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Orthogonally Blocked Second Order Response Surface Designs under Auto-Correlated Errors

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SUMMARY

Second order block rotatable response surface designs under auto-correlated errors were discussed in this paper. Rotatability conditions were derived for the proposed model under orthogonal blocks. The computed prediction variance was found to be increasing due to block effect and auto-correlation coefficient. D-optimality condition was derived and provided with a list of designs satisfying the derived rotatability conditions under auto-correlated errors.

Keywords: Second order, Block rotatability, D-optimality, Auto-correlated errors.

1. INTRODUCTION

In this manuscript, we have discussed D-optimal orthogonally blocked second order response surface designs under auto-correlated errors. Blocked experiments are normally carried out when experimental units are heterogeneous by formulating homogenous groups/blocks of experimental units. The variability that occurred due to blocking is accounted for by including the block effect in the proposed model. Khuri (1994) found that prediction variance increases due to the inclusion of the block effect in the model for estimating the mean response.

An orthogonally blocked second order response surface model for 'N' experimental units to estimate the mean response of $y_u, u = 1, 2, \cdots, N$, as a function of v input variables $x_1, x_2, \cdots, x_v$ can be written as

$$y_u = \beta_0 + \sum_{i=1}^{v} \beta_i x_{iu} + \Sigma \sum_{i<j=1}^{v} \beta_{ij} x_{iu} x_{ju} + \sum_{i=1}^{v} \beta_{ii} x_{iu}^2 + \sum_{l=1}^{b} \delta_l B_{lu} + \varepsilon_u,$$

$$1 \le i \le v;\ 1 \le u \le N;\ l \le 1 \le b,$$

$$\text{or}\quad \mathbf{Y} = \beta_0 \mathbf{1}_N + \mathbf{X}_1 \boldsymbol{\beta} + \mathbf{B}\boldsymbol{\delta} + \mathbf{e}, \tag{1}$$

where β_0 is intercept, $\mathbf{1}_N$ is a column vector **1** of order $N \times 1$, $\mathbf{X}_1$ is design matrix of order $N \times \left(2v + \binom{v}{2}\right)$,

$$\mathbf{X}_1 = \left(x_1, x_2, \cdots, x_v;\ x_1 \otimes x_2, \cdots,\ x_{(v-1)} \otimes x_v,\ x_1 \otimes x_1, \cdots,\ x_v \otimes x_v\right),$$

$$x_i = \left(x_{i1}, x_{i2}, \cdots, x_{iN}\right)',\quad x_i \otimes x_j = \left(x_{i1}x_{j1}, x_{i2}x_{j2}, \cdots, x_{iN}x_{jN}\right)';$$

$1 \le i, j \le v$, and $\otimes$ denotes the Hadamad product as defined by Das and Park (2009), $\boldsymbol{\beta} = \left(\beta_1, \cdots, \beta_v, \beta_{12}, \cdots, \beta_{(v-1)v}, \beta_{11}, \cdots, \beta_{vv},\right)'$ is a $\left(2v + \binom{v}{2}\right) \times 1$ vector of regression coefficients; $\boldsymbol{\delta} = (\delta_1, \delta_2, \cdots, \delta_b)'$, where δ_l is the effect of l^{th} block $(l = 1, 2, \cdots, b)$ and **B** is the block diagonal matrix of the form $\mathbf{B} = \mathrm{diag}\left(1_{n_1}, 1_{n_2}, \cdots, 1_{n_b}\right)$, where n_l is the size of the l^{th} block and **e** is $N \times 1$ vector of errors which follows a normal distribution with $E(\mathbf{e}) = 0$ and $D(\mathbf{e}) = \mathbf{V}$ with $\mathrm{rank}(\mathbf{V}) = N$ (Box and Draper (1987)).

The linear and quadratic effects of the model (1) can be estimated independent of the block effects, if the blocking is done orthogonally. Goos and Donev

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(2006) derived conditions for orthogonal blocking of experiments involved quantitative factors. The model in Equation (1) is not a full rank model since $B1_b = 1_N$. Therefore, Equation (1) can be written as given by Khuri and Cornell (1987) by putting restrictions on the elements of $\boldsymbol{\delta}$ to estimate β_0 independently of $\boldsymbol{\delta}$ assuming the sum of block effects constrained to zero, that is $\sum_{l=1}^{b} \delta_l = 0$.

We can get

$$\mathbf{Y} = \mathbf{X\theta} + \mathbf{e}, \text{ where } \mathbf{X} = [\mathbf{X_1} : \mathbf{B}] \text{ and } \boldsymbol{\theta} = [\boldsymbol{\beta} : \boldsymbol{\tau}],\ \boldsymbol{\tau} = \beta_0 \mathbf{1}_b + \boldsymbol{\delta}. \tag{2}$$

The parameter vector $\boldsymbol{\theta}$ is estimated by OLS method assuming the error vector $\mathbf{e}$ has a zero mean and variance-covariance matrix $\sigma^2 \mathbf{I}_N$.

Suppose experimental units in response surface methodology are arranged randomly in each block/group and observations are recorded from each block/group. The observations within each block/group are correlated, whereas observations from different blocks/groups are statistically uncorrelated. This might have happened due to either some ordering of experimental units within the blocks or time is included as an input factor in the model. For example, raw material from different species of prawn used for the production of chitin and chitosan may affect the quality and yield of the chitin and chitosan. Another example is the high-pressure processing of a particular species of fish for the production of fish sausage, the fish sample collected from different locations may affect the final quality parameters of the developed fish sausage. In the above examples, species of prawn and locations form the groups/block and all these experiments involved processed time and temperature as input variables; this may cause correlation among observed values of the response variables.

The OLS method of estimation is not valid when the observations are correlated and there have been few studies where rotatability conditions for first and second order response surface models under correlated errors were derived (Panda and Das (1994); Das (2003a, 2003b) and Varghese *et al.* (2013)). Incorporation of interefence effects, if any, into the first order blocked response surface model results in more precise estimates of the parameters (Varghese and Jaggi, 2011).Mann *et al.* (2010) introduced a generalized least squares estimator to construct robust designed experiments with blocks by considering correlated error in the linear model. Das *et al.* (2010) studied D-optimal robust second order rotatable and slope rotatable designs under correlated errors. They derived rotatability conditions for second order response surface designs under correlated errors along with a class of D-optimal robust second order slope rotatable designs for different types of correlated errors. Das and Park (2009) have given a measure for robust slope rotatability for second order response surface designs under auto-correlated errors.

Joshy and Balakrishna (2017) studied blocked first order response surface designs with interaction under correlated errors, where they have derived rotatability and D-optimality conditions for auto-correlated errors. In this paper, additional rotatabilty and D-optimality conditions were derived for blocked second order response surface designs under auto-correlated errors.

2. BLOCKED SECOND ORDER RESPONSE SURFACE MODELS UNDER AUTO-CORRELATED ERRORS

The best linear unbiased estimator of $\boldsymbol{\theta}$ in Equation (2) for a known variance – covariance matrix **(V)** of a blocked response surface model is obtained as $\hat{\boldsymbol{\theta}} = \left(\mathbf{X'V^{-1}X}\right)^{-1}\left(\mathbf{X'V^{-1}Y}\right)$ with dispersion matrix $\mathbf{D}\left(\hat{\boldsymbol{\theta}}\right) = \left(\mathbf{X'V^{-1}X}\right)^{-1}\sigma^2$, where $\mathbf{V} = \rho^{|i-j|}, 1 \le i, j \le N$ is the auto-correlated errors of order 1 for the elements of 'e' in the model (2) and $V_E^{-1} = \frac{1}{(1-\rho^2)}\left[\left(1+\rho^2\right)I_N - \rho^2 P_{N\times N} - \rho Q_{N\times N}\right]$, where ρ is auto-correlation coefficient, I_N is an $N\times N$ identity matrix, **P** is an $N\times N$ matrix with elements $p_{11} = p_{NN} = 1$ and all other elements zeros, and **Q** is an $N\times N$ matrix with $q_{ij} = 1$ for $|i-j| = 1$ and all other elements are zeros.

Now, the generalized least square estimator of $\boldsymbol{\theta}$ is

$\hat{\theta} = \left(X'V^{-1}X\right)^{-1} X'V^{-1}Y$, here

$$\mathbf{X'V^{-1}X} = \frac{1}{(1-\rho^2)}\left[\left(1+\rho^2\right)\mathbf{X'I_N X} - \rho^2\mathbf{X'PX} - \rho\mathbf{X'QX}\right], \tag{3}$$

where $\mathbf{I_N}$ is an $N \times N$ identity matrix, **P** is a matrix of order $N \times N$, which contains 'b' block-diagonal matrices of order $n \times n$ with elements $p_{111} = p_{1nn} = 1$ and the rest are zeros. Further, **Q** is a matrix of order $N \times N$, which contains 'b' block-diagonal matrices of order $n \times n$ with $q_{1ij} = 1$ for $|i-j| = 1$ on the diagonal and all other elements are zeros.

Thus, the best linear unbiased estimator of $\mathbf{Y}$ is $\hat{\mathbf{Y}} = \mathbf{X}\hat{\boldsymbol{\theta}}$. The estimated value of $\mathbf{Y}$ at the point $\mathbf{x}_0'$ and its prediction variance are given in Equations (4) and (5), respectively

$$\hat{y}(x) = \mathbf{x}_0'\hat{\theta},$$

where

$$x_0' = [\left(x_{10}, x_{20}, \cdots, x_{v0}, x_{10}x_{20}, \cdots, x_{(v-1)0}x_{v0}, x_{10}^2, x_{20}^2, \cdots, x_{v0}^2\right)' : \frac{1}{b}\mathbf{1}_b')] \tag{4}$$

$$V\left(\hat{y}(x)\right) = \mathbf{x}_0'\left(X'V^{-1}X\right)^{-1}\mathbf{x}_0\sigma^2. \tag{5}$$

2.1 Conditions for Orthogonal Parameter Estimation and Rotatability

The moment matrix $(X'V^{-1}X)$ is symmetric and an orthogonally blocked second order response surface design is said to be rotatable if all the odd moments of order up to four must be zero. Joshy and Balakrishna (2017) derived rotatabilty conditions for blocked first order response surface designs with interaction under auto-correlated errors in Section 4.1. The following additional restrictions ensure the rotatability of the second order response surface design $\mathbf{X}_1$ under orthogonal blocks

1. $w_{ii.j} = b\sum_{u=1}^{n} x_{iu}^2 x_{ju} + b\rho^2 \sum_{u=2}^{n-1} x_{iu}^2 x_{ju} - b\rho\left(\sum_{u=1}^{n-1} x_{iu}x_{i(u+1)}x_{ju} + \sum_{u=1}^{n-1} x_{iu}x_{i(u+1)}x_{j(u+1)}\right)$

   $= 0,\ 1 \le i, j \le v$

2. $w_{ii.js} = b\sum_{u=1}^{n} x_{iu}^2 x_{ju}x_{su} + b\rho^2 \sum_{u=2}^{n-1} x_{iu}^2 x_{ju}x_{su} - b\rho\left(\sum_{u=1}^{n-1} x_{iu}x_{i(u+1)}x_{j(u+1)}x_{su} + \sum_{u=1}^{n-1} x_{iu}x_{i(u+1)}x_{ju}x_{s(u+1)}\right)$

   $= 0,\ 1 \le i, j < s \le v$

and

3. $w_{i.i} = b\sum_{u=1}^{n} x_{iu}^2 + b\rho^2 \sum_{u=2}^{n-1} x_{iu}^2 - 2b\rho\left(\sum_{u=1}^{n-1} x_{iu}x_{i(u+1)}\right) = C_1,\ 1 \le i \le v$

4. $w_{ij.ij} = b\sum_{u=1}^{n} x_{iu}^2 x_{ju}^2 + b\rho^2 \sum_{u=2}^{n-1} x_{iu}^2 x_{ju}^2 - 2b\rho\sum_{u=1}^{n-1} x_{iu}x_{i(u+1)}x_{ju}x_{j(u+1)}$

   $= C_2,\ 1 \le i < j \le v$

5. $w_{ii.ii} = b\sum_{u=1}^{n} x_{iu}^4 + b\rho^2 \sum_{u=2}^{n-1} x_{iu}^4 - 2b\rho\sum_{u=1}^{n-1} x_{iu}^2 x_{i(u+1)}^2 = C_3,\ 1 \le i \le v$

6. $w_{ii.jj} = b\sum_{u=1}^{n} x_{iu}^2 x_{ju}^2 + b\rho^2 \sum_{u=2}^{n-1} x_{iu}^2 x_{ju}^2 - b\rho\left(\sum_{u=1}^{n-1} x_{iu}^2 x_{j(u+1)}^2 + \sum_{u=1}^{n-1} x_{i(u+1)}^2 x_{ju}^2\right)$

   $= C_4,\ 1 \le i \ne j \le v$

7. $w_{ii.b} = b\sum_{u=1}^{n} x_{iu}^2 + b\rho^2 \sum_{u=2}^{n-1} x_{iu}^2 - b\rho\left(\sum_{u=1}^{n} x_{iu}^2 + \sum_{u=2}^{n-1} x_{iu}^2\right) = C_5,\ 1 \le i \le v$

8. $w_{b.b} = (1-\rho)\left(n-(n-2)\rho\right) = C_6,$

where C_1, C_2, C_3, C_4, C_5 and C_6 are constants.

Therefore, the moment matrix is obtained as

$$\left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right) = \begin{bmatrix} \mathbf{A_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A_2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A_3} \end{bmatrix},$$

where

$$\mathbf{A_1} = C_1\mathbf{I}_v,\ \mathbf{A_2} = C_2\mathbf{I}_{\binom{v}{2}} \text{ and } \mathbf{A_3} = \begin{bmatrix} (C_3 - C_4)\mathbf{I}_v + C_4\mathbf{J}_v & C_5\mathbf{J}_{v\times b} \\ C_5\mathbf{J}_{b\times v} & C_6\mathbf{I}_b \end{bmatrix}. \tag{6}$$

Thus, the elements of the dispersion matrix is reduced to the form

$$\mathbf{D}\left(\hat{\boldsymbol{\theta}}\right) = \left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)^{-1} = \begin{bmatrix} \mathbf{A}_1^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_3^{-1} \end{bmatrix}, \text{ where}$$

$$\mathbf{A}_1^{-1} = \left(\frac{1}{C_1}\right)\mathbf{I}_v,\ \mathbf{A}_2^{-1} = \left(\frac{1}{C_2}\right)\mathbf{I}_{\binom{v}{2}} \text{ and}$$

$$\mathbf{A}_3^{-1} = \begin{bmatrix} (q_1 - p_1)\mathbf{I}_v + p_1\mathbf{J}_{v\times v} & p_2\mathbf{J}_{v\times b} \\ p_2\mathbf{J}_{b\times v} & (q_2 - p_3)\mathbf{I}_b + p_3\mathbf{J}_{b\times b} \end{bmatrix}, \tag{7}$$

$$p_1 = \frac{bC_5^2 - C_3C_6}{(C_3 - C_4)\left[C_6\left\{C_3 + (v-1)C_4\right\} - bvC_5^2\right]},$$

$$p_2 = \frac{-C_5}{\left[C_6\left\{C_3 + (v-1)C_4\right\} - bvC_5^2\right]},$$

$$p_3 = \frac{vC_5^2}{C_6\left[C_6\left\{C_3 + (v-1)C_4\right\} - bvC_5^2\right]},$$

$$q_1 = p_1 + \frac{1}{(C_3 - C_4)} \text{ and } q_2 = p_1 + \frac{1}{C_6}.$$

The quantity $\left[C_6\left\{C_3 + (v-1)C_4\right\} - bvC_5^2\right]$ is a factor of $\mathbf{A}_3^{-1}$ and appearing in the denominator; therefore, it gives an additional condition for rotatability, i.e.

$$\left[C_6\left\{C_3 + (v-1)C_4\right\} - bvC_5^2\right] > 0 \tag{8}$$

An orthogonally blocked second order response surface design will be second order block rotatable response surface design under auto-correlated errors if and only if

1. $w_{i.b} = 0$ and $w_{ij.b} = 0$ for all $i \neq j = 1, 2, \cdots, v$, $l = 1, 2, \cdots, b$,
2. $w_{ii.ii} = 2w_{ij.ij} + w_{ii.jj} \Rightarrow C_3 = 2C_2 + C_4$,

$$p_1 = \frac{bC_5^2 - C_6(2C_2 + C_4)}{(C_3 - C_4)\left[C_6\{2C_2 + vC_4\} - bvC_5^2\right]},$$

$$p_2 = \frac{-C_5}{\left[C_6\{2C_2 + vC_4\} - bvC_5^2\right]},$$

$$p_3 = \frac{vC_5^2}{C_6\left[C_6\{2C_2 + vC_4\} - bvC_5^2\right]},\ q_1 = p_1 + \frac{1}{2C_2} \text{ and}$$

$$q_2 = p_1 + \frac{1}{C_6}, \tag{9}$$

where C_1, C_2, C_3, C_4, C_5 and C_6 are defined as in Section 2.1.

The variance of the estimated parameters is given by $V(\hat{\boldsymbol{\theta}}) = \sigma^2 \left(\mathbf{X'V^{-1}X}\right)^{-1}$,

Now,

$$V(\hat{\beta}_i) = \sigma^2(1-\rho^2)\left(\frac{1}{C_1}\right)\mathbf{I}_v \tag{10}$$

$$V(\hat{\beta}_{ij}) = \sigma^2(1-\rho^2)\left(\frac{1}{C_2}\right)\mathbf{I}_{\binom{v}{2}} \tag{11}$$

$$V(\hat{\beta}_{ii}) = \sigma^2(1-\rho^2)\left\{(q_1 - p_1)\mathbf{I}_v + p_1\mathbf{J}_{v\times v}\right\} \tag{12}$$

$$V(\hat{\tau}_1) = \sigma^2(1-\rho^2)\left\{(q_2 - p_3)\mathbf{I}_b + p_3\mathbf{J}_{b\times b}\right\} \tag{13}$$

$$Cov(\hat{\beta}_{ii}, \hat{\beta}_{jj}) = \sigma^2(1-\rho^2)p_1\mathbf{J}_{v\times v} \tag{14}$$

$$Cov(\hat{\beta}_{ii}, \hat{\tau}_l) = \sigma^2(1-\rho^2)p_2\mathbf{J}_{v\times b} \tag{15}$$

The prediction variance of $\hat{y}(x)$ at a point $\mathbf{x}_0'$ is given in Equation (16)

$$V(\hat{y}(x)) = \sigma^2(1-\rho^2)\Bigg\{\left(\frac{1}{C_1} + 2p_2\right)\sum_{i=1}^{v} x_{i0}^2 + \left(\frac{1}{C_2} + 2p_2\right)\sum\sum_{i<j=2}^{v} x_{i0}^2 x_{j0}^2 + q_1\sum_{i=1}^{v} x_{i0}^4 + \frac{1}{b}(q_2 + 2p_3)\Bigg\} \tag{16}$$

This expression of prediction variance is a function of $\sum_{i=1}^{v} x_{i0}^2$, b and ρ. Therefore, for a given auto-correlation coefficient ρ, the estimated response will have the same variance for all such points **x** for which $\sum_{i=1}^{v} x_{i0}^2$ is some constant. The prediction variance $V(\hat{y}(x))$ for an orthogonally blocked second order response surface model under auto-correlated errors increases due to blocking. The prediction variance $V(\hat{y}(x))$ can be represented as

$$V(\hat{y}(x)) = V(\hat{y}_0(x)) + \sigma^2(1-\rho^2)\frac{1}{b}(q_2 + 2p_3), \tag{17}$$

Thus, $V(\hat{y}(x)) \geq V(\hat{y}_0(x))$ since $V(\hat{y}_0(x)) \geq 0$, where $V(\hat{y}_0(x))$ is the prediction variance under auto-correlated error structure when block effects are zero, that is $\boldsymbol{\delta} = \mathbf{0}$.

3. D-OPTIMALITY OF SECOND ORDER BLOCK ROTATABLE RESPONSE SURFACE DESIGNS

D-optimality for an experimental design **X**, **X**=[$\mathbf{X}_1$:**B**], where $\mathbf{X}_1$ is the extended design matrix and **B** is the given block structure, when the block effect is fixed and additive is obtained by maximizing $D = |\mathbf{X'X}|$. Based on the results derived by Goos and Donev (2006) for D-optimality criteria for blocked response surface designs, the design is D-optimal second order response surface designs under auto-correlated errors if $\left|\mathbf{X'V^{-1}X}\right|$ is maximum, that is

$$D = \left|\mathbf{X'V^{-1}X}\right| = \det\{\mathbf{A_1}\} * \det\{\mathbf{A_2}\} * \det\{\mathbf{A_3}\}$$

$$= C_1^v C_2^{\binom{v}{2}} (C_3 - C_4)^{v-1} (C_3 + (v-1)C_4) C_6^{b-1} \left(C_6 - \frac{bC_5^2}{(C_3 + (v-1)C_4)}\right) \tag{18}$$

3.1 List of Orthogonally Blocked Second Order Rotatable Designs under Auto-Correlated Errors

We have considered a set of computer-generated orthogonally blocked response surface designs with N non-central design points having b blocks with block size n_l and v input factors. D-optimal second order block rotatable designs under auto-correlated errors are obtained by extending N design points to $b(2n_l + 1)$ design points by incorporating $(n_l + 1)$ central points in each block. One of each $(n_l + 1)$ central point is added in between each set of design points in the sequence of each block and the remaining two central points are placed at the start and end of each block.

Example 1: Consider a central composite experimental design for v = 2 (x_1 and x_2) and $\alpha = 1.41$

with 8 non-central experimental runs arranged in two blocks each of size 4. A D-optimal second order block rotatable design under auto-correlated errors is obtained by adding 5 central points in each block as described above; this extends 8 experimental runs to 18 experimental runs in two blocks each of size 9.

Block 1	x_1	0	1	0	-1	0	1	0	-1	0
	x_2	0	1	0	1	0	-1	0	-1	0
Block 2	x_1	0	1.41	0	0	0	-1.41	0	0	0
	x_2	0	0	0	1.41	0	0	0	-1.41	0

Example 2: Consider a Box-Behnken experimental design of four input factors (x_1, x_2, x_3 and x_4) with 24 non-central experimental runs arranged in three blocks each of size 8. A D-optimal second order block rotatable design under auto-correlated errors is obtained by adding 9 central points in each block as described above, which gives 51 experimental runs in 3 blocks each of size 17.

The D-optimality values of designs mentioned in Example 1 and Example 2 are given in the Table 1. The values of D-optimality were found to be increasing when the values of $|\rho|$ was increasing from zero

Since blocking was done orthogonally to all the above designs, the rotatabilty conditions mentioned in Section 2.1 and D-optimality criterion mentioned in Equation (18) were satisfied and provide D-optimal second order optimum rotatable designs under auto-correlated errors. Similarly, Goos and Donev (2006) derived orthogonality conditions for blocked response surface designs. The orthogonal blocking ensures the homogeneity among the experimental units, especially when you have too many experimental runs and the addition of center points ensures the independence of observations. Das (2014) developed a method to construct randomized block designs with auto-correlated errors and similarly found that the robustness of the experimental design depends on auto-correlation coefficient ρ.

Table 1. D-optimality values for different values of ρ

Example 1		Example 2	
ρ	D-value	ρ	D-value
-0.9	2510260000	-0.9	6.8952E31
-0.8	62777091	-0.8	2.9701E26
-0.7	6810436	-0.7	1.9947E23
-0.6	1386401.8	-0.6	1.145E21
-0.5	411470.02	-0.5	2.2795E19
-0.4	160689.67	-0.4	1.0767E18
-0.3	78889.641	-0.3	1.0105E17
-0.2	47742.481	-0.2	1.7322E16
-0.1	35361.066	-0.1	5.2178E15
0.0	31995.909	0.0	2.7146E15
0.1	35361.066	0.1	5.2178E15
0.2	47742.481	0.2	1.7322E16
0.3	78889.641	0.3	1.0105E17
0.4	160689.67	0.4	1.0767E18
0.5	411470.02	0.5	2.2795E19
0.6	1386401.8	0.6	1.145E21
0.7	6810436	0.7	1.9947E23
0.8	62777091	0.8	2.9701E26
0.9	2510260000	0.9	6.8952E31

4. CONCLUSION

The effect of orthogonal blocking in second order response surface designs under auto-correlated errors has been studied in the paper. The rotatability conditions were derived for blocked second order

Block 1	x_1	0	0	0	0	0	0	0	0	0	1	0	-1	0	-1	0	1	0
	x_2	0	0	0	0	0	0	0	0	0	1	0	-1	0	1	0	-1	0
	x_3	0	1	0	-1	0	-1	0	1	0	0	0	0	0	0	0	0	0
	x_4	0	1	0	-1	0	1	0	-1	0	0	0	0	0	0	0	0	0
Block 2	x_1	0	0	0	0	0	0	0	1	0	1	0	0	0	-1	0	-1	0
	x_2	0	-1	0	1	0	1	0	0	0	0	0	-1	0	0	0	0	0
	x_3	0	-1	0	1	0	-1	0	0	0	0	0	1	0	0	0	0	0
	x_4	0	0	0	0	0	0	0	1	0	-1	0	0	0	1	0	-1	0
Block 3	x_1	0	1	0	0	0	0	0	1	0	-1	0	-1	0	0	0	0	0
	x_2	0	0	0	1	0	1	0	0	0	0	0	0	0	-1	0	-1	0
	x_3	0	1	0	0	0	0	0	-1	0	-1	0	1	0	0	0	0	0
	x_4	0	0	0	1	0	-1	0	0	0	0	0	0	0	1	0	-1	0

response surface designs under auto-correlated errors. The prediction variance of the estimated response was computed for the proposed model and found to be a function of $\sum_{i=1}^{v} x_{iu}^2$, the estimated response will have a constant variance for all such points **x** that is equi-distant from the design center. The prediction variance $V(\hat{y}(x))$ for an orthogonally blocked second order response surface model under auto-correlated errors increases due to blocking. The expression for D-optimality of orthogonally blocked second order design under auto-correlated errors has been given and a list of designs has been provided which satisfy the derived conditions.

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