Jour. Ind. Soc. Ag. Statistics
53(3), 2000 : 257-261

An Alternative Method of Construction of Border Circulant Plans

M.K. Sharma
Narendra Deva University of Agriculture & Technology
Kumarganj, Faizabad, U.P.
(Received : July, 1995)

SUMMARY

An alternative method of constructing the Border Circulant Plans of Arya [1] through circular designs of Das [3] has been proposed.

Key words : Partial diallel cross, Circular design, Circular plans, Border circulant plans.

1. Introduction

Partial diallel crossing plans of Kempthorne and Currnow [4] can be constructed for arbitrary values of n and s, where n denotes number of inbred and s (< n − 1) is the number of times each line is crossed with other lines, such that if n is even s must be odd, and vice versa. These plans have been termed as central circulant because these represent s crosses in the centre of the first line of table of crosses and crosses with rest of lines follow in a circulant manner. The name was first suggested by Arya [1] who distinguished his plan as Border one for the same reason.

Let there be total of n inbred lines numbered from 0 to n − 1. Arya [1] denoted a general circulant plan of crosses in which the O^{th} line has crosses with lines i, j, ..., q; n − q, ..., n − j, n − i by D (i, j, ..., q) where none of the integers i, j, ..., q is O and where i < j < ... < q < n/2. If s is even, the number of lines within parentheses is s/2. If s is odd, line O has an additional cross with line n/2 (n even) and there will be (s + 1) /2 integers in D (i, j, ..., q; n/2), crosses between rest of the lines are carried in a circulant manner.

By letting i = 1, j = 2 etc. in the above plan, the circulant plans D (1, 2, ..., s/2) for even s and any n and D (1, 2, ..., (s − 1)/2) for odd s but even n, are obtained. Arya [1] termed these plans as a border circulant ones and listed them in his Table A.1 from 13 to 24 and s from 3 to 12. Arya and Narain [2] listed some selected plans when n is odd and s is even and vice-versa for n upto 100 for comparison purpose to those of Kempthorne and Curnow [4]. They introduced the concept of asymptotic efficiency (AE) and best asymptotically efficient plans (BAE) to judge the superiority of border plans over general ones. According to them, a plan showing increasing efficiency with increased n for all s is said to be asymptotically efficient and an asymptotically efficient plan with maximum efficiency for all n and all s is said to be the best asymptotically efficient (BAE) plan.

In the present paper, an alternative method has been proposed to obtain border plans or Arya [1] using symmetrical circular designs evolved by Das [3].

2. *Definition*

According to Das [3], a circular design will be characterized by the following :

Let there be n equal arcs on the circumference of a circle denoted in order by a_1, a_2, ... , a_n. If we form bigger arcs of size A_{ki}, such that it is the sum of k consecutive small arcs starting with a_i (i = 1, 2, ... , n), we shall have in all n such arcs for n different values of i. Now we identify a_i with a set (s_{ij}), where j = 1, 2, ..., m of m treatments such that the different set (s_{ij}) are mutually exclusive, then the contents of the n arcs, A_{ki} will form an incomplete block design with n blocks, mn treatments, mk block size and k replications. The block contents of the design come out as below :

Block number

1. (s_{1j}) (s_{2j}) ... (s_{kj})
2. (s_{2j}) (s_{3j}) ... $(s_{k+1,j})$
3. (s_{3j}) (s_{4j}) ... $(s_{k+2,j})$
4. . . .

. . . .

. . . .

n. (s_{nj}) (s_{1j}) ... $(s_{k-1,j})$

According to Das [3], such designs can be constructed for any number of treatments and block sizes. Since the placement of the treatments make use of circular association scheme, hence these have been called circular designs.

It has also been stated by Das [3] that the Circular designs become symmetrical when m = 1 and all symmetrical circular designs are actually partially balanced incomplete block designs with number of associate classes $(n-1)/2$, when n is odd and each n_i^{th} associate $= 2$ $(i = 1, 2, \ldots, (n-1)/2)$. When n is even, the number of associate classes are n/2 and one of the i^{th} associate is 1 and rest are 2 $(i = 1, 2, \ldots, n/2)$. The n_i^{th} associate occur together λ_i times in the design.

where $\lambda_i = k - i$

$$= 0 \text{ when } k < i, \quad i = 1, 2, \ldots, (n-1)/2 \text{ or } n/2 \text{ when n is odd or even}$$

The construction of such designs with n treatments and k block size, where n and k are positive integers greater than 0, can be done in the case of $m = 1$, by arranging the treatments in ascending order on the circumference of a circle. The first block of the design will contain k treatments which are at a distance of say, q (= 1) in the same direction. The next block will have the $k-1$ treatments (taken in order from the second position) of the previous block and another treatment which is at a distance q (= 1) in the same direction. The process is to be repeated to obtain n blocks till we arrive at the first $k-1$ treatments of the first block with which we started. The distance between the last and first treatment is considered equal to 1.

Our choice for the construction of border plans will be symmetrical circular designs of the above type with block sizes $k > 2$.

Example 2.1 : The plan of symmetrical circular designs with parameters $n = 14 = b$, $r = k = 3$, $\lambda_1 = 2$, $\lambda_2 = 1$, $\lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0$, $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = 2$ and $n_7 = 1$ obtained by the above method, is given below

PLAN

1	2	3
2	3	4
3	4	5
4	5	6
5	6	7
6	7	8
7	8	9
8	9	10
9	10	11
10	11	12
11	12	13
12	13	14
13	14	1
14	1	2

3. Construction of Partial Diallel Crossing Plans

The method is simply stated as follows : Take for n lines under evaluation numbered randomly from 1 to n, a symmetrical circular design with n treatments and block size k (> 2 but less than n/2). Let the n lines be made correspond to the n treatments in the symmetrical design considered. The first line in each block is crossed with all the k-1 line in the block. Thus there will be a total of n(k – 1) crosses. These n (k – 1) crosses constitute the circular crossing plan for even s, where s = 2 (k – 1). In case n is even, then each i^{th} line is crossed with line $\left(\frac{n}{2}+i\right)^{th}$ mod n, neglecting reciprocal crosses, we obtain n(2k – 1)/2 crosses which constitute circular crossing plan for even n and odd s, where s = (2k – 1).

A symmetrical circular design of block size > n/2 gives n (n – 1)/2 crosses for odd n and $n^2/2$ crosses for even n. Out of $n^2/2$ crosses, n(n – 1)/2 crosses are distinct and n/2 crosses are reciprocal of the crosses of the type $\left\{i\left(\frac{n}{2}+i\right)\right\}$, where i = 1, 2, .. n/2.

It is observed that plans constructed by the above procedure are those of Arya [1].

Example 2.1 (Continued) : The circular crossing plans obtained from symmetrical circular design (Example 2.1) in Section 2, are given below.

Crossing Plans

	II n = 14 and s = 5	
I n = 14 and s = 4		
1 × 2	1 × 3	1 × 8
2 × 3	2 × 4	2 × 9
3 × 4	3 × 5	3 × 10
4 × 5	4 × 6	4 × 11
5 × 6	5 × 7	5 × 12
6 × 7	6 × 8	6 × 13
7 × 8	7 × 9	7 × 14
8 × 9	8 × 10	
9 × 10	9 × 11	
10 × 11	10 × 12	
11 × 12	11 × 13	
12 × 13	12 × 14	
13 × 14	13 × 1	
14 × 1	14 × 2	

The first 28 crosses under I constitute circular partial diallel crossing plan for n = 14 and s = 4. All 35 crosses under (I & II) constitute circular partial diallel crossing plan for n = 14 and s = 5.

4. *Efficiency*

The efficiency factor of border Circulant plans over central ones has been discussed by Arya and Narain [2] and it has been observed by them that the efficiency factor E > 1 for all combinations of n and s, implying that border circulant plans are better than central one. Further more the efficiency is higher for smaller values of s and for larger values of n as well.

ACKNOWLEDGEMENT

The author expresses deep sense of gratitude to the referee, whose suggestions led to substantial improvement in the draft as well as presentation of the paper.

REFERENCES

[1] Arya, A.S. (1983). Circulant plans for partial diallel crosses. *Biometrics*, **39**, 43-52.

[2] Arya, A.S. and Narain, P. (1990). Asymptotically efficient partial diallel crosses. *Theoretical and Applied Genetics*, 849-852.

[3] Das, M.N. (1959). Circular designs. *Jour. Ind. Soc. Agril. Stat.*, **12**, 45-56.

[4] Kempthorne, O. and Curnow, R.N. (1961). The partial diallel crosses. *Biometrics*, **17**, 229-250.