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On a New Type of Slope Rotatable Central Composite Design

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SUMMARY

A necessary and sufficient condition for the existence of the slope rotatable central composite design with c = 5 is α^2 integer. The number of replications of axial points is one or two according as t(v) odd or even.

Keywords: Response surface designs, Rotatable designs, Slope rotatable designs, Slope rotatable central composite designs.

1. Introduction

Hader and Park [2] introduced the property of slope rotatability for a second order response surface design in V factors, analogous to the rotatability property suggested by Box and Hunter [1]. For slope rotatability of central composite design (SRCCD) in V factors the value of α in the axial points of the type $(\pm\alpha, 0, \ldots, 0), \ldots, (0, 0, \ldots, \pm\alpha)$ is obtained by solving the biquadratic equation in α^2 given by

$$2n_a^2 (n_c + n_o)\alpha^8 - [4V\, n_a^2\, n_c]\alpha^6 - [n_a \,.\, n_c \{ n(4 - V) + V \,.\, n_c - 8(V-1)n_a\}]\alpha^4 + 8[(V-1)n_a n_c^2]\alpha^2 - 2n_c^2 (V-1)(n - n_c) = 0 \qquad (1.1)$$

where n_a : number of replications of axial points

n_o : number of replications of central point

n_c : number of factorial points

n : total number of design points

It was pointed out that replication of axial points rather than replication of central point only provide appreciable advantage in terms of efficiency of the estimates of the parameters of the model.

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The symmetry conditions to be satisfied by a general second order slope rotatable design (SOSRD) as given by Victor Babu and Narasimham [4] are

(i) $\Sigma X_{ui}^{a_1} X_{uj}^{a_2} X_{uk}^{a_3} X_{ul}^{a_4} = 0$

$i \neq j \neq k \neq l$, and any a_i $(i = 1, 2, 3, 4)$ being odd

(ii) $\Sigma X_{ui}^{2} = n\lambda_2 \ \forall\ i$

(iii) $\Sigma X_{ui}^{2} X_{uj}^{2} = n\lambda_4 \ \forall\ i \neq j$

(iv) $\Sigma X_{ui}^{4} = c\, n \lambda_4 \ \forall\ i$ (1.2)

(v) $\dfrac{\lambda_4}{\lambda_2^2} > \dfrac{V}{V + c - 1}$

(vi) $\lambda_4 [V(5 - c) - (c - 3)^2] + \lambda_2^2 [V(c - 5) + 4] = 0$

Victor Babu and Narasimham [5] introduced a new type of slope rotatable central composite design (SRCCD) by prefixing c in (1.2). In this case, it was shown that

$$\alpha^2 = [(c - 1)\, 2^{t(V) - 1}]^{1/2} \tag{1.3}$$

$$n_o = \frac{[2^{t(V)} + 2\alpha^2]^2 \, . \, [V(c - 5) + 4]}{2^{t(V)} \, . \, [V.(c - 5) + (c - 3)^2]} - 2^{t(V)} - 2V \tag{1.4}$$

where $2^{t(V)}$ is an appropriate faction of 2^V and n_o is integer.

It was pointed out that

(i) If V is such that t(V) is odd, integral solution for n_o exists and hence a new type of SRCCD exists for such V with c = 5.

A table of new type of SRCCDs for V = 3, 6, 9, 10, 11 $(V \leq 17)$ was given with values of a, α, n_o and n.

(ii) For V = 2, 4, 5, 7, 8, 13, 14, 15, 16, 17 $(V \leq 17)$ with c = 5, integral solution for n_o does not exist and suggested to take $[n_o]$ or $[n_o] + 1$ Central points to get nearly SRCCD.

Further, Victor Babu and Narasimham [6] derived Second order slope rotatable central composite design (SOSRCCD) by replicating the axial points of second order rotatable CCD (SORCCD).

In this paper a necessary and sufficient condition for the existence of the SRCCD with c = 5 is derived. It is shown that the SRCCD for any V exists with c = 5 by taking appropriate number of replications of the axial points. Formulae for n_a the number of axial points, n_o the number of central points and n the total number of design points have been derived. A table of new type of SRCCD for V = 2, 4, 5, 7, 8, is given for illustration.

2. A Necessary and Sufficient Condition for the Existence of New Type of SRCCD with c = 5

Let the axial points of type $(\pm\alpha, 0, \ldots, 0), \ldots, (0, 0, \ldots, \pm\alpha)$ of a CCD in V factors be replicated 2^k times (so that $n_a = 2^{k+1}V$) and the central point be replicated n_o times. Then prefixing c = 5, (1.3) and (1.4) become

$$\alpha^2 = 2^{\frac{t(V)-k+1}{2}} \tag{2.1}$$

$$n_o = \frac{(2^{t(V)} + 2\alpha^2 . 2^k)^2}{2^{t(V)}} - 2^{t(V)} - 2.\ V.2^k \tag{2.2}$$

which on simplification give

$$n_o = 2^{3+k} + 2^{\frac{t(V)+k+5}{2}} - V.2^{k+1} \tag{2.3}$$

Now, it can be seen that n_o is an integer, iff $\frac{t(V)+k+5}{2}$ is an integer

i.e.	$k = 0$	when	$t(V)$ is odd	(2.4)
and	$k = 1$	when	$t(V)$ is even	(2.5)

In this case, α^2 is also an integer.

Hence the following theorem :

Theorem 2.1: A necessary and sufficient condition for the existence of the SRCCD in V factors with c = 5 is that α^2 be integer.

Note:

(i) From (2.4) and (2.5), the number of axial points, n_a in the SRCCD with c = 5 is 2V or 4V according as t(V) is odd or even.

(ii) From (2.3) the number of central points

$$n_o = 2^{\frac{t(V)+5}{2}} + 8 - 2V \qquad \text{if } t(V) \text{ is odd}$$

$$= 2^{\frac{t(V)+6}{2}} + 16 - 4V \qquad \text{if } t(V) \text{ is even}$$

(iii) The total number of design points is

$$n = 2^{t(V)} + n_a + n_o$$

(iv) The SRCCDs with $c = 5$ obtained as above satisfy the symmetry conditions exactly.

3. Illustration

A list of SRCCD with $c = 5$ for some $V = 2, 4, 5, 7, 8$ is given in the following table for illustration of the method.

Table

V	$2^{t(V)}$	α^2	n_a	n_o	n
2	$2^2 = 4$	2	8	24	36
4	$2^4 = 16$	4	16	32	64
5	$2^4 = 16$	4	16	32	64
5	$2^5 = 32$	8	10	30	72
7	$2^5 = 32$	8	14	26	72
8	$2^6 = 64$	8	32	48	144

The resulting designs whenever large in size can be arranged in blocks, adopting the method given by Ramachandra Murthy and Murty [3].

4. A Special Case α Integer

It is clear from the above that for the existence of the SRCCD with $c = 5$, the number of replications of the axial points be such that α^2 is integer. But whenever α^2 is integer, α may not be an integer. When α is not an integer, the levels of the factors at the corresponding axial points cannot be accurately determined. Therefore, in order to have the levels of the factors at the axial points be exactly measured, it is desirable to have α as an integer.

In this case, from (2.1)

$$\alpha = 2^{\frac{t(V)-k+1}{4}} \tag{4.1}$$

and (2.2) remains same.

For α to be integer,

$$k \equiv t(V) + 1 \pmod 4 \tag{4.2}$$

i.e.

$$k = 0 \text{ for } t(V) \text{ is } 3, 7, 10, 15 \ldots \tag{4.3}$$

$$k = 1 \text{ for } t(V) \text{ is } 4, 8, 12, 16\ldots \tag{4.4}$$

$$k = 2 \text{ for } t(V) \text{ is } 5, 9, 13, 17 \ldots \tag{4.5}$$

$$k = 3 \text{ for } t(V) \text{ is } 6, 10, 14, 18 \ldots \tag{4.6}$$

Consequently

(i) $n_a = 2V$ for $t(V) = 3, 7, 11, 15 \ldots$ (4.7)

(ii) $n_a = 4V$ for $t(V) = 4, 8, 12, 16 \ldots$ (4.8)

(iii) $n_a = 8V$ for $t(V) = 5, 9, 13, 17, \ldots$ (4.9)

(iv) $n_a = 16V$ for $t(V) = 6, 10, 14, 18 \ldots$ (4.10)

In this case also, as the design size is large for large V, the design points can be arranged in blocks adopting the method given by Ramachandra Murthy and Murty [3].

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