarly, the variability in the values of a_q for a given year is also very less, which of course, make adjustment in the block estimates more accurately but the values of E_2 were found to be almost similar for all blocks in the given year. This was so because $E_2 = \left(\frac{1}{a_q^2}\right) \times 100$ and the differences in the efficiencies has occurred only after three digits from the decimal point.

Table 4. Block estimates of rice production based on different estimators and their relative efficiency during the year 1981-82

S. No.	Block	Actual Estimate* (000 Q)	Estimates of Rice Production (000 Q)		
			$\hat{Y}_q$	$\tilde{Y}_q^{(1)}$	$\tilde{Y}_q^{(2)}$
1.	Masodha	121.15	114.93	121.94	122.93
2.	Sohawal	114.90	109.01	115.66	117.01
3.	Purabazar	123.32	116.94	124.07	124.94
4.	Myabazar	143.21	135.78	144.06	143.78
5.	Amaniganj	125.95	119.39	126.67	127.38
6.	Tarun	170.63	161.72	171.58	169.72
7.	Milkipur	113.72	107.77	114.34	115.77
8.	Bikapur	127.03	120.51	127.86	128.51
9.	Haringtonganj	135.07	128.12	135.98	136.11
10.	Bhiti	129.35	122.60	130.08	130.60
11.	Bhiyaon	119.94	113.68	120.61	121.68
12.	Jalalpur	160.41	152.09	161.37	160.09
13.	Akbarpur	248.20	235.27	249.62	243.27
14.	Katehari	159.68	151.36	160.59	159.35
15.	Tanda	168.22	159.47	169.20	167.47
16.	Jahangirganj	95.69	90.67	96.20	98.67
17.	Baskhari	109.06	103.43	109.74	111.43
18.	Ramnagar	124.64	118.16	125.36	126.15

* Based on crop cutting experiments.

$$\text{Value of } a = \frac{Y}{\sum_{q=1}^{Q} \hat{Y}_q} = 1.061$$

N.B. : The relative efficiency of $\tilde{Y}_q^{(1)}$ and $\tilde{Y}_q^{(2)}$ over $\hat{Y}_q$, viz. E_1 and E_2 comes out to be same for all the blocks, i.e. 88.84 and 105.88 per cent, respectively.

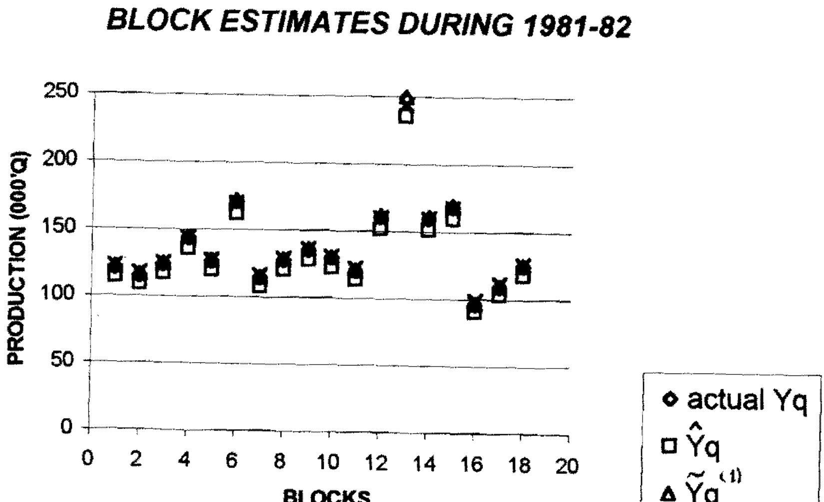
BLOCK ESTIMATES DURING 1981-82
PRODUCTION (000'Q)
250
200
150
100
50
0
0 2 4 6 8 10 12 14 16 18 20
BLOCKS
actual Yq
$\hat{Y}q$
$\tilde{Y}q^{(1)}$
$\tilde{Y}q^{(2)}$

Fig. 1

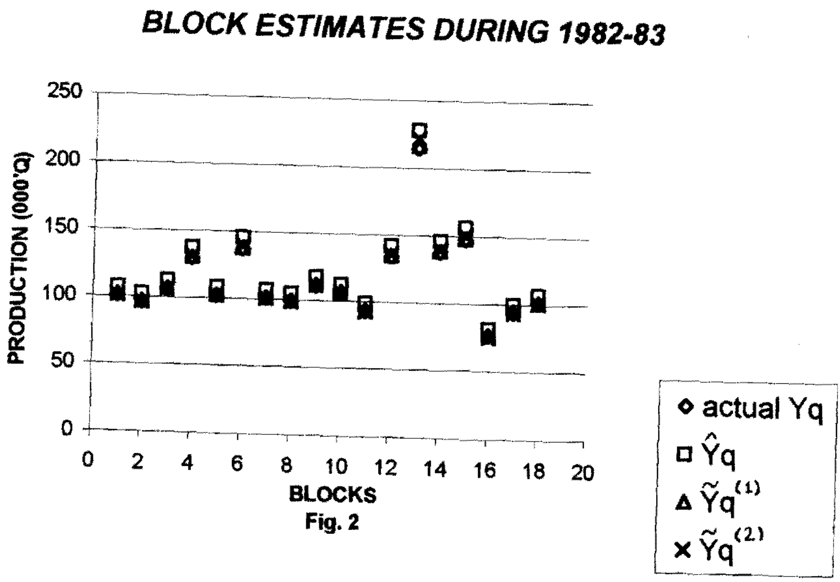
BLOCK ESTIMATES DURING 1982-83
PRODUCTION (000'Q)
250
200
150
100
50
0
0 2 4 6 8 10 12 14 16 18 20
BLOCKS
actual Yq
$\hat{Y}q$
$\tilde{Y}q^{(1)}$
$\tilde{Y}q^{(2)}$

Fig. 2

Table 5. Block estimates of rice production based on different estimators and their relative efficiency during the year 1982-83

S. No.	Block	Actual Estimate* (000 Q)	Estimates of Rice Production (000 Q)		
			$\hat{Y}_q$	$\tilde{Y}_q^{(1)}$	$\tilde{Y}_q^{(2)}$
1.	Masodha	100.00	107.07	101.72	100.94
2.	Sohawal	96.21	102.05	96.95	95.92
3.	Purabazar	105.63	112.03	106.44	105.91
4.	Myabazar	129.41	137.18	130.33	131.06
5.	Amaniganj	101.69	107.83	102.45	101.71
6.	Tarun	136.94	145.18	137.93	139.05
7.	Milkipur	100.56	106.62	101.29	100.49
8.	Bikapur	98.73	104.73	99.50	98.60
9.	Haringtonganj	110.47	117.11	111.26	110.98
10.	Bhiti	105.58	111.95	106.35	105.82
11.	Bhiyaon	92.65	98.27	93.36	92.14
12.	Jalalpur	133.60	141.70	134.62	135.57
13.	Akbarpur	214.23	227.10	215.75	220.97
14.	Katehari	137.70	145.98	138.69	139.85
15.	Tanda	147.28	156.16	148.36	150.04
16.	Jahangirganj	76.18	80.77	76.73	74.73
17.	Baskhari	93.74	99.45	94.48	93.32
18.	Ramnagar	100.25	106.31	100.99	100.18

* Based on crop cutting experiments.

$$\text{Value of } a = \frac{Y}{\sum_{q=1}^{Q} \hat{Y}_q} = 0.950$$

N.B. : The relative efficiency of $\tilde{Y}_q^{(1)}$ and $\tilde{Y}_q^{(2)}$ over $\hat{Y}_q$, viz. E_1 and E_2 comes out to be same for all the blocks, i.e. 110.80 and 105.88 per cent, respectively.

It is however important to point out some of the limitations of the present investigation. The first limitation was that rainfall data were not available at block level, therefore, the district-wise weekly rainfall data were used in developing the estimator at block level. The second limitation was about the fertilizer consumption, which is not being reported crop-wise by the reporting agencies. Thus, the total fertilizer consumption in a year was apportioned for rice crop. Almost one third of the total fertilizer consumption in a year was considered for rice production.

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