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An Empirical Study of Ratio-regression Type Estimators

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SUMMARY

An attempt has been made to compare empirically the relative performance of ratio, product, regression estimators, super population model based generalised least square (GLS) linear regression etimator, GLS with zero intercept linear estimator, GLS zero intercept reciprocal estimator, inclusion probability proportional to size estimator and some other estimators in respect of their bias, variance and mean square error. The various populations have been generated through computer satisfying different model assumptions and distributions and a simulation study of the sampling distributions of the estimators has been made drawing a large number of random samples. The conditions under which different estimators give better performance have been noted.

Key words: Simulation studies, Ratio estimator, Regression estimator, Product estimator, Generalised least square estimator, Super population models.

1. Introduction

The technique of ratio estimation has been widely used for improving the precision of simple mean estimator when information on an auxiliary variable X having high positive correlation with study variable Y, is available. Sukhatme & Sukhatme [4] have proved ratio estimator to be optimum having minimum variance when V (Y/X) is proportional to X and the regression line passes through origin. Hartley & Ross [6] proposed an unbiasedly ratio type estimator by estimating the bias of ratio estimator $\bar{r}.\,\bar{x}$ unbiased and subtracting the same from the estimator. When regression line of Y on X does not pass through origin, the regression estimator has been suggested (Cochran [3]).

From the standard regression theory, and best linear unbiased property of estimators based on least square error regression, the linear regression estimator is optimal when regression between Y & X is linear and V(Y|X) is constant. Johnston [7] suggested the use of generalized least square (GLS) technique when the variance of error terms is not constant. Goodman [4] proposed a product estimator as complementary to ratio estimator when Y & X are negatively correlated. Murthy [8] developed unbiased product type estimators.

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Srivenkataramana [13] proposed a new product type estimator and Sahoo [12] studied efficiency of this estimator under Super Population Model $Y_i = \beta X_i + e_i$ with $E(e_i \mid X_i) = 0$ and $V(e_i \mid X_i) = \gamma X_i^g$ where e_i's are uncorrelated with $0 < \gamma < \infty$ and $0 \leq g \leq 2$. It has been observed that no estimator is uniformly superior over other estimators. Comparison of estimators under Super Population Models satisfying certain broad framework of conditions, were first employed by Cochran [2] to overcome complex algebraic expressions which often occur in ratio/regression/product type estimators. Cassel, Sarndal and Wretman [1] provide a comprehensive account of such models. Rao and Bayless [9] compared the stabilities of estimators and stabilities of variance estimators using Super Population Models. Reddy [10] suggested an estimator of type $\hat{Y}_\theta = \hat{Y}X/(X + \theta(\hat{X} - X))$ for any scalar θ and studied the bias and mean square error under a super population model and found that a ratio type estimator with a suitable transformation on the supplementary variance resulted in a smaller absolute bias and smaller mean square error under fairly wide conditions. Gupta and Kothwala [5] studied various ratio-product type estimators and discussed stability of these estimators with the help of live data.

In this work an attempt has been made to study through sampling experiments, the relative performance of ratio, product, regression and some other estimators in respect of their bias, variance and mean square errors for various kinds of population generated through computer, satisfying different model assumptions based on Super Population Model

$$Y_i = \alpha + \beta X_i + e_i \text{ with } E(Y_i \mid X_i) = 0,\ V(Y_i \mid X_i) = \gamma X_i^g \quad \text{for } i = 1, 2, \ldots, N \qquad (1)$$

where α, β, γ, g are the population parameters of the infinite Super-population. The relative performance of the various estimators under different model situations has been assessed through sampling experiments so that a stable estimator over model fluctuations can be identified.

A new product estimator based on harmonic means rather than arithmetic means has been proposed whose performance against the usual product estimator has been studied. The impact of sample size in reducing sampling error has also been studied.

2. *Estimators Considered*

1. Ratio Estimator $$\bar{y}_1 = \frac{\bar{y}}{\bar{x}} \cdot \bar{X} \qquad (2)$$

2. Product Estimator $$\bar{y}_2 = \frac{\bar{y}\,.\,\bar{x}}{\bar{X}} \tag{3}$$

3. Regression Estimator $$\bar{y}_3 = \bar{y} + b\,(\bar{X} - \bar{x}) \tag{4}$$

where $b = s_{xy}/s_{xx}$ is estimated from the sample and $\bar{X}$ is assumed to be known.

4. Generalized Least Square Linear Estimator

One can develop an estimator based on generalized least square (Johnston [7]) with underlying model

$$Y_i = \alpha + \beta\, X_i + e_i \text{ with } E\,(e_i|\, X_i) = 0 \text{ and } V\,(e_i|X_i) = \gamma\, X_i^g \quad \text{for } i = 1, 2, \ldots, N \tag{5}$$

and obtaining Normal equations

$$\begin{aligned} \sum Y_i X_i^{-g} &= \tilde{\alpha} \sum X_i^{-g} + \tilde{\beta} \sum X_i^{1-g} \text{ and} \\ \sum Y_i X_i^{1-g} &= \tilde{\alpha} \sum X_i^{1-g} + \tilde{\beta} \sum X_i^{2-g} \end{aligned} \tag{6}$$

giving $$\tilde{\beta} = \frac{\sum Y_i X_i^{1-g} - \dfrac{(\sum Y_i X_i^{-g})(\sum X_i^{1-g})}{\sum X_i^{-g}}}{\sum X_i^{2-g} - \dfrac{(\sum X_i^{1-g})^2}{\sum X_i^{-g}}} \text{ and} \tag{7}$$

$$\tilde{\alpha} = \frac{\sum Y_i X_i^{-g} - \tilde{\beta} \sum X_i^{1-g}}{\sum X_i^{-g}}$$

Based on these estimators of $\tilde{\alpha}$ and $\tilde{\beta}$ and a given value of g, we may consider an estimator

$$\bar{y}_4 = \tilde{\alpha} + \tilde{\beta}\,.\,\bar{X} \tag{8}$$

$$= \frac{\sum_{i=1}^{n} Y_i X_i^{-g}}{\sum_{i=1}^{n} X_i^{-g}} + \tilde{\beta} \left(\bar{X} - \frac{\sum_{i=1}^{n} X_i^{1-g}}{\sum_{i=1}^{n} X_i^{-g}} \right) \tag{9}$$

It may be observed that the usual regression estimator $\bar{y}_3$ is a particular case of $\bar{y}_4$ for $g = 0$.

5. GLS Zero Intercept Linear Estimator

If the zero intercept linear model $Y_i = \beta X_i + e_i$ is assumed, we have only one coefficient to be estimated by the generalized least square method, and consequently, due to lesser variability in using an estimator based on the single estimated coefficient, the same is expected to outperform the estimator $\bar{y}_4$ when such a situation actually holds. Proceeding on the analogous lines, we obtain another estimator, namely $\bar{y}_5$

$$\bar{y}_5 = \tilde{\beta} . \bar{X} = \frac{\sum^{n} Y_i X_i^{1-g}}{\sum^{n} X_i^{2-g}} . \bar{X} \tag{10}$$

We note the following deductions

(i) For $g = 0$, we obtain $\bar{y}_5 = \dfrac{\sum Y_i X_i}{\sum X_i^2} . \bar{X}$, the usual linear regression estimator with zero intercept

(ii) For $g = 1$, we obtain $\bar{y}_5 = \dfrac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} . \bar{X} = \bar{y}_1$, the ratio estimator

(iii) For $g = 2$, we obtain $\bar{y}_5 = \dfrac{\sum_{i=1}^{n} y_i/x_i}{n} . \bar{X} = \bar{r} . \bar{X}$

where $\bar{r} = \dfrac{1}{n} \sum_{i=1}^{n} r_i = \dfrac{1}{n} \sum \dfrac{y_i}{x_i}$

Then $\bar{y}_5 = \dfrac{1}{n} \sum_{i=1}^{n} \dfrac{y_i}{x_i} . \dfrac{X}{N} = \dfrac{1}{n} \sum_{i=1}^{n} \dfrac{y_i}{N . P_i}$ where $P_i = X_i/X$

which is a well known estimator for inclusion probabilities proportional to size, assuming ofcourse that the sample is accordingly selected with varying probabilities.

6. GLS Zero Intercept Reciprocal Estimator

Consider a model $Y_i = \frac{\beta}{X_i} + e_i$ with $E(e_i|X_i) = 0$, $V(e_i|X_i) = \gamma X_i^g$. This model establishes a hyperbolic relationship between Y and X. Using GLS method, as before we obtain another estimator

$$\bar{y}_6 = \frac{1}{N}\cdot\tilde{\beta}\sum_{i=1}^{N}\frac{1}{X_i} = \frac{\sum_{i=1}^{n} y_i x_i^{-(1+g)}}{\sum_{i=1}^{n} x_i^{-(2+g)}}\cdot\frac{1}{\tilde{X}} \tag{11}$$

where $\tilde{X} = \frac{N}{\sum_{i=1}^{N}\frac{1}{X_i}}$ = Harmonic mean of population for variable X

7. Inclusion Probability Proportional to Size Estimator

The well known πps estimator

$$\bar{y}_7 = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{N.P_i} \text{ where } P_i = X_i/X_{..}, \text{ can be rewritten as } \bar{y}_7 = \bar{r}.\bar{X} \tag{12}$$

We note that for $g = 2$, this estimator is same as $\bar{y}_5$

8. Hartley-Ross Ratio-Type Estimator

The design bias of $\bar{r}.\bar{X}$ when the sample has been selected with srswor, can be estimated unbiasedly and the estimator can be adjusted to obtain a design unbiased estimator, as suggested by Hartley and Ross [6].

$$\bar{y}_8 = \bar{r}.\bar{X} + \frac{n(N-1)}{N(n-1)}.(\bar{y} - \bar{r}.\bar{x}) \tag{13}$$

9. Product Estimator using Harmonic Means

We finally consider a special case of $\bar{y}_6$ when $g = -1$. The estimator reduces to

$$\bar{y}_9 = \frac{\bar{y}.\tilde{x}}{\tilde{X}} \tag{14}$$

where $\tilde{x}$ and $\tilde{X}$ are the sample and population harmonic means respectively.

This is a product type estimator using harmonic means rather than the arithmetic means for the auxiliary variables.

3. Population Generation

In order to study the performance of the above nine estimators, we generate the following populations for variables X and Y respectively.

1. General Linear Transformation Model

The values of X variables are generated by considering the deterministic model $X_i = a + b$, $i = 1, 2 \ldots, N$ where a and b are given constants, giving a well spread over range of values for X and a flat frequency distribution.

The values for Y variables are generated by considering a stochastic model

$$Y_i = \alpha + \beta X_i + e_i \text{ with } E(e_i \mid X_i) = 0, V(Y_i \mid X_i) = E(e_i^2 \mid X_i) = \gamma X_i^g \quad \text{for } i = 1, 2, \ldots, N \tag{15}$$

where α, β, γ, g are the given constants and X_i values are those generated earlier. The stochastic error term e_i is obtained so that it follows a Normal distribution with $e_i \sim N(0, \gamma X_i^g)$.

The random number z_1 is obtained between 0 and 1 as generated by computer after setting a random seed for pseudo random number generator. Taking the random numbers obtained as the distribution function value from a Normal Distribution, we solve for the standard normal variate z such that

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2} \, dx = z_1 \tag{16}$$

For the purpose we have utilized Area under the Normal Curve table by suitably transforming z_1 so that the area values from 0 to z may be utilized rather than from $-\infty$ to z as the equation above requires. The search technique employed is to read two successive values denoted as xx and yy and test whether the random number z_1 falls in between. If it falls between these two values, the standard normal variate z is computed by linear interpolation and finally the appropriate sign, positive or negative is attached to z, depending upon whether, $z_1 > 0.5$ or $z_1 < 0.5$ respectively.

The standard normal variate obtained above is utilized to obtain the stochastic term, $e_i = z. \sqrt{(\gamma X_i^g)}$, $i = 1, 2, \ldots, N$ so that e_i follows a normal distribution $N(0, \gamma X_i^g)$. In the end, the Y values are generated using the formula,

$$Y_i = \alpha + \beta X_i + z. \sqrt{(\gamma X_i^g)} \quad i = 1, 2, \ldots, N$$

so that
$$E(Y_i \mid X_i) = \alpha + \beta X_i \quad \text{and} \quad V(Y_i \mid X_i) = \gamma X_i^g \tag{17}$$

2. Quadratic Transformation Model

The values of X variables are generated from the deterministic model

$$X_i = a + bi + di^2, i = 1, 2, ..., N \tag{18}$$

giving more spread or less spread over the range of values of X as compared to the linear transformation model depending upon as d is positive or negative, respectively.

The values of Y variables are generated from the stochastic model

$$Y_i = \alpha + \beta X_i + \delta X_i^2 + e_i$$

with $$E(e_i | X_i) = 0, V(Y_i | X_i) = E(e_i^2 | X_i) = v X_i^g, \quad i = 1, 2, \ldots, N \tag{19}$$

where α, β, v, g are the given constants. The X_i values used here are generated earlier. The stochastic error term e_i follows a Normal distribution with $e_i \sim N(0, v X_i^g)$.

The standard normal variate z is obtained through the process described earlier in order to generate Y-values using the relationship

$$Y_i = \alpha + \beta X_i + \delta X_i^2 + z\sqrt{(v X_i^g)} \quad i = 1, 2, \ldots, N \tag{20}$$

Such models are special cases of Royall and Herson [11] polynomial regression models, and provide for an opportunity to test for the stability of estimator in respect of its performance over different model situations when there is departure from the linearity.

3. Reciprocal Transformation Models

The values of X variables are generated from the deterministic model

$$X_i = a + b/i, i = 1, 2, ..., N \tag{21}$$

where a and b are given constants. For this model, X_i values are steadily decreasing values as i increases when b is positive and are increasing values as i increases when b is negative. When a = 0, the relation becomes a hyperbola.

The values of Y variables are generated from the stochastic model

$$Y_i = \alpha + \beta/X_i + e_i, E(e_i | X_i) = 0, \; V(Y_i | X_i) = E(e_i^2 | X_i) = v X_i^g \quad i = 1, 2, \ldots, N \tag{22}$$

where α, β, v, g are the given constants and the error term e_i follows a Normal distribution, $N(0, v\ X^g_i)$.

4. Normal Population

The values of X variables are generated by drawing random samples from a Normal Population with given mean m_x and variance v_x. The standard normal variate z is obtained in the manner described earlier and then the values of X are computed as

$$X_i = m_x + z\sqrt{(v_x)} \tag{23}$$

the values of Y variables are generated similarly from a Normal population with given mean m_y and variance v_y.

The possible combinations of models for X and Y gives a total of 16 combinations under which the estimators were compared. These are listed on the next page.

In order to have a manageable number of simulations, we have assumed the following values of parameters as fixed.

$a = 0$	$b = 1$	$d = 0.5$	$\beta = 5$	$\delta = 1$	$v = 1$
$N = 100$	$m_x = 50$	$v_x = 0$	$m_y = 100$	$v_y = 20$	No. of Samples = 500

and varied the values of parameters

$\alpha = 0, 100$	$g = -1, 0, 1, 2$	$n = 10, 20, 30$

Infact for each sample size there are eight combinations of α and g values for which the performance of the estimators over different 16 models has been computed. Note that model numbers 4, 8, 12 and 16 do not involve α and g values at all. The results for these simulations have been represented by an abbreviation such as $g\,(x, y)\,\alpha$, where $g = -1, 0, 1, 2$ and $\alpha = 0, 100$ with x taking values 1, 2, 3, N and y also taking values 1, 2, 3, N the digit standing respectively for the type of model used and alphabet N standing for the Normal distribution model.

Five hundred (500) random samples were selected independently by simple random sampling without replacement (srswor) from the populations thus generated. The characteristics of the population such as Population Means $\overline{Y}, \overline{X}$, Sum of Squares, S_{xx} and S_{yy}, Sum of Products S_{xy}, Population Correlation Coefficient ρ, the expression $\rho\, C_y / C_x$ are computed. The bias, variance, Mean Square Error (MSE), relative bias and relative error for each of the estimators have been computed and the estimators are compared for lower bias and greater efficiency.

Sl. No.	Model for X variable	Model for Y variable		Notation
1.	$X_i = a + b\,i$	$Y_i = \alpha + \beta X_i + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = 1, y = 1$
2.	$X_i = a + b\,i$	$Y_i = \alpha + \beta X_i + \delta X_i^2 + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = 1, y = 2$
3.	$X_i = a + b\,i$	$Y_i = \alpha + \beta / X_i + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = 1, y = 3$
4.	$X_i = a + b\,i$	$Y_i \sim N(m_y, v_y)$		$x = 1, y = N$
5.	$X_i = a + b\,i + di^2$	$Y_i = \alpha + \beta X_i + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = 2, y = 1$
6.	$X_i = a + b\,i + di^2$	$Y_i = \alpha + \beta X_i + \delta X_i^2 + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = 2, y = 2$
7.	$X_i = a + b\,i + di^2$	$Y_i = \alpha + \beta / X_i + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = 2, y = 3$
8.	$X_i = a + bi + di^2$	$Y_i \sim N(m_y, v_y)$		$x = 2, y = N$
9.	$X_i = a + b / i$	$Y_i = \alpha + \beta X_i + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = 3, y = 1$
10.	$X_i = a + b / i$	$Y_i = \alpha + \beta X_i + \delta X_i^2 + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = 3, y = 2$
11.	$X_i = a + b / i$	$Y_i = \alpha + \beta / X_i + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = 3, y = 3$
12.	$X_i = a + b / i$	$Y_i \sim N(m_y, v_y)$		$x = 3, y = N$
13.	$X_i \sim N(m_x, v_x)$	$Y_i = \alpha + \beta X_i + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = N, y = 1$
14.	$X_i \sim N(m_x, v_x)$	$Y_i = \alpha + \beta X_i + \delta X_i^2 + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = N, y = 2$
15.	$X_i \sim N(m_x, v_x)$	$Y_i = \alpha + \beta / X_i + e_i$,	$E(e_i \mid X_i) = 0, V(e_i \mid X_i) = v X_i^g$	$x = N, y = 3$
16.	$X_i \sim N(m_x, v_x)$	$Y_i \sim N(m_y, v_y)$		$x = N, y = N$

4. *Results and Discussions*

The trend of performance of various estimators does not fluctuate drastically with sample sizes chosen i.e., 10, 20 and 30 except that the MSE decreases with increasing sample size. Based on the MSE for the estimators, Table 1 lists the model situations for which each estimator performs best in terms of lowest MSE value. Taking these 100 different model situations to be a very spectrum of real life situations, following conclusions can be drawn:

1. Ratio Estimator $(\bar{y}_1)$ has the smallest mean square error in 5% of the 100 model simulations studied for each sample size. However, it is close competitor of the best estimator in 19% of the cases. Thus estimator can be recommended in 24% situations.
2. Product Estimator $(\bar{y}_2)$ has the smallest mean square error in 2% situations namely for - 1 (2, 1) and 1 (2, 3)0. In 9% of situations it is close competitor of the best estimator. Thus it can be recommended in 11% of the cases.
3. Regression Estimator $(\bar{y}_3)$ has the smallest MSE in 37% of the cases and in 35% cases it is close competitor of the best one. It can therefore be recommended in 72% of the cases and is thus the **most stable** estimator.
4. GLS Linear Estimator $(\bar{y}_4)$ is the best in 38% cases and is a close competitor of the best estimator in 25% cases. This estimator can thus be recommended in 63% of the cases.
5. GLS Zero Intercept Linear Estimator $(\bar{y}_5)$ has lowest MSE in 16% cases and is close competitor of the best estimator in 12% cases. Thus the estimator could be suitable in 28% cases.
6. GLS Zero Intercept Reciprocal Estimator $(\bar{y}_6)$ has the smallest MSE in 12% situations studied. It is close competitor of the best estimator in 9% cases thereby can be recommended in 21% cases.
7. Inclusion Probability Proportional to Size Estimator $(\bar{y}_7)$ is best in 3% cases and close to the best in 11% cases and can therefore be recommended in 14% cases.
8. Hartley-Ross Ratio-Type Estimator $(\bar{y}_8)$ has the smallest MSE in only 1% situations namely, 0(N,1)0 and it is close competitor of best estimator in 4% of the model situations.
9. New Product Estimator $(\bar{y}_9)$ using harmonic means rather than the arithmetic means of the auxiliary characters is the best in 13% of the

Table 1. Model situations for which the estimator has lowest MSE

Sl. No.	Estimators	Model for which MSE is lowest					
1.	Ratio estimator ($\overline{y}_1$)	1 (1, 1) 0	−1 (1, 2) 100	1 (3, 1) 0	1 (N, 1) 0	2 (N,1) 100	
2.	Product estimator ($\overline{y}_2$)	−1 (2, 1)	1 (2, 3) 0				
3.	Regression estimator ($\overline{y}_3$)	−1 (1, 1) 100, 1 (1, 2) 100, −1 (2, 2) 0, 2 (2, 2) 100, 0 (3, 2) 100, 0 (N, 2) 0, 0 (N, 3) 100	0, (1, 1) 100, 2 (1, 2) 0, 0 (2, 2) 0, −1 (2, 3) 100, 1 (3, 2) 0, 0 (N, 2) 100	−1 (1, 2,) 0, 2 (1, 2) 100, 0 (2, 2) 100, 0 (2, 3) 100, (3, N), 1 (N, 2) 0	0 (1, 2) 0, 0 (1, 3) 100, 1 (2, 2) 0, (2 , N) −1 (N, 1) 100, 1 (N, 2) 100	0 (1, 2) 100, (1, N), 1 (2, 2) 100, 0 (3, 1) 100, 0 (N, 1) 100, 2 (N, 2) 0	1, (1, 2) 0, 0 (2, 1) 100, 2 (2, 2) 0, 2 (3, 1) 100, 1 (N, 1) 100, −1 (N, 3) 100
4.	GLS Linear estimator ($\overline{y}_4$)	0 (1, 1) 100, 0 (1, 3) 100, 2 (2, 1) 100, 1 (2, 3) 100, −1 (3, 2) 100, −1 (N, 2)0, 0 (N, 3) 100	1 (1, 1) 100, 1 (1, 3) 100, −1 (2, 2) 100, 1 (2, 3) 100, 0 (3, 2) 100, −1 (N, 2) 100, (3, N)	2 (1, 1) 100, 2 (1, 3) 100, 0 (2, 2) 0, −1 (3, 1) 100, 1 (3, 2) 100, 0 (N, 2) 0	0 (1, 2) 0, −1 (2, 1) 100, 0 (2, 3) 100, 0 (3, 1) 100, 2 (3, 2) 100, 0 (N, 2) 100	0 (1, 2) 100, 0 (2, 1) 100, −1 (2, 3) 100, 1 (3, 1) 100, −1 (N, 1) 100, 2 (N, 2) 100	−1 (1, 3)100, 1 (2, 1) 100, (3, N), 2 (3, 1) 100, 0 (N, 1) 100, − (N, 3) 100
5.	GLS Zero Intercept Linear estimator ($\overline{y}_5$)	−1(1,1)0 1(2,1)0, 2(3,2)0	0(1,1)0 2(2,1)0, −1(N,1)0	1(1,1)0 1(3,1)0, 1(N,1)0	2(1,1)0 2(3,1)0, 2(N,1)0	−1(2,1)0, −1(3,2)0,	0(2,1)0, 0(3,2)0,
6.	GLS Zero Intercept Linear Reciprocal estimator ($\overline{y}_6$)	−1(1,3)0, 0(3,3)100	−1(3,1)0, 1(3,3)0	0(3,1)0, 1(3,3)100	−1(3,3)0, 2(3,3)0	−1(3,3)100, 2(3,3)100	0(3,3)0, 2(N,3)100
7.	IPPS estimator ($\overline{y}_7$)	2(1,1)0,	2(3,1)0,	2(3,2)0			
8.	Hartley-Ross Ratio type estimator ($\overline{y}_8$)	0(N,1)					
9.	New Product estimator ($\overline{y}_9$)	−1(1,2)0, 2(2,3)0, 2(N,3)0	0(1,3)0, 2(2,3)100	1(1,3)0, −1(N,3)0,	2(1,3)0, 0(N,3)0	−1(2,3)0, 1(N,3)0	0(2,3)0, 1(N,3)100

model situations studied. In 4% of the model situations it is close competitor of the best estimator. This estimator can thus be recommended in 17% situations.

Further the regression estimator is the most stable estimator among the different model situations studied. The New Product estimator based on harmonic means is biased and underestimates the population to a considerable extent. However, the sampling fluctuations are of much lower magnitude than those of the usual product estimator. The new product estimator is generally best for Y model 3 for which it is infact intended.

REFERENCES

[1] Cassel, C.M., Sandal, C.E. and Wretman, J.H., 1977. *Foundations of Inference in Survey Sampling*. John Wiley and Sons, New York.

[2] Cocharn, W.G., 1946. Relative efficiency of systematic and stratified random samples for a certain class of population. *Ann. Math. Statist.*, **17**, 164-177.

[3] Cochran, W.G., 1977. *Sampling Techniques*. 3rd edition, John Wiley and Sons, New York.

[4] Goodman, L.A., 1960. On the exact variance of products. *J. Amer. Statist. Assoc.*, **55**, 708-713.

[5] Gupta, P.C. and Kothwala, N.H., 1990. A study of second order approximation for some product type estimators. *J. Ind. Soc. Agril. Statist.*, **XLII(2)**, 171-185.

[6] Hartley, H.O. and Ross, A., 1954. Unbiased ratio estimates. *Nature*, **174**, 270-271.

[7] Johnston, J., 1972. *Econometric Methods*. 2nd edition, Mc-Graw Hill, New York.

[8] Murthy, M.N., 1964. The precision of unbiased ratio type estimators. *J. Amer. Statist. Assoc.*, **53**, 491-508.

[9] Rao, J.N.K. and Bayless, D.L., 1969. An empirical study of the stabilities of estimators and variance estimators in P.P.S. Sampling, *Proceedings of Social Stat. Section of Amer. Stat. Assoc.*

[10] Reddy, V.N., 1974. On a transformed Ratio Method of Estimator. *Sankhya (C)*, **36**, 59-70.

[11] Royall, R.M. and Herson, J., 1973. Robust estimation in finite populations: I, *J. Amer. Statist. Assoc.*, **68**, 880-889.

[12] Sahoo, L.N., 1986. A note on the Efficiency of a Product Type Estimator under a Super Population Model. *J. Ind. Soc. Agril. Statist.*, **38**, 383-387.

[13] Srivenkataramana, T., 1980. A dual to ratio estimator in sample survey. *Biometrika*, **67**, 199-204.

[14] Sukhatme, P.V. and Sukhatme, B.V., 1970. *Sampling Theory of Surveys with Applications*. Asia Publishing House, Bombay.