Available online at www.isas.org.in/jisas
JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 75(1) 2021 63–73

Construction of Two Level Balanced and Nearly Balanced Optimal Supersaturated Designs

Parvez Mallick[1], Jit Shankar Basak[1], A. Dutta[1], H. Das[2] and A. Majumder[1]

[1]Bidhan Chandra Krishi Viswavidyalaya, Nadia
[2]ICAR – Indian Institute of Soil Science, Bhopal

Received 17 November 2020; Revised 02 August 2021; Accepted 17 August 2021

SUMMARY

Supersaturated designs (SSDs) are very useful for screening experiments with many factors using only a few runs or design points. The widely accepted criteria for optimality of two level SSD is the $E(s^2)$ measure, where the design matrix X_d has the restriction that either each column sum will be zero for balanced supersaturated designs or each column sum will be ±1 for nearly balanced designs (Gupta, 2010). Several researchers have constructed many two level balanced and nearly balanced SSDs for different combinations of m and n (m stands for number of factors and n number of runs; $m \geq n$). The solutions of almost all the available balanced and nearly balanced SSDs are presented in 'Design Resource Server' of IASRI website. Some new methods are developed for construction of new balanced and nearly balanced SSDs. In the first part of the article, some new methods of construction of balanced and nearly balanced supersaturated designs have been presented. The methods yield some new balanced and nearly balanced optimum supersaturated designs which are not yet reported in the available literature. Many available supersaturated designs can also be constructed from these methods; in the sense these methods are more general. The developed designs are examined by sharper lower bounds of $E(s^2)$ measures (Suen and Das, 2010). The design points or solutions of some designs are given in Appendix I. In the second part, new methods for construction of two level balanced and nearly balanced supersaturated designs (master SSDs) involving maximum possible number (mmax) of factors for any particular number of runs (n), are presented. A series of new SSDs are constructed from those master SSDs after deleting the similar columns of available SSDs.

Keywords: Hadamard matrix, Supersaturated designs and Lower bounds of supersaturated designs.

1. INTRODUCTION

In a multi-factor experiment with m two level factors, we require at least n runs for the estimation of all main effects and the general mean where ($n > m+1$). But in a supersaturated designs, n is always less than $m+1$($n < m+1$). Such designs are useful for factor screening experiments which can provide considerable cost reduction involving a few runs for a bigger number of factors. Under the assumption of effect sparsity that among the total factors only a small number of factors are active. Identification of those active factors is one of the most important objectives of supersaturated designs. In addition, the estimation of maximum number of active factors is also very important for supersaturated designs. Cost reduction is another beneficial role of such designs as they require a lesser number of runs for a large number of factors. The concept of supersaturated designs was initiated by Satterthwaite (1959) as random balanced designs. Booth and Cox (1962) suggested a systematic method for construction of supersaturated designs as multifactor experiments, where the number of runs (n) (or combination of different levels of different factors) is less than the number of factors (m). Later, several authors have developed many methods for constructions of supersaturated designs. Interested readers may go through the review article on supersaturated designs by Georgiou (2014).

2. USEFUL DEFINITIONS, MODEL AND PRELIMINARIES

Definition 2.1: A supersaturated Design (SSD) is essentially a fractional factorial design in which the degrees of freedom for all its main effects and

Corresponding author: Anurup Majumder
E-mail address: anurupbckv@gmail.com

the intercept term exceed the total number of distinct factor level combinations of the design. Alternatively, SSDs are also fractional factorial designs of which the numbers of columns for allocating factors are greater than those for ordinary orthogonal designs.

Definition 2.2: A design is said to be balanced if the number of times each level appears in a column or in a factor of a supersaturated Design is equal. Therefore, in a two level supersaturated design the number of run will be an even number. Otherwise the design is called unbalanced.

Definition 2.3: Gupta (2010) defined nearly balanced supersaturated designs for odd number of runs. A supersaturated design is said to be nearly balanced if the frequencies of occurrences of level +1 and -1 differ in a column of the design matrix ($\mathbf{X}_d$) at most by one in such a way that in each of the first $[\frac{m}{2}]$ columns of $\mathbf{X}_d$, the frequencies of the occurrence of levels +1 and -1, in each column, are $[\frac{n}{2}]$ and n-$[\frac{n}{2}]$, respectively. Similarly, in each of the remaining m-$[\frac{m}{2}]$ columns of $\mathbf{X}_d$, the frequency of occurrence of levels +1 and -1 are n-$[\frac{n}{2}]$ and $[\frac{n}{2}]$, respectively, where [.] denotes the greatest integer function.

Definition 2.4: A supersaturated design is said to be optimal if λ_{st} is a constant, where λ_{st} is the number of coincidences between the s^{th} and t^{th} row of a two level Supersaturated designs D(n,m) with n rows and m columns. The value of λ_{st} can be estimated by making the sum of elements of Kronecker product of s^{th} and t^{th} row of D(n,m) after substituting the product value -1 by 0. The lower bound of a supersaturated designs will be attained only when λ_{st} will be constant, where $s \neq t = 1, 2, \ldots, n$

2.1 Model

Let us consider a supersaturated design **d** with m two level factors and n number of runs. Let $\mathbf{X}_d$ be the design matrix of **d** of order n x m (2 < n < m+1) with elements +1 or -1.

For the design **d**, the linear main effects model will be:

$$\mathbf{y} = [\mathbf{1}_n : \mathbf{X}_d]\boldsymbol{\beta} + \mathbf{e};\ \mathbf{e} \sim \mathbf{N}_n(\mathbf{0}_n, \boldsymbol{\sigma}^2\mathbf{I}_n), \qquad 2.1$$

where **y** be n x1 response vector and $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, \ldots, \beta_m)'$ is the parameter vector representing the general mean effect and the m main effects. Here, $\mathbf{Y}_d = [\mathbf{1}_n : \mathbf{X}_d]$ is the n x m+1 model matrix. Here, $\mathbf{X}_d = (x_{ij})$ be an n x m design matrix for the above model of a factorial experiment in m factors in n runs. Thus, xij = +1 or -1. Suppose $\mathbf{u}$ = [u1, u2, …, un]′and $\mathbf{v}$ = [v1, v2, …, vn]′ be two different columns of $\mathbf{X}_d$. Then, i) $\mathbf{u} \neq \pm\mathbf{v}$ and ii) $\mathbf{u} \neq \mathbf{v} \neq \mathbf{1}_n$. Thus the design matrix $\mathbf{X}_d$ has m distinct columns.

The design matrix $\mathbf{X}_d$ will be called **orthogonal** or **saturated** if $\mathbf{X}_d'\mathbf{X}_d$ is a diagonal matrix. But in a two level supersaturated designs the inner product of any two distinct columns of $\mathbf{X}_d$ does not always zero (as n < m+1) i.e. the matrix $\mathbf{X}_d'\mathbf{X}_d$ will never be a diagonal matrix.

Booth and Cox (1962) proposed the idea of $\mathbf{E(s^2)}$ criterion for choice of such designs. Their criterion for selection of design was minimization of $E(\mathbf{s^2})$, where,

$$E(s^2) = \sum_{1 \le i \le j \le m} s_{ij}^2 \Big/ \binom{m}{2}, \qquad 2.2$$

where s_{ij} be the element of the matrix $\mathbf{X}_d'\mathbf{X}_d$. Actually $\mathbf{E(s^2)}$ is the measure of non- orthogonality between two columns of $\mathbf{X}_d$.

Nguyen (1996) obtained a lower bound of $\mathbf{E(s^2)}$ as:

$$E(s^2) \geq \frac{n^2(m-n+1)}{(m-1)(n-1)} \qquad 2.3$$

After that many authors improved the lower bounds (sharper lower bounds) considering different values of m and n, for construction of optimal two level supersaturated designs, e.g. Bulutoglu and Cheng (2004), Das *et al.* (2008), Suen and Das (2010), etc.

Given m and n, all supersaturated designs $\mathbf{X}_d$ has the restriction that the sum of each column is either zero (balanced) for even number of runs (n) or ±1 (nearly balanced) for odd number of runs (n). Such designs constitute a restricted class of supersaturated designs and is denoted by D_R(n, m). Again, the supersaturated designs without the above restrictions constitute an unrestricted class and is denoted by D_U (n, m).

A supersaturated design **d*** is said to be $\mathbf{E(s^2)}$ optimal if $Ed^*(s^2) \leq Ed(s^2)$ for any $d \in D_R(n, m)$.

Literature survey reveals that several researchers developed two level optimal supersaturated designs for different parameters (n, m). List of two level optimal supersaturated designs are also presented by many authors. The solutions of most of the SSDs with bigger m (number of factors) are generated through computer search methods ('Design Resource Server' of IASRI website). No specific rule has been developed for getting the solutions of the listed available designs. However, it is an uphill task to prepare a complete catalogue of supersaturated optimal designs. The goal of the present article is to explore new areas to develop new supersaturated designs, new in the sense that the designs which are not yet listed in available literature.

Section three describes some new methods of construction of two level optimal balanced and nearly balanced supersaturated designs using Hadamard matrices $H_m (m > 4)$. In section four, methods to construct two level optimal balanced and nearly balanced supersaturated designs $D_R(n, m)$ for maximum m (m_{max}) for any n, has been described. These designs will be used as master designs for developing many new designs which ultimately make some additions to the available catalogues. Concluding part is presented in section five. The solutions of some designs are available in Appendix I.

3. METHOD OF CONSTRUCTION

Let us take one Hadamard matrix of order m (≥ 8). $H_m = (1_m, H_m^*)$, where H_m^* of order m x (m-1) with all elements either +1 or -1 and 1_m is a column vector with all entries +1. Let us now construct an array B of order m x (2m-1) as: $B = (1_m : H_m^* \vdots H_m^{**}) = (1_m \vdots B^*)$, where H_m^{**} of order m x (m-1) is constructed from H_m^* by reshuffling (m-1) rows except the first row (with all elements one) of H_m^*. Then, we calculate the correlation matrix of B^*. If any cell of the correlation matrix shows +1, then we may proceed for new reshuffling of (m-1) rows of H_m^{**}. If the values +1 or -1 in the correlation matrix of B^* are not removed completely by repeated reshuffling of (m-1) rows of H_m^*, we construct a new array C by deleting the any column of the pair of columns which are responsible for the correlation values +1 or -1 from H_m^{**}. Thus the array C of order m x (2m-p-1); (where p = number of deleted columns) has been constructed and p ≥ 1. Then,

$$C = (1_m : H_m^* : H_{(m-p)}^{***}) = (1_m : C^*)$$

If we add s (≥ 1) sets of (m-1) columns from different reshuffling schemes of H_m^* either in B or in C we get arrays of elements +1 and -1 of order either m x {(s+1)m-s-1} or m x {(s+1)m-s-p-1} , respectively.

Remark 3.1: Hadamard matrix of order 4 i.e. H_4 cannot be used for the construction of array B or array C, because reshuffling of 3 rows (except the first row) of H_4^* can yield same array, i.e. H_4^* and H_4^{**} are similar.

3.1 Construction of balanced optimal SSD

Theorem 3.1: If there exists a Hadamard matrix of order m (≥ 8), then there exists a balanced optimal SSD with k (= 2m-q-1) factors and n (= m) runs, where q is a positive integer and (q ≥1) or a balanced SSD with k (=2m-q-p-1) factors and n (= m) runs where p and q both are positive integers and p ≥1; q ≥1.

Proof (by construction)**:** Firstly, we construct the matrix B of order m x (2m-1) from H_m^* of order m x (m - 1). Next, the correlation matrix of B* will be prepared. If any cell of the correlation matrix shows +1, then we may proceed for new reshuffling of (m-1) rows of H_m^{**}. Now, the resulting B* from reshuffled H_m^{**} will be a balanced optimum SSD with order m x (2m - 2) where q = 1 with minimum $E(s^2)$ value (Das *et al.*).

Again, if we delete one column of B* and consider the new matrix as B** of order m x (2m-3) where q = 2. Then B** will also give another balanced optimum SSD with (2m-3) factors and n(=m) run. The operation of deletion of one more column from B** again may be repeated to construct separate balanced optimum SSD of order m x (2m-4) where q = 3. **This operation of deletion can be repeated q times on developed SSD till it gives the minimum $E(s^2)$ (Das et. al.).**

If the values +1 or -1 in the correlation matrix of B^* are not removed completely by reshuffling of (m-1) rows of H_m^{**}, we construct a new array C by deleting the any column from each of the p pairs of columns which are responsible for the correlation values +1 or -1 from H_m^{**}. The new array C with deleted columns be denoted as

$$C = (1_m : H_m^* : H_{(m-p)}^{***}) = (1_m : C^*)$$

Here several SSDs will come with order m x (2m - q - p - 1), where p is the number of deleted columns and q as defined earlier.

Example 3.1: Let us start with H_{16}. We get the following balanced optimum SSDs in Table 3.1. Here, the designs are developed from the matrix B. The design points of serial number 1 in table 3.1 are given in Appendix I.

Example 3.2: Let us start with H_8. We get the following balanced optimum SSDs in Table 3.2. Here, the correlation value of 7^{th} and 12^{th} column of B matrix has shown +1. Therefore, we delete 12^{th} column from B and call the new matrix as C. The design points of serial number 1 of Table 3.2 are given in Appendix I.

3.2 Construction of nearly balanced optimal SSD

Theorem 3.2: If there exists a Hadamard matrix of order m (≥ 8), then there exists a nearly balanced optimal SSD with (m-1) runs and either (2m-q-1) factors, where q is a positive integer and (q ≥1) or (2m-q-p-1) factors where p and q both are positive integers and p ≥1; q ≥1.

Proof (by construction)**:** Firstly, delete the last row of the Hadamard matrix and then the constructional procedure is same as Theorem 3.1.

Example 3.3: Let us start with H_{16}. We get the following balanced optimum SSDs in Table 3.3. Here, the designs are developed from the matrix B. The design points of serial number 1 in Table 3.3 are given in Appendix I.

Example 3.4: Let us start with H_8. We get the following balanced optimum SSDs in Table 3.2. Here, the correlation value of 7^{th} and 12^{th} column of B matrix

Table 3.1. Balanced optimum supersaturated design for 16 runs developed from Theorem 3.1

Sl. No.	Number of Runs	Number of Factors	q	Deleted column	$E(s^2)$ as calculated	$E(s^2)$ Das *et al.*	$\lvert r_{max}\rvert$	f_{max}	Efficiency
1*	16	30	1	-	8.828	8.828	0.8	5	1.000
2*	16	29	2	30^{th}	8.828	8.828	0.8	4	1.000
3*	16	28	3	29^{th}	8.804	8.804	0.8	4	1.000
4*	16	27	4	28^{th}	8.752	8.752	0.8	4	1.000

(*) New designs

Table 3.2. Balanced optimum supersaturated design for 8 runs developed from Theorem 3.1

Sl. No.	Number of Runs	p	Number of Factors	Deleted column	q	$E(s^2)$ as calculated	$E(s^2)$ Das *et al.*	$\lvert r_{max}\rvert$	f_{max}	$\lvert *r_{max}\rvert$ (IASRI)	$*f_{max}$ (IASRI)	Efficiency
1	8	1	13	-	1	4.923	4.923	0.5	24	0.5	24	1.000
2	8	1	12	13^{th}	2	4.848	4.848	0.5	20	0.5	20	1.000
3	8	1	11	12^{th}	3	4.655	4.655	0.5	16	0.5	16	1.000
4	8	1	10	11^{th}	4	4.267	4.267	0.5	12	0.5	12	1.000

(*) Parameters of available designs in Design resource server prepared by IASRI, New Delhi.

Table 3.3. Nearly Balanced optimum supersaturated design for 15 runs developed from Theorem 3.2

Sl. No.	Number of Runs	Number of Factors	Deleted column	q	$E(s^2)$ as calculated	$E(s^2)$ Suen and Das	$\lvert r_{max}\rvert$	f_{max}	Efficiency
1*	15	30	-	1	8.724	8.724	0.86	2	1.000
2*	15	29	30^{th}	2	8.724	8.724	0.86	2	1.000
3*	15	28	29^{th}	3	8.703	8.703	0.86	2	1.000

(*) New designs

Table 3.4. Nearly Balanced optimum supersaturated design for 7 runs developed from Theorem 3.2

Sl. No.	Number of Runs	p	Number of Factors	Deleted column	q	$E(s^2)$ as calculated	$E(s^2)$ Das *et al.*	$\lvert r_{max}\rvert$	f_{max}	$\lvert **r_{max}\rvert$ (IASRI)	$**f_{max}$ (IASRI)	Efficiency
1	7	1	13	-	1	4.69	4.69	0.75	6	0.75	6	1.00
2	7	1	12	13^{th}	2	4.64	4.64	0.75	5	0.75	5	1.00
3	7	1	11	12^{th}	3	4.49	4.49	0.75	4	0.75	4	1.00
4*	7	1	10	11^{th}	4	4.2	4.2	0.75	3			1.00
5	7	1	9	10^{th}	5	3.667	3.677	0.75	2	0.75	2	1.00

(*) New Design; (**) Parameters of available designs in Design resource server prepared by IASRI, New Delhi.

has shown +1. Therefore, we delete 12^{th} column from B and call the new matrix as C. The design points of serial number 1 in Table 3.4 are given in Appendix I.

Corollary 3.1: If there exists a Hadamard matrix of order m (≥ 8) and then we add s (≥ 1) sets of (m-1) columns from different reshuffling schemes of H_m^* either in B or in C we get balanced optimum SSD for m runs with {(s+1)m-s –p-q-1}number of factors.

Proof: The proof is similar to proof of theorem 3.1.

Example 3.5: Let us start with H8. We get the following balanced optimum SSDs in Table 3.5. Here, the correlation value of 7^{th} and 12^{th} column of B matrix has shown +1. Therefore, we delete 12^{th} column from B and call the new matrix as C. Then repeat for two times with different reshuffling of rows of each H_8^* to get a balanced SSD. Again the correlation values of 3^{rd} vs. 17^{th} column of C matrix and 12^{th} vs. 15^{th} column of C matrix have shown +1. Therefore, we delete 17^{th} and 15^{th} columns from C. Thus we get a balanced SSD with 8 runs and 18 factors where (s = 2, p = 3 and q =1). Ultimately, we get the following balanced optimum SSDs in Table 3.5.

4. CONSTRUCTION OF TWO LEVEL BALANCED OPTIMAL SUPERSATURATED DESIGNS D(N, M) WITH MAXIMUM M

4.1 Case 1 (n is an even number)

Let us consider an array $\mathbf{A}_1 = [a_{ij1}]$ of order n* x m1 where a_{ij1} will be numerical numbers from 1 to n arranged in ${}^{n-2}C_{[(n-2)/2]}$ ways. Let us consider another array $[a_{ij1}]$ $\mathbf{A}_2 = [a_{ij2}]$of order (n*-1)x m2 where a_{ij2} will be numerical numbers from 1 to n arranged in ${}^{n-2}C_{[(n-2)/2]-1}$ ways. In $\mathbf{A}_1$, n* = [(n-2)/2] and m1 = ${}^{n-2}C_{[(n-2)/2]}$. In $\mathbf{A}_2$, n* = [(n-2)/2]-1 and m2 = ${}^{n-2}C_{[(n-2)/2]-1}$.

Theorem 4.1: If there exist the above mentioned arrays $\mathbf{A}_1$ and $\mathbf{A}_2$ with elements as numerical numbers from 1 to n, then there exists an optimal supersaturated design $\mathbf{D}$(n, m_{max}) with a constant $\lambda_{st} = {}^{n-2}C_{[(n-2)/2]-1}$, $s \neq t$ = 1, 2, …, n where $m_{max} = {}^{n-2}C_{[(n-2)/2]} + {}^{n-2}C_{[(n-2)/2]-1} = (1/2)\ ({}^{n-2}C_{(n/2)})$.

Proof: Let us arrange n numerical numbers following the formula of combination of basic algebra. As n is even, and the design is balanced, there are equal number of +1s and -1s in each column of D**.** If we impose the restriction that the last row of D will comprise only +1 and the first element of the column of D started with -1 then the remaining n-2 elements of any column should have (n-2)/2 number +1 and (n-2)/2 number -1. Thus the number of such columns will be ${}^{n-2}C_{[(n-2)/2]}$. Then we convert the numbers present in A_1 in each arrangement as -1 and numbers absent to +1. Then we place ${}^{n-2}C_{[(n-2)/2]}$ columns of converted A_1 in D between first and last rows as defined. Thus ${}^{n-2}C_{[(n-2)/2]}$ columns are prepared in D with n rows. For the remaining columns we impose the restriction that the last row comprises only +1 and the first element as also +1. Again we convert the numerical figures of A_2 as -1 and +1 for numbers absent and present, respectively. Now we place the ${}^{n-2}C_{[(n-2)/2]-1}$ columns of converted A_2 in remaining columns of D between first and last rows having all elements +1. Thus the Design D is constructed with ${}^{n-2}C_{[(n-2)/2]} + {}^{n-2}C_{[(n-2)/2]-1} = (1/2)\ ({}^{n-2}C_{(n/2)})$ columns and n rows. These columns are the maximum possible columns for a particular number of n (even) rows of a balanced design. Thus a two level balanced optimal supersaturated design D(n, m_{max}) has been constructed with a constant λ_{st} (= ${}^{n-2}C_{[(n-2)/2]-1}$, $s \neq t$ = 1, 2, …, n) which attains the lower bound of $E(s^2)$. For easy understanding, readers may go through the Example 4.1.

Table 3.5. Balanced optimum supersaturated design for developed from corollary 3.1

Sl. No.	Number of Runs	p	Number of Factors	Deleted column	$E(s^2)$ as calculated	$E(s^2)$ Das *et al.*	$\lvert r_{max}\rvert$	f_{max}	$\lvert *r_{max}\rvert$ (IASRI)	$*f_{max}$ (IASRI)	Efficiency
1	8	3	18	-	6.274	6.274	0.5	60	0.5	60	1.00
2	8	3	17	14^{th}	6.118	6.118	0.5	52	0.5	52	1.00

(*) Parameters of available designs in Design resource server prepared by IASRI, New Delhi.

Example 4.1: Let n = 8 and m_{max} =35.

A1=

1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	4
2	2	2	2	3	3	3	4	4	5	3	3	3	4	4	5	4	4	5	5
3	4	5	6	4	5	6	5	6	6	4	5	6	5	6	6	5	6	6	6

A2=

1	1	1	1	1	2	2	2	2	3	3	3	4	4	5
2	3	4	5	6	3	4	5	6	4	5	6	5	6	6

D(8, 35) =

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1
-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1
1	-1	1	1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1
1	1	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1
1	1	1	-1	1	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1	-1	-1	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	1	-1	-1	-1	1	1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1	1	-1	-1	1	1	-1
-1	-1	-1	1	-1	-1	-1	1	-1	-1	1	-1	1	-1	1
-1	-1	-1	-1	1	-1	-1	-1	1	-1	-1	1	-1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

λ_{st} = 15 and $E(s^2)$ = 7.53, r_{max} = 0.5, f_{max} = 280.

Remark 4.1: If the restrictions of design D(n, m_{max}) are not imposed, the design can be constructed from (1/2) x ${}^{n}C_{n/2}$ combinations.

Example 4.2: Solution of D(8,35)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1
-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
-1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	1
1	-1	1	1	1	-1	1	1	1	-1	-1	-1	1	1	1	-1	1	1	1	-1
1	1	-1	1	1	1	-1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	-1
1	1	1	-1	1	1	1	-1	1	1	-1	1	-1	1	-1	1	1	-1	1	1
1	1	1	1	-1	1	1	1	-1	1	1	-1	1	-1	-1	1	1	1	-1	1

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1
-1	-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1
1	1	-1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1
-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1
1	-1	1	-1	-1	1	1	-1	1	-1	-1	1	-1	-1	-1

λ_{st} = 15 and $E(s^2)$ = 7.53, , rmax=0.5, fmax=280.

4.2 Case 2 (n is an odd number)

Let us consider an array **A** = $[a_{ij}]$ of order n* x m where aij will be numerical numbers from 1 to n arranged in ${}^{n}C_{[(n-1)/2]}$ ways. In A, n* = [(n-1)/2] and m = ${}^{n}C_{[(n-1)/2]}$

Theorem 4.2: If there exist the above mentioned array **A** with elements as numerical numbers from 1 to n, then there exists an optimal supersaturated design **D**(n, m_{max}) with a constant

$$\lambda_{st} = {}^{n-2}C_{[(n-1)/2]} + {}^{n-2}C_{[(n+1)/2]},\ s \neq t = 1, 2, \ldots, n$$

where $m_{max} = {}^{n}C_{(n-1)/2}$.

Proof: Let us arrange n numerical numbers following the formula of combination of basic algebra. As n is odd, and the design can be nearly balanced, there are unequal number of +1s and -1s in each column of **D.** Here, the frequencies of occurrences of level +1 and -1 differ in a column of the design matrix ($\mathbf{X}_d$) at most by one in such a way that in each of the first $[\frac{m}{2}]$ columns of $\mathbf{X}_d$, the frequencies of the occurrence of levels +1 and -1 in each column, will be $[\frac{n}{2}]$ and n-$[\frac{n}{2}]$, respectively. Similarly, in each of the remaining m-$[\frac{m}{2}]$ columns of $\mathbf{X}_d$, the frequency of occurrence of levels +1 and -1 are n- $[\frac{n}{2}]$ and $[\frac{n}{2}]$, respectively, where [.] denotes the greatest integer function.

Then we convert the numbers present in first $[\frac{m}{2}]$ columns of A in each arrangement as -1 and numbers absent to +1. Again we convert the numerical figures of remaining m-$[\frac{m}{2}]$ columns of **A** as +1 and -1 for numbers present and absent, respectively. Now, we consider all the converted m columns as columns of design D. Thus the design D is constructed with $^{n}C_{[(n-1)/2]}$ columns and n rows. These columns are the maximum possible columns for a particular number of n odd rows. Thus a two level supersaturated design $D(n, m_{max})$ has been constructed with a constant λ_{st} (= $^{n-2}C_{[(n-1)/2]} + {}^{n-2}C_{[(n+1)/2]}$, $s \neq t = 1, 2, \ldots, n$) which attains the lower bound of $E(s^2)$. For easy understanding, readers may go through the Example 4.3.

Example 4.3: Let n = 7 and m_{max} =35.

A=

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2
2	2	2	2	2	3	3	3	3	4	4	4	5	5	6	3	3	3
3	4	5	6	7	4	5	6	7	5	6	7	6	7	7	4	5	6

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
2	2	2	2	2	2	2	3	3	3	3	3	3	4	4	4	5
3	4	4	4	5	5	6	4	4	4	5	5	6	5	5	6	6
7	5	6	7	6	7	7	5	6	7	6	7	7	6	7	7	7

D(7,35)=

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1
-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1
-1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1
1	-1	1	1	1	-1	1	1	1	-1	-1	-1	1	1	1	-1	1	1
1	1	-1	1	1	1	-1	1	1	-1	1	1	-1	-1	1	1	-1	1
1	1	1	-1	1	1	1	-1	1	1	-1	1	1	1	-1	1	1	-1
1	1	1	1	-1	1	1	1	-1	1	1	-1	1	-1	-1	1	1	1

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1
-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	1	1	-1
-1	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1	1	1	-1	1
-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1
1	-1	-1	1	-1	1	1	-1	-1	1	-1	1	1	-1	1	-1	1

λ_{st} = 15 and $E(s^2)$ = 6.647 rmax =0.75, fmax = 70, Efficiency= 1.00.

4.3 Construction of two level optimal Supersaturated designs as sub designs of $D(n, m_{max})$

Theorem 4.3: Under the realization of theorem 4.1 and 4.2 and existence of two level optimal supersaturated designs D(n, m), their exist some two level optimal supersaturated designs D*(n*, m*), where n* = n and m* = m_{max} – m.

Proof: These designs can be constructed either from theorem 4.1 or theorem 4.2, the designs with maximum possible factors or m_{max}. All available two level optimal supersaturated designs D(n, m) are actually sub-designs of $D(n, m_{max})$ developed from either theorem 4.1 or 4.2. Let us select any available two level optimal supersaturated design from available catalogue of IASRI design resource server for any particular n. The solution of the design will also be collected from the above source. Next, the elements of the columns present in the listed design will be converted to numerical digits as is done in case of the theorems 4.1 or 4.2. Then the columns of the listed design are removed from the columns of $D(n, m_{max})$. The residual design D* with remaining columns of $D(n, m_{max})$ will be a new two level supersaturated design as D*(n*=n, m*= m_{max} - m).

Example 4.4: From example 4.2, D(8, 35), the columns numbered 22, 3, 13, 25, 27, 9, 2, 17, 33 and 15 are removed. As these columns are used in D(8, 10). Residual columns of D(8, 35) will produce a two level optimal supersaturated design D*(8, 25). Solution of D(8, 25) after deletion of the columns numbered 22, 3, 13, 25, 27, 9, 2, 17, 33 and 15 is given below:

D(8, 25) =

1	4	5	6	7	8	10	11	12	14	16	18	19	20	21	23	24
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1
-1	-1	-1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1
-1	1	1	-1	-1	-1	1	1	1	1	-1	-1	-1	1	1	1	1
1	1	1	-1	1	1	-1	-1	-1	1	-1	1	1	-1	-1	1	1
1	1	1	1	-1	1	-1	1	1	-1	1	1	1	-1	1	-1	-1
1	-1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1
1	1	-1	1	1	1	1	1	-1	-1	1	1	-1	1	1	1	-1

26	28	29	30	31	32	34	35
-1	-1	-1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
-1	-1	-1	-1	-1	1	1	1
-1	-1	1	1	1	-1	-1	1
-1	1	-1	-1	1	-1	1	-1
1	1	-1	1	-1	-1	-1	-1
1	-1	1	-1	-1	1	-1	-1

$E(s^2) = 7.04$, Efficiency 1.00, $r_{max} = 0.5$, $f_{max} = 132$.

Remark 4.2: The designs developed from theorems 4.1 and 4.2 are the optimal designs with maximum possible factors (m_{max}). More designs can be constructed from the developed designs by successive deletion of columns from them. The residual designs may or may not be optimal. Optimality can be achieved by searching methods for which the $E(s^2)$ values will attain the lower bound. A list of developed two level optimal designs from theorem 4.3 is given below.

5. CONCLUSION

The problem of construction of SSD has some special importance because of its beneficial role of incorporating a large number of factors but with a lesser number of runs which ultimately reduce the cost involvement. The methods stated above can yield many bigger designs. Popular computer search method has the demerit of developing a series of SSDs as the procedure has no specific rule. The theorems, 3.1, 3.2, 4.1, 4.2 and 4.3 will be helpful to develop a suitable computer algorithm for construction of different series of balanced or nearly balanced optimal two level supersaturated designs as they are constructed by following the above methods or theorems. These methods are more general construction methods in the sense that they include many available optimum SSDs even if the designs are developed by computer search methods. The article is enriched with useful examples for easy understanding of the methods. Some developed SSDs are given in Appendix I.

ACKNOWLEDGEMENTS

Authors are very much thankful to the referee for his valuable and constructive suggestions for preparation of final manuscript with some extra points related to the constructional methods of balanced and nearly balanced supersaturated new designs.

Table 4.1. List of two level optimal supersaturated designs (n ≤ 10)

Sl. No.	No. of Runs	No. of Factors	$D(n, m_{max}) – D(n, m)$ either from IASRI catalogue or from designs in the article	Number of aliased columns to be deducted from $D(n. m_{max})$
1	7	26	D(7, 35) – D(7,9)	9
2	7	25	D(7, 35) – D(7,10)	10
3	7	24	*D(7, 34) – D(7,10)	10
4	7	21	*D(7, 34) – D(7,13)	13
5	7	18	*D(7, 34) – D(7, 16)	16
6	7	17	D(7, 35) – D(7, 18)	18
7	7	14	D(7, 35) – D(7, 21)	21
8	8	28	D(8, 35) – **D(8, 7)	7
9	9	113	D(9, 126) – D(9, 13)	13
10	9	112	D(9, 126) – D(9, 14)	14
11	9	111	D(9, 126) – D(9, 15)	15
12	9	110	D(9, 126) – D(9, 16)	16
13	9	109	*D(9, 125) – D(9, 16)	16
14	9	108	D(9, 126) – D(9, 18)	18
15	9	107	D(9, 126) – D(9, 19)	19
16	9	105	*D(9, 125) – D(9, 20)	20
17	9	104	*D(9, 125) – D(9, 21)	21
18	9	103	D(9, 126) – D(9, 23)	23
19	9	102	D(9, 126) – D(9, 24)	24
20	9	101	D(9, 126) – D(9, 25)	25
21	9	100	*D(9, 125) – D(9, 25)	25
22	9	21	D(9, 126) – D(9, 105)	105
23	9	22	D(9, 126) – D(9, 104)	104
24	10	113	D(10, 126) – D(10, 13)	13
25	10	107	D(10, 126) – D(10, 19)	19

Table 4.1. List of two level optimal supersaturated designs ($n \leq 10$) (Contd.)

Sl. No.	No. of Runs	No. of Factors	$D(n, m_{max}) – D(n, m)$ either from IASRI catalogue or from designs in the article	Number of aliased columns to be deducted from $D(n. m_{max})$
26	10	106	D(10, 126) – D(10, 20)	20
27	10	105	D(10, 126) – D(10, 21)	21
28	10	104	D(10, 126) – D(10, 22)	22
29	10	103	D(10, 126) – D(10, 23)	23
30	10	102	D(10, 126) – D(10, 24)	24
31	10	101	D(10, 126) – D(10, 25)	25
32	10	100	D(10, 126) – D(10, 26)	26
33	10	99	D(10, 126) – D(10, 27)	27
34	10	98	D(10, 126) – D(10, 28)	28
35	10	97	D(10, 126) – D(10, 29)	29
36	10	96	D(10, 126) – D(10, 30)	30
37	10	95	D(10, 126) – D(10, 31)	31
38	10	94	D(10, 126) – D(10, 32)	32
39	10	93	D(10, 126) – D(10, 33)	33
40	10	92	D(10, 126) – D(10, 34)	34
41	10	91	D(10, 126) – D(10, 35)	35
42	10	112	D(10, 126) – D(10, 14)	14
43	10	111	D(10, 126) – D(10, 15)	15
44	10	109	D(10, 126) – D(10, 17)	17
45	10	108	D(10, 126) – D(10, 18)	18
46	10	107	D(10, 126) – D(10, 19)	19
47	10	106	D(10, 126) – D(10, 20)	20
48	10	105	D(10, 126) – D(10, 21)	21
49	10	104	D(10, 126) – D(10, 22)	22

* design from remark 4.2., ** Orthogonal Columns are taken from H_8

REFERENCES

Booth, K.H.V. and Cox, D.R. (1962). Some systematic supersaturated designs. Technometrics., **4**: 489-495.

Bulutoglu, D.A. and Cheng, C.S.(2004). Construction of $E(s^2)$-optimal supersaturated designs. Ann. Stat., **32**(4): 1662-1678.

Cheng, C.S. (1997). $E(s^2)$ Optimal supersaturated designs. Stat. Sin., **7**: 929-939.

Das, A., Dey, A., Chan, L.Y. and Chatterjee, K. (2008). $E(s^2)$-optimal supersaturated designs. J. Stat. Plan. Inference., **138**(12): 3749-3757.

Das, U., Gupta, S. and Gupta, S. (2014). Screening active factors in supersaturated designs. Comput Stat Data Anal., **77**: 223-232.

Georgiou, S.D. (2014). Supersaturated designs: A review of their construction and analysis. J. Stat. Plan. Inference., **144**: 92-109.

Gupta, V.K., Parsad, R., Kole, B. and Bhar, L.M. (2008). Computer-Generated Efficient Two-level Supersaturated Designs. J. Indian Soc. Agric. Stat., **62**(2): 183-194.

Gupta, V.K., Singh, P., Kole, B. and Parsad, R. (2010). Construction of Efficient Balanced and Nearly Balanced Two-level Supersaturated Designs. J. Stat. Appl., **5**(2): 179.

Jones, B. and Majumdar, D. (2014). Optimal supersaturated designs. J. Am. Stat. Assoc.,**109**(508): 1592-1600.

Lin, D.K.J. (1993). A new class of supersaturated designs. Technometrics., **35**(1): 28-31.

Nguyen, N.K. (1996). An algorithmic approach to constructing supersaturated designs. Technometrics., **38**: 69-73.

Ryan, K.J. and Bulutoglu, D.A. (2007). $E(s^2)$-Optimal supersaturated designs with good minimax properties. J. Stat. Plan. Inference.,**137**(7): 2250-2262.

Satterthwaite, F. (1959). Random balance experimentation. Technometrics., **1**(2): 111-137.

Singh, R. and Das, A. (2015). On optimal two-level supersaturated designs. Technical Report, Indian Institute of Technology Bombay, Mumbai.

Suen, C.S. and Das, A. (2010). $E(s^2)$-optimal supersaturated designs with odd number of runs. J. Stat. Plan. Inference.,**140**(6):1398-1409.

Tang, B. and Wu, C.F.J. (1997). A method for constructing supersaturated designs and its $E(s^2)$-optimality. Can. J. Stat., **25**(2): 191-201.

APPENDIX I

1. SSD with 30 factors in 16 runs:

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1
-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1
-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1
-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1

2. SSD with 13 factors in 8 runs:

1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	-1	-1	-1	-1	1	-1	-1	-1	1	1
-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1
1	-1	-1	-1	-1	1	1	-1	1	-1	-1	1	-1
-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	-1
-1	1	-1	1	-1	1	-1	-1	-1	1	-1	-1	1
1	-1	-1	1	1	-1	-1	1	1	1	-1	-1	-1
-1	-1	1	1	-1	-1	1	-1	1	-1	1	-1	1

SSD with 11 factors in 8 runs:

1	1	1	1	1	1	1	1	1	1	1
1	1	1	-1	-1	-1	-1	1	-1	-1	-1
-1	1	-1	-1	1	-1	1	-1	-1	1	1
1	-1	-1	-1	-1	1	1	-1	1	-1	-1
-1	-1	1	-1	1	1	-1	1	-1	-1	1
-1	1	-1	1	-1	1	-1	-1	-1	1	-1
1	-1	-1	1	1	-1	-1	1	1	1	-1
-1	-1	1	1	-1	-1	1	-1	1	-1	1

3. SSD with 30 factors in 15 runs:

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1
-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1
-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1
-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1

4. SSD with 18 factors in 8 runs:

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	-1	1
-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	-1	-1	-1	-1
1	-1	-1	-1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	-1	-1	-1	-1	1	1
-1	1	-1	1	-1	1	-1	-1	-1	1	-1	-1	1	-1	-1	1	1	-1
1	-1	-1	1	1	-1	-1	1	1	1	-1	-1	-1	1	1	-1	1	-1
-1	-1	1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	-1