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Estimation of Current Population Ratio in Successive Sampling

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SUMMARY

The problem of estimation of the population ratio for the current occasion based on the samples selected over two occasions has been considered. Expressions for optimum estimator and its variance have been derived. The values of optimum matched proportion has been tabulated. The gain in efficiency of the proposed estimate over the direct estimate using no information gathered on the first occasion is computed. The proposed strategy has been compared with other sampling strategies and an empirical study is made to study the performance of the proposed strategy.

Key words : Successive sampling, Gain in efficiency, Matching fraction.

1. Introduction

Usually, in many national sample surveys, information collected regularly on the same population from one period to the next. In such repetitive surveys, three possible sampling procedures may be used

1. Extracting a new sample on each occasion (repeated sampling)

2. Using the same sample on every occasion ((panel sampling)

3. Performing a partial replacement of units from one occasion to another (sampling on successive occasions, which is also called rotation sampling when the units are constructed in the number of stages in which they are to become part of the sample, as it happens with the EPA–Spanish Survey of Working Population–which are performed quarterly, and most of the family surveys carried out by the INE–Spanish Statistics Institute).

The third possibility, has been discussed extensively by several authors in the case of estimating the population mean (total) (Jessen [15], Tikkiwal [27], Rao and Mudholkar [21], Artes and Garcia ([3], [7])).

The use of ratio method of estimation in successive sampling was first introduced by Avdhani [1] and later Sen *et al.* [22]. Gupta [13] has suggested the use of product method and later Artes *et al.* [2], Artes and Garcia [9] and Artes and Garcia [10].

However, in many practical situations an estimate of the population ratio of two characters for the most recent occasion may be of considerable interest, for example, the ratio of corn acres to wheat acres, the ratio of expenditures on labor to total expenditures, or the ratio of liquid assets to total assets.

In this paper we have presented a sampling strategy for estimating, by a linear estimate, the population ratio of two characters under two-stage sampling over two occasions (or sampling with partial replacement of units).

The theory of estimation of the population ratio of two characters over two occasions has been considered by Rao *et al.* [21], Rao and Pereira (1968), Thipathi and Sinha (1976) (Okafor and Arnab [18]), Okafor [19], Artes and Garcia [8], among others.

Okafor [19] gave some estimators of the population ratio when sampling is done with partial replacement of units. In this case, the estimate of the population total of the character y_1 on the recent occasion is first obtained by a suitable combination of two independent estimates of the population totals from the matched and unmatched samples. The estimate of the population total of y_2 is similarly obtained. These two estimates of the population totals of y_1 and y_2 are then used to derive the estimate of the population ratio.

2. *Selection of the Sample*

Suppose that the samples are of size n on both occasions, we use a simple random sampling and the size of the population N is sufficiently great for the factor of correction be ignored.

Let a simple random sample of size n be selected on the first occasion from a universe of size N. The measurements are taken on two characteristics y and x in each of two occasions in bivariate normal population. When selecting the second sample, we assume that $m = pn$ $(0 < p < 1)$ of the units of the selected sample on the first occasion are retained for the second occasion (matched sample) and the remaining $u = n - m = qn$, $(q = 1 - p)$ units are replaced by a new selection from the universe of $N - m$ units left after omitting m units.

3. Notation Used

x_i (y_i) : the variable x(y) on i^{th} occasion, i = 1, 2

$R_1 = \frac{\overline{Y}_1}{\overline{X}_1} \left(R_2 = \frac{\overline{Y}_2}{\overline{X}_2} \right)$: the population ratio on the first (second) occasion

$\hat{R}_1 = \frac{\overline{y}_1}{\overline{x}_1} \left(\hat{R}_2 = \frac{\overline{y}_2}{\overline{x}_2} \right)$: the estimator of the population ratio on first (second) occasion

ρ_1 (ρ_2) : the correlation coefficients between the variables y_1 and x_1 (y_2 and x_2)

ρ_3 (ρ_4) : the correlation coefficients between the variables y_2 and x_1 (y_1 and x_2)

ρ_5 (ρ_6) : the correlation coefficients between the variables x_1 and x_2 (y_1 and y_2)

$\hat{R}_{1m}$ $(\hat{R}_{2m})$: the estimator of the population ratio on the first (second) occasion based on the matched sample of m units

$\hat{R}_{1u}$ $(\hat{R}_{2u})$: the estimator of the population ratio on the first (second) occasion based on the unmatched sample of u units

We wish to estimate, R_2, the population ratio for the second period by a linear estimate (Hansen *et al.* [12]) of the form

$$\hat{R}'_2 = a\hat{R}_{1u} + b\hat{R}_{1m} + c\hat{R}_{2m} + d\hat{R}_{2u}$$

Since (Cochran [10])

$$E\left(\hat{R}_{1u}\right) = E\left(\hat{R}_{1m}\right) = R_1 \text{ and } E\left(\hat{R}_{2u}\right) = E\left(\hat{R}_{2m}\right) = R_2$$

we find that

$$E\left(\hat{R}'_2\right) = (a + b)R_1 + (c + d)R_2$$

If we now require that $\hat{R}'_2$ be an unbiased estimate of R_2, we must have

$$a + b = 0 \quad \text{and} \quad c + d = 1$$

so that

$$\hat{R}'_2 = a\left(\hat{R}_{1u} - \hat{R}_{1m}\right) + c\hat{R}_{2m} + (1-c)\hat{R}_{2u}$$

The variance of $\hat{R}'_2$ is

$$V\left(\hat{R}'_2\right) = a^2\left(\frac{1}{q} + \frac{1}{p}\right)\frac{1}{n\bar{x}_1^2}A + c^2\frac{1}{pn\bar{x}_2^2}B + (1-c)^2\frac{1}{qn\bar{x}_2^2}B - 2ac\,\mathrm{Cov}\left(\hat{R}_{1m}, \hat{R}_{2m}\right)$$

where

$$A = S^2_{y_1} + \hat{R}_1^2 S^2_{x_1} - 2\hat{R}_1\,\mathrm{Cov}(y_1, x_1)$$

$$B = S^2_{y_2} + \hat{R}_2^2 S^2_{x_2} - 2\hat{R}_2\,\mathrm{Cov}(y_2, x_2)$$

$$\mathrm{Cov}\left(\hat{R}_{1m}, \hat{R}_{2m}\right) = \frac{1}{pn\bar{x}_1\bar{x}_2}\left[\mathrm{Cov}(y_1, y_2) - \hat{R}_1\,\mathrm{Cov}(y_2, x_1) - \hat{R}_2\,\mathrm{Cov}(y_1, x_2) + \hat{R}_1\hat{R}_2\,\mathrm{Cov}(x_1, x_2)\right]$$

We wish to choose values of a and c that minimize $V\left(\hat{R}'_2\right)$ Equating to zero the derivatives of $V\left(\hat{R}'_2\right)$ with respect to a and c, it follows that the optimum values are

$$a_{opt} = \frac{pq\bar{x}_1\,ABC}{A^2B\bar{x}_2 - q^2\bar{x}_2AC^2}$$

$$c_{opt} = \frac{pAB}{AB - q^2C^2}$$

where

$$C = \left[\mathrm{Cov}(y_1, y_2) - \hat{R}_1\,\mathrm{Cov}(y_2, x_1) - \hat{R}_2\,\mathrm{Cov}(y_1, x_2) + \hat{R}_1\hat{R}_2\,\mathrm{Cov}(x_1, x_2)\right]$$

Thus, the estimate with optimum values for a and c may be written as

$$\hat{R}'_2 = \frac{pq\bar{x}_1\,ABC}{\left(AB - q^2C^2\right)A\bar{x}_2}\left(\hat{R}_{1u} - \hat{R}_{1m}\right) + \frac{pAB}{AB - q^2C^2}\hat{R}_{2m} + \left(1 - \frac{pAB}{AB - q^2C^2}\right)\hat{R}_{2u} \tag{1}$$

and its variance is

$$V\left(\hat{R}'_2\right) = \frac{B}{\bar{x}_2^2 n}\,\frac{AB - qC^2}{AB - q^2C^2} \tag{2}$$

Note that if $q = 0, p = 1$, complete matching or $p = 0$, $q = 1$, no matching this variance (2) has the same value

$$V\left(\hat{R}_2'\right)=\frac{1}{\bar{x}_2^2 n}\left(S_{y_2}^2+\hat{R}_2^2 S_{x_2}^2-2\hat{R}_2 \operatorname{Cov}\left(y_2, x_2\right)\right)$$

Thus, for current estimates, equal precision is obtained either by keeping the same sample or by changing it on every occasion.

If $\bar{x}_1=\bar{x}_2$, the estimate given by (1) is somewhat simplified

$$\hat{R}_2'=\frac{pqABC}{\left(AB-q^2C^2\right)A}\left(\hat{R}_{1u}-\hat{R}_{1m}\right)+\frac{pAB}{AB-q^2C^2}\hat{R}_{2m}+\left(1-\frac{pAB}{AB-q^2C^2}\right)\hat{R}_{2u}$$

but its variance is unchanged, that is

$$V\left(\hat{R}_2'\right)=\frac{B}{\bar{x}_2^2 n}\frac{AB-qC^2}{AB-q^2C^2}$$

Note also that an estimate for the first occasion is given by (1) simply by interchanging R_1's and R_2's if the estimate for the first occasion can await a time until data for both occasions are available.

$$\hat{R}_1'=\frac{pq\bar{x}_2 ABC}{\left(AB-q^2C^2\right)\bar{x}_1 B}\left(\hat{R}_{2u}-\hat{R}_{2m}\right)+\frac{pAB}{AB-q^2C^2}\hat{R}_{1m}+\left(1-\frac{pAB}{AB-q^2C^2}\right)\hat{R}_{1u} \tag{3}$$

Its variance is

$$V\left(\hat{R}_1'\right)=\frac{A}{\bar{x}_1^2 n}\frac{AB-qC^2}{AB-q^2C^2}$$

Equating to zero the derivative of $V\left(\hat{R}_2'\right)$ with respect to q, we find that the variance of $\hat{R}_2'$ will have its minimum value if we choose

$$q_{opt}=\frac{AB-\sqrt{A^2B^2-C^2AB}}{C^2} \tag{4}$$

and

$$V_{min}\left(\hat{R}_2'\right)=\frac{B}{\bar{x}_2^2 n}\frac{AB+\sqrt{A^2B^2-C^2AB}}{2AB} \tag{5}$$

However, if only the estimate using information gathered on the second occasion is considered, the estimator of the population ratio is

$$\hat{R}=p\hat{R}_{2m}+q\hat{R}_{2u}$$

and its variance is

$$V(\hat{R}) = \frac{1}{\bar{x}_2^2 n}\left(S_{y_2}^2 + \hat{R}_2^2 S_{x_2}^2 - 2\hat{R}_2 \operatorname{Cov}(y_2, x_2)\right) = \frac{B}{\bar{x}_2^2 n} \tag{6}$$

We find

$$\frac{B}{\bar{x}_2^2 n}\,\frac{AB + \sqrt{A^2B^2 - C^2AB}}{2AB} \le \frac{B}{\bar{x}_2^2 n} \tag{7}$$

We can compute the gain in precision G of the estimate obtained by using a linear estimate over the direct estimate using no information gathered on the first occasion

$$G = \frac{V(\hat{R})}{V(\hat{R}_2')} = \frac{AB - q^2C^2}{AB - qC^2} \tag{8}$$

or $$G_{opt} = \frac{V(\hat{R})}{V_{min}(\hat{R}_2')} = \frac{2AB}{AB + \sqrt{A^2B^2 - C^2\,AB}} \tag{9}$$

If now A, B and C are rewritten in terms of the correlation coefficients and the coefficients of variation

$$A = \bar{y}_1^2\left(C_{y_1}^2 + C_{x_1}^2 - 2\rho_1 C_{y_1} C_{x_1}\right)$$
$$B = \bar{y}_2^2\left(C_{y_2}^2 + C_{x_2}^2 - 2\rho_2 C_{y_2} C_{x_2}\right)$$
$$C = \bar{y}_1\bar{y}_2\left(\rho_6 C_{y_1} C_{y_2} - \rho_3 C_{x_1} C_{y_2} - \rho_4 C_{x_2} C_{y_1} + \rho_5 C_{x_1} C_{x_2}\right)$$

and assuming that

$$C_{y_1} = C_{y_2} = C_{x_1} = C_{x_2} = C_0$$
$$\rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho$$
$$\rho_5 = \rho_6 = \rho_0$$

the expressions (4), (8) and (9), become

$$q_{opt} = \frac{(1-\rho)^2 - \sqrt{(1-\rho)^4 - (\rho_0 - \rho)^2 (1-\rho)^2}}{(\rho_0 - \rho)^2}$$
$$G = \frac{\left(1 + \rho^2 - 2\rho\right) - q^2\left(\rho_0^2 + \rho^2 - 2\rho_0\rho\right)}{\left(1 + \rho^2 - 2\rho\right) - q\left(\rho_0^2 + \rho^2 - 2\rho_0\rho\right)}$$
$$G_{opt} = \frac{2(1-\rho)}{(1-\rho) + \sqrt{(1-\rho)^2 - (\rho_0 - \rho)^2}}$$

Table 1. Gain in precision, G, of the e imate proposed over the direct estimate

	$\rho = 0.3$			$\rho = 0.6$			$\rho = 0.9$		
q \ ρ_0	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.73
0.3	1.00	1.00	1.00	1.14	1.19	1.19	0.22	0.47	0.71
0.6	1.04	1.05	1.04	1.00	1.00	1.00	–0.11	0.35	0.68
0.9	1.20	1.29	1.32	1.14	1.19	1.19	1.00	1.00	1.00

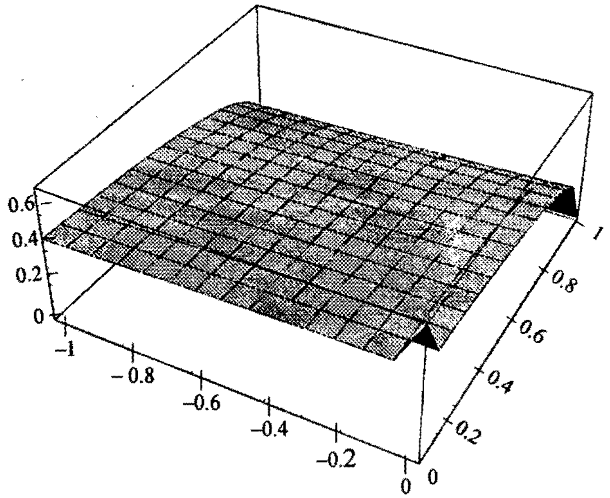

Figure 1. Optimum matching fraction, $1 - q_{opt}$

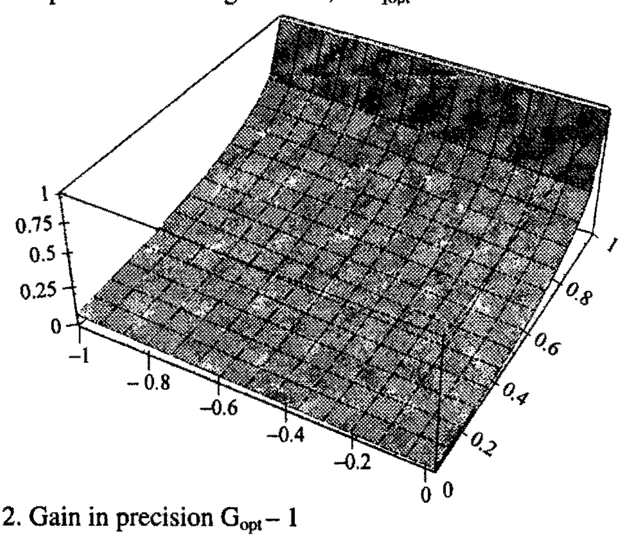

Figure 2. Gain in precision $G_{opt} - 1$

If, also we assume that

$$C_{y_1} = C_{y_2} = C_{x_1} = C_{x_2} = C_0$$

$$\rho_0 = -\rho$$

the expressions (4) and (9), become

$$q_{opt} = \frac{(1-\rho)^2 - \sqrt{(1-\rho)^4 - 4\rho^2 (1-\rho)^2}}{4\rho^2}$$

$$G_{opt} = \frac{2(1-\rho)}{(1-\rho) + \sqrt{(1-\rho)^2 - 4\rho^2}}$$

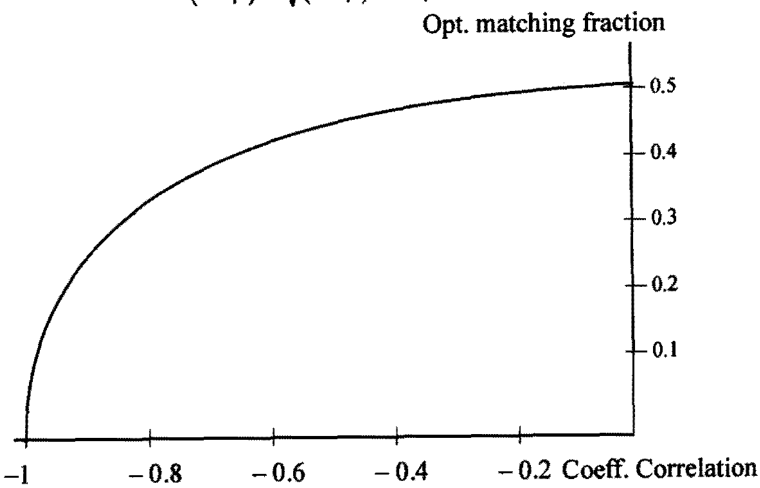

Figure 3. Optimum matching fraction, when $\rho_0 = -\rho$

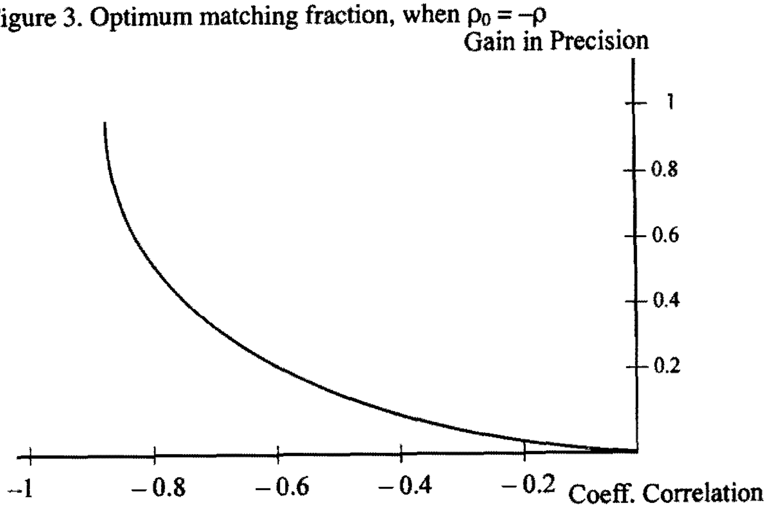

Figure 4. Gain in precision, G_{opt}, when $\rho_0 = -\rho$

The figures (3) and (4) show for a series of values of ρ the optimum that should be matched and the gain in precision compared with no matching. The best percentage to match never exceeds 50% (Patterson [20], Kulldorff [16], Tikkiwal [31]) and decrease steadily as ρ increases. The greatest attainable gain in precision is 100% when $\rho = -1$. Unless ρ (absolute value) is high, the gains are modest.

Although the optimum percentage to match varies with ρ, only a single percentage can be used in practice for all items in a survey.

4. Comparison with Other Strategies

The strategy proposed (5), is more efficient than the Rao (1957) (Okafor and Arnab [18]) strategy when

$$\frac{4(1-\rho_0)}{(1-\rho)+\sqrt{(1-\rho)^2-(\rho_0-\rho)^2}} \geq 1$$

Also, the strategy proposed (5), is more efficient than the Rao and Pereira (1968) (Okafor and Arnab [18]) strategy when

$$\frac{4(1+\rho_0-2\rho)}{(1-\rho)+\sqrt{(1-\rho)^2-(\rho_0-\rho)^2}} \geq 1$$

Finally, the strategy proposed (5), is more efficient than Thipathi and Sinha (1976) (regression-type ratio estimator) (Okafor and Arnab [18]) strategy when

$$\frac{\dfrac{2+\rho_0[\rho_0(1-\rho)+2(\rho-\rho_0)]}{(1-\rho)+\sqrt{1+\rho_0^2(1-\rho)^2+2\rho_0(1-\rho)(\rho-\rho_0)}}}{\dfrac{(1-\rho)+\sqrt{(1-\rho)^2-(\rho_0-\rho)^2}}{1-\rho}} \geq 1$$

In order to compare our strategy $\hat{R}'_2, (2)$, with the Okafor [19] strategy, $\hat{R}_2(i), i = 1, 2, 3, 4, 5$ we are going to compare the variances in the next way

$$V\left(\hat{R}_2(i)\right) - V\left(\hat{R}'_2\right) \geq 0$$

The last expression happens when

$$DF + A_i\, E \geq 0 \quad \text{for} \quad i = 1, 2, 3, 4, 5$$

where

$$E = 1 + \rho^2 - 2\rho, \; F = \rho_0^2 + \rho^2 - 2\rho_0\rho, \; D = 1 - \rho$$

and A_i, i = 1, 2, 3, 4, 5 are given as follows

Ratio-type ratio estimator $\hat{R}_2(1)$

$$A_1 = 1 + \rho - 2\rho_0$$

Product-type ratio estimator $\hat{R}_2(2)$

$$A_2 = 1 - 3\rho + 2\rho_0$$

Difference-type ratio estimator $\hat{R}_2(3)$

$$A_3 = \rho_0 \left(2\rho - \rho_0 - \rho_0\rho\right)$$

Difference-cum-ratio-type ratio estimator $\hat{R}_2(4)$

$$A_4 = \frac{1}{2}\left(1 - \rho_0^2\right) - \rho_0 + \rho$$

Difference-cum-product-type ratio estimator $\hat{R}_2\ (5)$

$$A_5 = \frac{1}{2}\left(1 - \rho_0^2 + 2\rho_0\right) + \rho(2\rho_0 - 1)$$

5. An Empirica! Study

The data under consideration was taken from census 1951 and census 1961, West Bengal, District Census Hand Book, Midnapore (Das [12]).

The characters x_1 and x_2 are numbers of houses for 1951 and 1961 respectively and the characters y_1 and y_2 are numbers of literate persons for 1951 and 1961 respectively. For this population we obtained

$\bar{x}_1 = 38.3696$	$\hat{C}_{x_1} = 1.3916$	$\hat{\rho}_5 = 0.7990$	$\hat{\rho}_3 = 0.5471$
$\bar{x}_2 = 50.4321$	$\hat{C}_{x_2} = 1.0585$	$\hat{\rho}_6 = 0.5392$	$\hat{\rho}_4 = 0.7028$
$\bar{y}_1 = 31.4321$	$\hat{C}_{y_1} = 2.2129$	$\hat{\rho}_1 = 0.9187$	
$\bar{y}_2 = 42.5761$	$\hat{C}_{y_2}$ 1.5048	$\hat{\rho}_2 = 0.7952$	

From this data we can state that

$$\hat{V}_{min}\left(\hat{R}'_2\right) = 0.99\frac{B}{\bar{x}_2^2 n} < \frac{B}{\bar{x}_2^2 n} = \hat{V}\left(\hat{R}\right)$$

(G_{opt} = 1.01%) which means a gain in precision of 1% of the proposed estimator over the usual estimator.

We have also calculated the optimum matching fraction

$$\hat{p}_{opt} = 49.58\%$$

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