Jour. Ind. Soc. Ag. Statistics
50(2), 1997 : 156–167

Robustness of Block Designs against Interchange of Treatments

P.K. Batra, P.R. Sreenath and Rajender Parsad
Indian Agricultural Statistics Research Institute, New Delhi-110012
(Received : July, 1996)

SUMMARY

This article investigates the robustness aspects of binary variance balanced block designs against interchange of a pair of treatments in two blocks of the design using connectedness and relative efficiency criterion. The different classes of binary variance balanced block (VBD) designs viz. randomised complete block designs, balanced incomplete block designs, non-proper unequireplicated VBD's of Kageyama [12] and non-proper equireplicated VBD's of Gupta and Jones [7] have been found robust.

Key words: Interchange of a pair of treatments, Robustness, Connectedness, Variance balance, Relative efficiency, Block designs.

1. *Introduction*

During the course of the layout of a planned experiment, each of the pair of experimental units belonging to different blocks may receive, by accident, treatment originally designated for the other i.e., interchange of treatments may occur. Such discrepancy was first reported by Pearce [16]. Gomez and Gomez [6] have listed this type of error among mechanical errors while Pearce [17] has termed this as an error in the application of the treatments. Pearce [16] has discussed the analysis of an experiment in randomised complete block (RCB) design with interchange of treatments occuring in two of its replications. The design may loose its optimal properties under such disturbances. For example, the BIB design (1,2); (2,3); (1,3) results into a disconnected design when treatment 1 in block 1 gets interchange with treatment 2 in block 2. Hence, it is desirable to study the robustness of block designs against interchange of two treatments. Several studies have been undertaken on the robustness of block designs against (1) missing observation(s) (2) presence of outlier(s) (3) model inadequacy and (4) presence of systematic trends among observations within a block. For an uptodate bibliography and review one may refer to Dey *et al.* [4]. Other important references are Kageyama [13], [14], Mukerjee and Kageyama [15], Bhaumik and

Whittinghill [1], Gupta and Srivastava [8] and Dey [3]. From the persual of literature it appears that no work has been undertaken to study the robustness of block designs against the interchange of the treatments. In this article, the robustness of certain classes of binary variance balanced block designs against the interchange of a pair of treatments in two blocks has been studied using the criteria (1) connectedness property (Ghosh [5]) of the resulting design (2) Relative efficiency (John [10]) of the resulting design with respect to the original design.

For the present investigation, under general binary block design set up we have obtained the information matrix C^* of the resulting design (d^*) in terms of the original design in Section 2. The eigenvalues of C^* when d is binary variance balanced block design have been obtained in Section 3. The robustness of RCB designs, BIB designs, non-proper equireplicated binary variance balanced block designs of Gupta and Jones [7] and non-proper and unequireplicated designs of Kageyama [12] has been investigated in Section 4.

2. *Interchange of Treatments in a General Block Design*

Consider a binary block design $d \in D(v, b, \mathbf{r}, \mathbf{k})$ in which v treatments are allotted to experimental units arranged in b blocks with block size vector $\mathbf{k}' = (k_1, \ldots, k_b)$ and replication vector $\mathbf{r}' = (r_1, \ldots, r_v)$. Let $\mathbf{N} = ((n_{ij}))$ be a $v \times b$ incidence matrix. The $v \times v$ information matrix

$$\mathbf{C} = \mathbf{R} - \mathbf{N}\mathbf{K}^{-1}\mathbf{N}' \tag{2.1}$$

where $\mathbf{R} = \mathrm{diag}(r_1, \ldots, r_v)$ and $\mathbf{K} = \mathrm{diag}(k_1, \ldots, k_b)$. For a connected block design Rank $(\mathbf{C}) = v - 1$. We shall denote the v treatments in the design d as 1, 2, ..., v and b blocks as 1, 2, ..., b. Let treatment i_1 in block j_1 be interchanged with treatment i_2 in block j_2. When $i_1 = i_2$ or $j_1 = j_2$, the design remains unaltered. The only case of interest would be when $i_1 \neq i_2$ and $j_1 \neq j_2$. Without loss of generality let us assume that $i_1 = j_1 = 1$ and $i_2 = j_2 = 2$. We shall denote the resulting design by d^*, the two treatments 1 and 2 involved in the interchange as the affected treatments and other $(v-2)$ treatments as unaffected treatments. Similarly the two blocks involved in the interchange of the treatments are termed as the affected blocks and other $(b-2)$ blocks as unaffected blocks.

In order to investigate the possible forms of $\mathbf{N}^*$, the incidence matrix of d^*, we re-write the incidence matrix $\mathbf{N}$ with elements 0, 1 of design d as

$$\mathbf{N} = [\mathbf{n}_1 \ \mathbf{n}_2 \ \mathbf{N}_1] \tag{2.2}$$

$$\text{where } \mathbf{n}_1 = \begin{bmatrix} 1 \\ x_1 \\ \mathbf{u}_1 \end{bmatrix} \text{ and } \mathbf{n}_2 = \begin{bmatrix} x_2 \\ 1 \\ \mathbf{u}_2 \end{bmatrix} \tag{2.3}$$

x_1 = 1 if treatment 2 is present in block 1

= 0 otherwise

x_2 = 1 if treatment 1 is present in block 2

= 0 otherwise

$\mathbf{u}_t$ = $(v-2)\times 1$ incidence vector of $(v-2)$ unaffected treatments in block t $(t = 1, 2)$

Thus, depending upon the incidence of interchanged treatments in the two affected blocks, we have the following four possible cases for the choice of affected blocks.

$$\begin{array}{lcccc} & \text{Case I} & \text{Case II} & \text{Case III} & \text{Case IV} \\ \mathbf{n}_1 = & \begin{bmatrix} 1 \\ 0 \\ \mathbf{u}_1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ \mathbf{u}_1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ \mathbf{u}_1 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ \mathbf{u}_1 \end{bmatrix} \\ \mathbf{n}_2 = & \begin{bmatrix} 0 \\ 1 \\ \mathbf{u}_2 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ \mathbf{u}_2 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ \mathbf{u}_2 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ \mathbf{u}_2 \end{bmatrix} \end{array} \tag{2.4}$$

Remark 2.1:

2.1.1. When affected blocks are of equal sizes, then, Case III and Case IV are identical in the sense that one case can be obtained from the other simply by the renumbering of treatments and blocks.

2.1.2. In a given block design, it may not always be possible to have all the four choices of the affected blocks e.g. for a BIB design d(v, b, r, k, λ) with λ = 1, it is not possible to have a pair of affected blocks for Case II.

Let c be the total number of common treatments in the two affected blocks of which ρ is the number of unaffected treatments, thus $\mathbf{n}'_1 \mathbf{n}_2 = c$ and $\mathbf{u}'_1 \mathbf{u}_2 = \rho$. These v treatments can be grouped as given in Table 2.1.

Table 2.1

Group	Nature of grouping	No. of treatments
1.	Affected treatments	2
2.	Common unaffected treatments in the affected blocks	ρ
3.	Unaffected treatments in block 1 and not in block 2	$k_1 - c - 1 + x_2 (= v_1)$, say
4.	Unaffected treatments in block 2 and not in block 1	$k_2 - c - 1 + x_1 (= v_2)$, say
5.	Treatments neither occuring in block 1 nor in block 2	$v - k_1 - k_2 + c (= v_0)$, say

Here, $v_0 + v_1 + v_2 + \rho = v - 2$. Without loss of generality, after suitably renumbering of treatments if necessary, the vectors $\mathbf{n}_1, \mathbf{n}_2$ defined in (2.3) can be written as

$$\mathbf{n}_1 = [1\ x_1\ \mathbf{1}'_\rho\ \mathbf{1}'_{v_1}\ \mathbf{0}'_{v_2}\ \mathbf{0}'_{v_0}]'$$

$$\mathbf{n}_2 = [x_2\ 1\ \mathbf{1}'_\rho\ \mathbf{0}'_{v_1}\ \mathbf{1}'_{v_2}\ \mathbf{0}'_{v_0}]' \tag{2.5}$$

The rows of $\mathbf{N}$ and $\mathbf{N}^*$ defined in (2.2) are also rearranged according to the arrangement of treatments in vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ in (2.5). We have the following results.

Lemma 2.1: With $\mathbf{N}, \mathbf{N}_1, \mathbf{n}_1$ and $\mathbf{n}_2$ defined in (2.2), $\mathbf{u}_1, \mathbf{u}_2$ as defined in (2.3), the following relations hold

(i) $\mathbf{n}'_1 \mathbf{n}_1 = k_1$; $\mathbf{n}'_2 \mathbf{n}_2 = k_2$

(ii) $\mathbf{u}'_1 \mathbf{u}_1 = k_1 - 1 - x_1$; $\mathbf{u}'_2 \mathbf{u}_2 = k_2 - 1 - x_2$

(iii) $\mathbf{n}'_1 \mathbf{n}_2 = \mathbf{u}'_1 \mathbf{u}_2 + x_1 + x_2$, i.e., $c = \rho + x_1 + x_2$ (2.6)

(iv) $\mathbf{N}\mathbf{K}^{-1}\mathbf{N}' = (\mathbf{n}_1\mathbf{n}'_1/k_1) + (\mathbf{n}_2\mathbf{n}'_2/k_2) + \mathbf{N}_1 \mathbf{K}_1^{-1} \mathbf{N}'_1$

where $\mathbf{K}_1 = \text{diag}\,(k_3, k_4, \ldots, k_b)$

Thus the incidence matrix $\mathbf{N}^*$ of the resulting design d* can be written as

$$\mathbf{N}^* = [\mathbf{n}_1 - \mathbf{e}_1 + \mathbf{e}_2 \quad \mathbf{n}_2 - \mathbf{e}_2 + \mathbf{e}_1 \quad \mathbf{N}_1] \tag{2.7}$$

where $\mathbf{e}_1 = (1\ 0\ \mathbf{0}'_{v-2})'$; $\mathbf{e}_2 = (0\ 1\ \mathbf{0}'_{v-2})'$

The parameters of d* are same as that of d, with incidence matrix as given in (2.7). Thus, the information matrix of d* is

$$C^* = R - N^* K^{-1} N^{*\prime} \tag{2.8}$$

Using Lemma 2.1, it can easily be seen that

$$C^* = C - A \tag{2.9}$$

where

$$A = \begin{bmatrix} -1/k_1 + (2x_2+1)/k_2 & -x_1/k_1 - x_2/k_2 & -u'_1/k_1 + u_2'/k_2 \\ -x_1/k_1 - x_2/k_2 & (2x_1+1)/k_1 - 1/k_2 & u'_1/k_1 - u_2'/k_2 \\ -u_1/k_1 + u_2/k_2 & u_1/k_1 - u_2/k_2 & 0 \end{bmatrix}$$

For further investigation, we shall assume design d to be a binary variance balanced block design.

3. *Eigenvalues of C**

For a connected binary variance balanced block design $d \in D(v, b, r, k)$, the information matrix is $C = \mu[I - (1/v)\, 1_v\, 1'_v]$, where μ is unique non-zero eigenvalue of C. Hence the information matrix of d* using (2.9) is

$$C^* = \mu\,[I - (1/v)\, 1_v\, 1'_v] - A \tag{3.1}$$

The eigenvalues of A defined in (2.9) are

(i) $\theta_0 = 0$ with multiplicity $(v-2)$

(ii) $\theta_1 = a_1 + (a_1^2 + a_2)^{1/2}$

(iii) $\theta_2 = a_1 - (a_1^2 + a_2)^{1/2}$ (3.2)

where $a_1 = x_1k_1^{-1} + x_2k_2^{-1}$ and $a_2 = (x_1k_1^{-1} - x_2k_2^{-1})^2 - (k_1^{-1} - k_2^{-1})\,\beta + 2\alpha$

$\beta = \{(2x_2+1)k_2^{-1} - (2x_1+1)k_1^{-1}\}$ and

$\alpha = (k_1 - 1 - x_1)k_1^{-2} + (k_2 - 1 - x_2)k_2^{-2} - 2(c - x_1 - x_2)k_1^{-1}k_2^{-1}$

It is seen that C and A commute and thus the following theorem.

Theorem 3.1: The eigenvalues of information matrix C* (3.1) are

(i) 0 with multiplicity one

(ii) $\theta_1^* = \mu$ with multiplicity $(v-3)$

(iii) $\theta_2^* = \mu - \theta_1$ and

(iv) $\theta_3^* = \mu - \theta_2$

Therefore, efficiency (E) of d* relative to d, is the ratio of harmonic mean of non-zero eigenvalues of C* to that of C and given by

$$E = \frac{(v-3)\,\theta_2^*\,\theta_3^*}{(v-1)\,\theta_2^*\,\theta_3^* + (\theta_1^* + \theta_2^*)\,\theta_1^*} \tag{3.3}$$

We shall now study the robustness of various classes of binary variance balanced block designs against interchange of two treatments between two different blocks.

4. *Particular Cases*

4.1 Randomised Complete Block Designs

For a RCB design with v treatments and r replications, the unique non-zero eigenvalue of C is r with multiplicity $v-1$. Then the non-zero eigenvalues of C* of d* using Theorem 3.1 are

(i) 0 with multiplicity one

(ii) $\theta_1^* = \theta_2^* = r$ with multiplicity $(v-2)$ and (4.1)

(iii) $\theta_3^* = r - 4/v$

Therefore, d* is connected if $r \neq 4/v$. Hence we have the following theorem.

Theorem 4.1: Randomised complete block designs (except with $r = 2$ and $v = 2$) are robust against the interchange of a pair of treatments in two blocks according to connectedness criterion.

The efficiency (E) of d* relative to d is obtained using (3.3) and (4.1). The efficiencies of RCB designs with parameters satisfying $2 \leq v \leq 15$, $2 \leq r \leq 15$ and $4 < n \leq 60$ have been worked out. It is observed that this efficiency increased with increase of either v or r. The frequency distribution of these designs for various ranges of efficiencies is presented in Table 4.1.

Table 4.1

Range of efficiencies	0.33	0.50 – 0.70	0.70 – 0.80	0.80 – 0.90	0.90 – 0.95	≥ 0.95
No. of designs	1	4	5	12	15	66

For designs with $v \geq 8$ the efficiency is greater than 95%. The minimum number of replications required for the efficiency to be more than 95% in case of RCB designs with $2 < v < 8$ are as given in Table 4.2.

Table 4.2

v	3	4	5	6	7
Minimum	13	7	5	4	3

Thus, using efficiency criterion, RCB designs are robust against interchange of treatments for $v \geq 8$. For $v < 8$, it is robust if number of replications is greater than or equal to as given in Table 4.2.

4.2 Balanced Incomplete Block Designs

Consider d, a BIB design with parameters v, b, r, k, λ and the information matrix **C** with unique non-zero eigenvalue as $\lambda v/k$ with multiplicity $v - 1$. Using Theorem 3.1, the non-zero eigenvalues of **C*** are

(i) $\theta_1^* = \lambda v/k$ with multiplicity $v - 3$

(ii) $\theta_2^* = \lambda v/k - [(x_1 + x_2) + 2(k - c - 1 + x_1 + x_2)^{1/2}]/k$

(iii) $\theta_3^* = \lambda v/k - [(x_1 + x_2) - 2(k - c - 1 + x_1 + x_2)^{1/2}]/k$

We shall study in sequel the robustness aspects of BIB design d using various criteria.

Connectedness Criterion

For studying the connectedness of d*, we shall examine the structure of **C***. Substituting $k_1 = k_2 = k$ in (2.9), the **C*** can be written as

$$kC^* = \begin{bmatrix} P & Q \\ Q' & S \end{bmatrix} \tag{4.2}$$

where

$$P_{2\times 2} = \begin{bmatrix} \lambda(v-1) - 2x_2 & -\lambda + x_1 + x_2 \\ -\lambda + x_1 + x_2 & \lambda(v-1) - 2x_1 \end{bmatrix}$$

$$Q_{2\times v-2} = [Q_1 \quad Q_2 \quad Q_3 \quad Q_4]$$

$$Q_1 = \begin{bmatrix} -\lambda 1'_{\rho} \\ -\lambda 1'_{\rho} \end{bmatrix} \qquad Q_2 = \begin{bmatrix} -(\lambda-1)1'_{v1} \\ -(\lambda+1)1'_{v1} \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} -(\lambda+1)1'_{v2} \\ -(\lambda-1)1'_{v2} \end{bmatrix} \qquad Q_4 = \begin{bmatrix} -\lambda 1'_{v0} \\ -\lambda 1'_{v0} \end{bmatrix}$$

$$S_{v-2\times v-2} = \lambda\, v\, [I_{(v-2)} - (1/v)\, 1_{(v-2)}\, 1'_{(v-2)}]$$

Utilising (4.2) and characterization of connectedness due to Chakraborti [2], we have the following result.

Theorem 5.2: A BIB design d(v, b, r, k, λ) with $v > 3$ is robust against the interchange of two distinct treatments between its two blocks according to connectedness criterion.

Proof: Consider the last row of information matrix, identify the non-zero elements of this row. Now, if at least one element in any row above these elements is non-zero, then design is connected (Chakraborti [2]). On observing the information matrix C* (4.2), it is clear that submatrix S has all non-zero elements. It is sufficient to show that there is at least one non-zero entry in any row of the submatrix Q of C* also. For any BIB design d, $\lambda \geq 1$. Using $v_0 + v_1 + v_2 + \rho = v - 2$ and $\rho + v_0 + v_1 + v_2 > 1$, for $v > 3$, atleast one of ρ, v_0, v_1, v_2 is non-zero. If either one of ρ or v_0 or both v_1 and v_2 is non-zero, then all the entries in Q_1 or Q_4 are non-zero and thus the design d* is connected. When $\rho = v_0 = 0$, then atleast one of v_1 and v_2 is non zero. Then for $k_1 = k_2 = k$ in Table 2.1 and relation $c = \rho + x_1 + x_2$ (2.6) we get

$$v = 2k - c \quad \text{and} \quad c = x_1 + x_2 \tag{4.3}$$

We shall now consider different cases for the choice of affected blocks,

Case I: $x_1 = x_2 = 0$. From (4.3) and Table 2.1, we get

$$c = 0,\ v = 2k \text{ and } v_1 = v_2 = k - 1$$

Since $k \geq 2$, therefore v_1 and v_2 are positive integers and hence d* is connected.

Case II: $x_1 = x_2 = 1$. From (4.3) and Table 2.1, we get

$$c = 2,\ v = 2(k-1) \text{ and } v_1 = v_2 = k - 2$$

Now $v > 3$ and $v = 2(k-1)$ implies $k \geq 3$ and hence v_1 and v_2 are positive integers thus d* will be connected.

Case III: One of the $x_t (t = 1, 2)$ is 1 and other is 0 then from (4.3), we get $c = 1$, $v = 2k - 1$. Then we have either of the following values for v_1 and v_2, $v_1 = k - 1$ and $v_2 = k - 2$ or $v_1 = k - 2$ and $v_2 = k - 1$. Since $v > 3$, $v = 2k - 1$ implies that $k > 2$. Hence, v_1 and v_2 are positive integers and d* is connected.

Remark 4.1: If $v = 3$, $k = 2$ either of v_1 or v_2 is necessarily 0 and with $\rho = v_0 = 0$, only one of Q_1 or Q_2 exists and for $\lambda = 1$, we shall have zero element in one of the rows of Q and thus design is not connected. Thus the only BIB design for which design d* is disconnected is d(3, 3, 2, 2, 1).

Efficiency Criterion

We have proved that a BIB design $v > 3$ is robust against the interchange of treatments according to connectedness criterion against interchange of a pair of treatments in two blocks of the design. The efficiency of design d* relative to d using (3.3) is

$$E = 1 - X/[X + Y] \tag{4.4}$$

where $X = 8(k - c - 1) - 2(x_1 + x_2)(3 + \lambda v) - 4x_1x_2$ and

$$Y = (v-1)[\lambda^2 v^2 - 4(k - c - 1) - (x_1 + x_2)(3 + \lambda v) + 2x_1x_2]$$

The relative efficiencies of the existent BIB designs from Raghavarao [18], Kageyama [11] and Hall [9] have been studied for all three cases of incidences of interchanged treatments using connectedness criterion. Since the expression of E in (4.4) is dependent upon 'c' the number of common treatments in two blocks and also the values of x_1 and x_2, the exact efficiencies have been worked out for the following three classes of BIB designs:

1. Symmetric, $c = \lambda$ [Case I and III for designs with $\lambda = 1$ and all the three cases for designs with $\lambda > 1$].
2. Resolvable (affected blocks belong to same replication $c = 0$, only Case I is possible].
3. Affine resolvable [Group (a), $c = 0$ when affected blocks belong to the same replication only Case I is possible]; Group(b), $c = k^2/v$ when affected blocks belong to different replications ($c = 1$; Case I & III; $c > 1$; Case I, II & III].

For other BIB designs not belonging to sub-classes 1 to 3, lower and upper bounds of relative efficiencies using the lower and upper bounds of 'c' viz. Max $\{0, 2k - v, k - r + \lambda\} \leq c \leq$ Min $\{k - 1, [\{r(r - k - \lambda) + 2\lambda k\}/r]\}$; where [.] denotes the greatest integer function, has been obtained.

It has been observed that for the design in classes 1 to 3 in most of the designs the loss in information is less than 1% and hence are robust except for very few designs where the loss of efficiency is more than 5%. The list of such designs is given in Table 4.3.

Table 4.3

v	b	r	k	λ	Subclass	% loss in efficiency		
						Case I	Case II	Case III
4	4	3	3	2	symmetric	—	33	14
5	5	4	4	3	symmetric	—	10	—
7	7	3	3	1	symmetric	—	—	15
4	6	3	2	1	affine resolvable			
					Case (a)	19	—	—
					Case (b)	—	—	49

For the designs not belonging to subclasses 1 to 3, it has been observed that the lower bound on efficiency for the design with $v > 7$ is greater than 0.95 and hence are robust.

4.3 Equireplicated Non-proper Variance Balanced Block Designs of Gupta and Jones [7]

Here we have considered the designs with two distinct block sizes, when affected blocks are of (i) same sizes either are identical in treatments or disjoint (ii) disjoint blocks of different sizes. For each of such cases the efficiency is found to be greater than 0.95 except for the designs with parameters $v = 6$, $b = 18$, $r = 8$, $k_1 = 2$, $k_2 = 3$; $v = 6$, $b = 18$, $r = 8$, $k_1 = 2$, $k_2 = 4$; $v = 6$, $b = 24$, $r = 9$, $k_1 = 2$, $k_2 = 3$; $v = 8$, $b = 12$, $r = 6$, $k_1 = 2$, $k_2 = 4$ and $v = 8$, $b = 20$, $r = 9$, $k_1 = 2$, $k_2 = 4$.

In these designs the loss in efficiency is about 10 to 12% when the blocks are identical and are of size 2. Thus we conclude that the designs with $v > 8$ belonging to this class are robust.

4.4 Unequireplicated and Non-proper VBD's of Kageyama [12]

The designs in this series have the parameters $v = p+2$, $b = m+p+1$, $\mathbf{r}' = (p+1, (m+1)1'_{p+1})$, $\mathbf{k}' = ((p+1)1'_m, 21'_{p+1})$ where $p > 1$ is an odd natural number and $m = (p+1)/2$. Here, we have either blocks of size 2 or $(p+1)$ and consequently the pair of affected blocks could be each of size 2 (Case I), size $p+1$ (Case II) and of size 2 or $(p+1)$ (Case III). For each of such cases using (3.3) the relative efficiencies of resulting design d* have been worked out for odd p in $3 \le p \le 45$. The relative efficiencies have been found to be greater than 0.95 for $p > 5$. For $p = 3$, it varied from 0.72 to 0.85 in different cases of affected blocks (except for Case III when affected blocks are disjoint and of different sizes). For $p = 5$, the relative efficiency is found to be 0.90 (Case I) and 0.89 (Case III). Thus, we conclude that this series of binary variance balanced designs is robust for $p > 5$. However, it is interesting to note that this series is also robust according to connectedness criterion against interchange of a pair of treatments.

5. *Conclusion and Recommendations*

In any experiment it is assumed that each plot receives the treatment allotted to it but due to mechanical error or by accident it may not happen. The interchange of treatment between two blocks is one of such possibilities. Various classes of binary variance balanced block designs viz. RCBD, BIBD, VBD's of Gupta and Jones [7] and of Kageyama [12] are robust against the interchange of a pair of treatments between any two blocks of the design except for few designs in various classes with $v \le 8$ and $k \le 4$. However, the study of robustness aspects of VBD's when there is interchange of more than one pair of treatments is under investigation. Similar studies for other classes of designs viz. partially balanced block designs or row column design would be of interest.

ACKNOWLEDGEMENT

Authors are grateful to the anonymous referee for valued suggestion which helped in the improvement of the earlier version of the manuscript.

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