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# Partial Diallel Crosses Through Circular Designs

M.K. Sharma
*N.D.U.A. & Technology, Kumarganj, Faizabad (U.P.)*
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SUMMARY

A method of obtaining partial diallel crosses in incomplete block designs as environment designs through symmetrical circular designs with v treatments and block size $k \geq 3$ ($\leq v/2$) is proposed. This method is compared with a method of embedding partial diallel crosses in incomplete block designs as environment designs.

*Key Words* : Symmetrical circular designs, Partially balanced incomplete block designs, Partial diallel crosses.

## *1. Introduction*

Partial diallel experiments are successfully being used to screen a large number of inbred lines or homozygous parents for their combining abilities at manageable and low levels of resources. These experiments give information on the genetic architecture of the breeding material to aid in selecting appropriate strategies for further breeding programs.

Such plans were constructed using different association schemes, such as triangular and rectangular (Fyfe and Gilbert), group divisible (Hinkelmann and Kempthorne), (Arya and Narain) and truncated triangular association schemes (Narain and Arya). The draw back with these plans is that they are usually carried out by accommodating all crosses in block of randomized block designs (RBD).

In such experiments, the blocks may be large enough leading to considerable decrease in the efficiency of the design due to large intra-block variance. It is for this reason that the use of incomplete block designs as environment designs has been advocated by e.g. Braaten [4], Ceranka and Kielezewska [5], Ceranka and Mejza [6], Agarwal and Das [1], [2], Divecha and Ghosh [7], Singh and Hinkelmann [12] and Sharma [10], [11].

In this paper, we have dealt the above situation by giving method of obtaining partial diallel crosses in incomplete block designs as environment designs using symmetrical circular designs with v treatments and block size

$k \geq 3$ ($\leq v/2$). The analysis of such designs is also presented along with their efficiency factors.

## 2. *Correspondence between Circular Design and PDC's*

A circular design will be characterized by the following.

*Definition 1* : Let there be n equal arcs on the circumference of a circle denoted in order by $a_1, a_2, \ldots, a_n$. If we form bigger arcs of size $A_{ki}$, such that it is the sum of k consecutive small arcs starting with $a_i$, we shall have in all n such arcs for n different values of i. Now we identify $a_i$ with a set $(t_{ij})$, where $j = 1, 2, \ldots, m$, of m treatments such that the different sets $(t_{ij})$ are mutually exclusive, then the contents of the n arcs, $A_{ki}$ $(i = 1, 2, \ldots, n)$ will form an incomplete block design with n block, mn treatment, mk block size and k replications.

If the independent treatments be denoted by $t_{ij}$ $(i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$ and the set of all the m treatments on the arc $a_i$ be denoted by $t_{ij}$, the block contents of the design come out as below.

*Block number*

1. $(t_{1j})$ $(t_{2j})$ – – $(t_{kj})$
2. $(t_{2j})$ $(t_{3j})$ – – $(t_{k+1,j})$
3. $(t_{3j})$ $(t_{4j})$ – – $(t_{k+2,j})$

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– – – – – –

n. $(t_{nj})$ $(t_{ij})$ – – $(t_{k-1,j})$

Such designs can be obtained for any number of treatments and block sizes. Since the placements of the treatments are on the arc of a circle, these have been called *Circular designs*. All the designs become symmetrical when $n = 1$ i.e. when the set $(t_{ij})$ contains one distinct treatment for each i. Our choice for the construction of partial diallel crosses (PDC) will be symmetrical circular designs of block sizes $k \geq 3$.

It is well known that there exists a relationship between diallel crosses and incomplete block designs and also between partial diallel crosses (PDC) and a partially balanced incomplete block designs (PBIBD) with v treatments, b block, r replication, block size $k = 2$, $n_i$th associates occurring together $\lambda_i$ times in the same block ( $i = 1, 2, \ldots, m$).

The circular designs are actually partially balanced incomplete block designs (PBIBD) with associate classes of half or just less than half the number of treatments, when the number of treatments is odd, each $n_i = 2$ ($i = 1, 2, \ldots (v-1)/2$) and when the number of treatments is even one of the $n_i$ ($i = 1, 2, \ldots v/2$) is 1 and rest are 2. The $n_i$th associates occur together $\lambda_i$ times in the design. Thus circular design with v treatments corresponds to partial diallel cross with v lines, each line being crossed with 2(k-1) distinct lines and occurs r (k–1) times among the crosses in PDC. Let $n_c$ be total possible distinct cross and ith cross appears $r_{di}$ times ($i = 1, 2, 3, \ldots, n_c$) in a partial diallel cross design. This implies that

$$v^* = n_c = v(k-1),\ b^* = v,\ r_d$$

$$k^* = k(k-1)/2 \text{ and } s = 2(k-1)$$

where $i = 1, 2, \ldots, (v-1)/2$ or $v/2$ when v is odd or even

and $r_d = (r_{d1}, r_{d2}, \ldots, r_{dnc})$

then $1_{nc}\ r_d = n =$ total number of crosses

Actually the appearance of cross i x j in PDC design depends on the association scheme of lines i and j in a circular design.

Hence replication vector $r_d$ can only take distinct values $\lambda_1, \lambda_2, \ldots, \lambda_{(v-1)/2}$ when v is odd and $\lambda_1, \lambda_2, \ldots, \lambda_{v/2}$ when v is even

where $\lambda_i = k - i$ ($i = 1, 2, \ldots, (v-1)/2$ or $v/2$, when v is odd or even)

$= 0$ when $k - i$ is –ve

## 3. *Method of Blocking*

The method is simply stated as follows : Take for v lines under evaluation and numbered randomly a symmetrical circular design with parameters $v = b$, $r = k$ ($\geq 3$), $\lambda_i$, $n_i$ ($i = 1, 2, 3, \ldots, (v-1)/2$ or $v/2$ when v is odd or even).

A block of the partial cross plan is formed by putting all k(k–1)/2 crosses obtained from the k line in block, thus bk(k–1)/2 crosses get arranged in b blocks each containing k(k–1)/2 crosses. Since some of the $\lambda_i$'s are greater than 1, it leads to repetition of some crosses without making the design complex. We shall refer to the circular designs that leads to PDC plans as auxiliary designs or A-designs. If N be the incidence matrix of the A-design with v treatments, then the incidence matrix of the resulting design, which is of order $b \times v$, is (k-1)N. Thus the resulting design is an incomplete block design for PDC plan. We shall call the resulting design mating-environment design or simply M-E design. The actual number of crosses, which are being used in the plan is v(k–1) and out of these $v_{ni}/2$ crosses appear in the design $\lambda_i$ times ($i = 1, 2, \ldots, v-1/2$ or $v/2$, when v is odd or even).

## 4. *Analysis*

The model for data from PDC plan in incomplete block is

$$y_{ijk} = m + g_i + g_j + b_k + e_{ijk} \tag{1}$$

$$i < j = 1, 2, \ldots, v; k = 1, 2 \ldots b$$

where $y_{ijk}$ is the yield from the cross involving lines i and j in the $k^{th}$ block of the design, m is the overall mean, $g_i$ ($g_j$) is the g.c.a. effect for the $i^{th}$ ($j^{th}$) lines, $b_k$ is the effect of the block k and $e_{ijk}$ is an error term. We assume further that $\Sigma\, g_i = 0$ and $\Sigma\, b_k = 0$, and that the $e_{ijk}$ are i.i.d random variables with mean 0 and variance $\sigma_e^2$.

Let us label the experimental units i.e. the unit on which the offspring of a cross are grown by u =1, 2, ..., n, where n = vk (k–1)/2. Now the model (1) can be written in matrix notation as follows:

$$y = m\mathbf{I} + X_g\, \mathbf{g} + X_b\, \mathbf{b} + \mathbf{e} \tag{2}$$

where y is the $n \times 1$ vector of observations, I is an $n \times 1$ vector of unity ; $X_g = (x_{gui})$ is an $n \times v$ matrix with elements $x_{gui} = 1$ if one of the parents of the cross on unit u is line i and 0 otherwise; $X_b = (x_{buk})$ is an $n \times b$ matrix with elements $x_{buk} = 1$ if unit u is contained in block k and 0 otherwise ; $\mathbf{g}' = (g_1, g_2, \ldots, g_v)'$ is the vector of general combining abilities; $\mathbf{b}' = (b_1, b_2, \ldots, b_v)'$ is the vector of block effects; and **e** is the vector of errors with $E(\mathbf{e}) = 0$ and $Cov(\mathbf{e}) = \sigma^2 I_n$.

From the foregoing discussion, it is now easy to see that

$$X'_g\, I = r\,(k-1)\, I,\ \ I' X_g = r\,(k-1)\, I \text{ and } I' X_b = rI$$

The normal equations for model (2) are

$$\begin{pmatrix} I'I & I'X_g & I'X_b \\ X'_g I & X'_g X_g & X'_g X_b \\ X'_b I & X'_b X_g & X'_b X_b \end{pmatrix} \begin{pmatrix} \hat{m} \\ \hat{g} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} I' y \\ X'_g y \\ X'_b y \end{pmatrix} \quad (3)$$

with our choice of M-E design, it follows immediately that

$$X'_g X_g = N N' - r(2-k) I$$

$$X'_g X_b = (k-1) N$$

$$X'_b X_b = \frac{k(k-1)}{2} I$$

Now the reduced normal equations for the g.c.a. are given by

$$C_g \hat{g} = Q_g \quad (4)$$

where

$$C_g = (2-k)/k \; N N' - r(2-k) I \quad (5)$$

$$Q_g = X'_g y \frac{(k-1)}{r} N X'_b y$$

Since $C_g$ is a symmetric circulant doubly-centred matrix of order v, and circular designs are also connected designs. Therefore, Rank $(C_g) = v - 1$. Hence Doubly-centred $g^+$ inverse of $C_g$ will be

$$C_g^+ = (C_g + v^{-1} JJ)^{-1}$$

That is, in order to obtain $C_g^+$, (1/v), Rao and Mitra [9], is added to each of the elements of $C_g$ and the resultant non-singular matrix is inverted.

Hence solution of (4) is given by

$$\hat{g} = C_g^+ Q_g \quad (6)$$

The estimator for any contrast $C_g$ among the g.c.a. is then given by

$$c' \hat{g} = c' C_g^+ Q_g \quad (7)$$

with $$\text{var}(c' \hat{g}) = c' C_g^+ c \; \sigma_e^2 \quad (8)$$

Finally the analysis of variance associated with model (1) is as given in Table 1.

**Table 1. ANOVA**

| Source | d.f. | SS | E(MS) |
|---|---|---|---|
| Blocks | v−1 | $\frac{2}{k(k-1)}\sum_{j=1}^{v} B_j - \frac{2G^2}{vk(k-1)}$ | |
| GCA | v−1 | $\hat{g}' Q_g$ | $\sigma_e^2 + g' C_g^{+} g/v-1$ |
| Error | | By Subtraction | $\sigma_e^2$ |
| Total | vk (k − 1)/2 − 1 | $\sum y_{ijk}^2 - \frac{2G^2}{vk(k-1)}$ | |

*Example 4.1*: We consider a circular design with parameters $v = b = 9$, $r = k = 3$, $\lambda_1 = 2$, $\lambda_2 = 1$, $\lambda_3 = \lambda_4 = 0$, $n_1 = n_2 = n_3 = n_4 = 2$

| AUXILIARY DESIGN | | | MATING – ENVIRONMENT + DESIGN | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 × 2 | 1 × 3 | 2 × 3 |
| 2 | 3 | 4 | 2 × 3 | 2 × 4 | 3 × 4 |
| 3 | 4 | 5 | 3 × 4 | 3 × 5 | 4 × 5 |
| 4 | 5 | 6 | 4 × 5 | 4 × 6 | 5 × 6 |
| 5 | 6 | 7 | 5 × 6 | 5 × 7 | 6 × 7 |
| 6 | 7 | 8 | 6 × 7 | 6 × 8 | 7 × 8 |
| 7 | 8 | 9 | 7 × 8 | 7 × 9 | 8 × 9 |
| 8 | 9 | 1 | 8 × 9 | 8 × 1 | 9 × 1 |
| 9 | 1 | 2 | 9 × 1 | 9 × 2 | 1 × 2 |

The information matrix $C_g$ is given below

$$\frac{1}{3}\begin{bmatrix} 6 & -2 & -1 & 0 & 0 & 0 & 0 & -1 & -2 \\ -2 & 6 & -2 & -1 & 0 & 0 & 0 & 0 & -1 \\ -1 & -2 & 6 & -2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 6 & -2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2 & 6 & -2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -2 & 6 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -2 & 6 & -2 & -1 \\ -1 & 0 & 0 & 0 & 0 & -1 & -2 & 6 & -2 \\ -2 & -1 & 0 & 0 & 0 & 0 & -1 & -2 & 6 \end{bmatrix}$$

### *5. Measure of efficiency*

We have described one method of employing partial diallel crosses in an incomplete block design. There are several methods of employing partial diallel crosses in incomplete block designs. Whenever one has choices, it is useful to compare competing designs. This can be accomplished by means of the efficiency factors of those designs. For example if A and B are two incomplete block designs employing partial diallel crosses having the same v and the same s, the efficiency factor can be developed as below.

For a complete diallel cross (CDC) in a randomised complete block design (RCBD) with r replication and one off-spring per experimental unit, we have (Griffing [8])

$$\text{var}\ (c'\hat{g}) = c'c\ \sigma_e^2/[r(v-2)] \tag{9}$$

In the context of our proposed design where we have different replications for each associate group of mating but this, of course, does not affect the efficiency factor. We then have

$$\text{var}\ (\hat{g}_i - \hat{g}_j) = \frac{\sigma_e^2}{v-2} \tag{10}$$

for all i and j ($i \neq j$). For the corresponding M-E design, we have (v-1)/2 variances when v is odd otherwise v/2 variances of the difference of general combining ability for any line i (i = 1, 2, ..., v)

$$\text{var}\ (\hat{g}_i - \hat{g}_{i+j}) = (c^{ii} + c^{i+j,\,i+j} - 2c^{i,\,i+j})\ \sigma_e^2$$

$$= v_{i,\ i+j}\ \sigma_e^2$$

where j = 1, 2, ..., (v-1)/2 or v/2 when v is odd or even

and $C_g^+ = (C^{i,\,i+j})$

Averaging our all possible comparisons we obtain

$$\text{Av . Variance} = \frac{2}{v-1} \sum_{j=1}^{(v-1)/2} v_{i,\,i+j}\ \sigma_e^2 \tag{11}$$

when v is odd

$$= \frac{2}{v} \sum_{j=1}^{v/2} v_{i,\,i+j}\ \sigma_e^2$$

when v is even

If we denote for short the expressions in (9) and (10) by $var_c$ and $var_p$ respectively, then the efficiency factor of M-E design is given by

$$E_p = \frac{var_c}{var_p}$$

$$= \frac{(v-1)}{2(v-2)} \sum_{j=1}^{(v-1)/2} v_{i,\,i+j} \qquad \text{when v is odd} \qquad (12)$$

$$= \frac{v}{2(v-2)} \sum_{j=1}^{v/2} v_{i,\,i+j} \qquad \text{when v is even}$$

Since the CDC and PDC involve different number of crosses i.e. v(v–1)/2 vs v(k–1), we shall follow Hinkelmann and Kempthorne and define the efficiency factor $E^*$ on per cross basis as

$$E^* = E_p \frac{(v-1)}{2(k-1)}$$

In Table 2 we list the efficiency factor $E^*$ of M-E design with $v < 27$, $s = 10$ using symmetrical circular designs.

*Example 4.1.* Continued: From the generalised inverse $C^+$ of information matrix $C_g$, the (v–1)/2 variances of the difference of general combining abilities for line 1 come out as

(i) $var(\hat{g}_1 - \hat{g}_2) = 0.8104$

(ii) $var(\hat{g}_1 - \hat{g}_3) = 1.0457$

(iii) $var(\hat{g}_1 - \hat{g}_4) = 1.2941$

(iv) $var(\hat{g}_1 - \hat{g}_5) = 1.3986$

Av. var = 1.1372

Hence the efficiency factor $E^*$ on per cross basis is obtained from (12) as

$$E^* = .4164$$

**Table 2**

| Sl. No. | v = b | k = r | s | $v_p$ | $E^*$ | $E^*_{S-H}$ |
|---|---|---|---|---|---|---|
| 1 | 6 | 3 | 4 | .9222 | .6777 | .5515 |
| 2 | 7 | 3 | 4 | .9755 | .6150 | – |
| 3 | 8 | 3 | 4 | 1.0848 | .5376 | .6806 |
| 4 | 9 | 3 | 4 | 1.1372 | .4164 | .4762 |
| 5 | 10 | 3 | 4 | 1.2509 | .4496 | – |
| 6 | 11 | 3 | 4 | 1.3000 | .4270 | – |
| 7 | 12 | 3 | 4 | 1.4167 | .3235 | – |
| 8 | 13 | 3 | 4 | 1.4811 | .3682 | – |
| 9 | 9 | 4 | 6 | .6484 | .5874 | .5275 |
| 10 | 10 | 4 | 6 | .6915 | .6695 | .5192 |
| 11 | 11 | 4 | 6 | .7116 | .5203 | – |
| 12 | 12 | 4 | 6 | .7565 | .4846 | .6050 |
| 13 | 13 | 4 | 6 | .7751 | .4691 | .4909 |
| 14 | 14 | 4 | 6 | .8219 | .4391 | – |
| 15 | 15 | 4 | 6 | .8409 | .4267 | .4734 |
| 16 | 16 | 4 | 6 | .8877 | .4022 | .4762 |
| 17 | 17 | 4 | 6 | .9064 | .3920 | – |
| 18 | 18 | 4 | 6 | 1.0050 | .3512 | – |
| 19 | 10 | 5 | 8 | .4774 | .5890 | .5463 |
| 20 | 11 | 5 | 8 | .4869 | .5705 | – |
| 21 | 12 | 5 | 8 | .5082 | .5411 | .5186 |
| 22 | 13 | 5 | 8 | .5186 | .5258 | – |
| 23 | 14 | 5 | 8 | .5402 | .5013 | – |
| 24 | 15 | 5 | 8 | .5496 | .4898 | .5048 |
| 25 | 16 | 5 | 8 | .5739 | .4083 | .5740 |
| 26 | 17 | 5 | 8 | .5853 | .4556 | .4952 |
| 27 | 18 | 5 | 8 | .5918 | .4485 | – |
| 28 | 19 | 5 | 8 | .5986 | .4421 | – |
| 29 | 20 | 5 | 8 | .6175 | .4272 | – |

| | | | | | | |
|---|---|---|---|---|---|---|
| 30 | 12 | 6 | 10 | .3851 | .5712 | .5398 |
| 31 | 13 | 6 | 10 | .3903 | .5589 | – |
| 32 | 14 | 6 | 10 | .4024 | .5382 | – |
| 33 | 15 | 6 | 10 | .4078 | .5280 | .5140 |
| 34 | 16 | 6 | 10 | .4204 | .5097 | .5143 |
| 35 | 17 | 6 | 10 | .4257 | .5011 | – |
| 36 | 18 | 6 | 10 | .4380 | .4843 | – |
| 37 | 19 | 6 | 10 | .4436 | .4773 | – |
| 38 | 20 | 6 | 10 | .5077 | .4157 | .5571 |
| 39 | 21 | 6 | 10 | .5000 | .4210 | .4971 |
| 40 | 22 | 6 | 10 | .4754 | .4416 | – |
| 41 | 23 | 6 | 10 | .4807 | .4358 | – |
| 42 | 24 | 6 | 10 | .4941 | .4229 | – |
| 43 | 25 | 6 | 10 | .4991 | .4180 | – |
| 44 | 26 | 6 | 10 | .5140 | .3740 | .4872 |
| 45 | 27 | 6 | 10 | .5201 | .3998 | .4794 |
| 46 | 15 | 7 | 12 | .3304 | .5432 | – |
| 47 | 16 | 7 | 12 | .3333 | .5357 | – |
| 48 | 17 | 7 | 12 | .3357 | .5294 | – |
| 49 | 18 | 7 | 12 | .3447 | .5136 | – |
| 50 | 19 | 7 | 12 | .3477 | .5074 | – |
| 51 | 20 | 7 | 12 | .3558 | .4943 | – |
| 52 | 21 | 7 | 12 | .3590 | .4880 | – |
| 53 | 22 | 7 | 12 | .3671 | .4767 | – |
| 54 | 23 | 7 | 12 | .3703 | .4713 | – |
| 55 | 24 | 7 | 12 | .3788 | .4598 | – |
| 56 | 25 | 7 | 12 | .3819 | .4552 | – |
| 57 | 26 | 7 | 12 | .3903 | .4447 | – |
| 58 | 27 | 7 | 12 | .3921 | .4417 | – |

$E^*_{S-H}$ is the efficiency factor on a per cross basis given by Singh and Hinkelmann [12].

## 6. Conclusion

Singh and Hinkelmann [12] reported that the intuitive method of accommodating the crosses from a PDC in an E-design, namely using BIBD, requires in general a large number of blocks. The M-E design which we have proposed here through symmetrical circular design, the number of blocks is equal to the number of lines, v, involved in the diallel. The M-E designs proposed by Singh and Hinkelmann use PBIB designs. It requires that the breeder must have a list of PBIB design such as, for example, given by Clatworthy. Moreover PBIB design are not available for every v lines. In our proposed method, the circular design are available for any v lines and also easy to construct. Our method is not applicable when we use circular designs with block size k = 2.

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