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A Series of Factorial Row-Column Designs with Incomplete Rows and Columns

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SUMMARY

Row-column designs are advantageously used in experimental situations when the heterogeneity in the experimental material is due to two cross-classified sources. If the experimenter wants to study the effect of two or more factors simultaneously under a row-column set up, a statistically more efficient and informative method is to use a factorial row-column design (RCD), than to run separate experiments for each factor. Factorial RCDs not only need fewer experimental units but can also address the issue of interactions between the factors. As the number of factors or the number of levels of the factors increases, the number of treatment combinations increases considerably and, hence, it may be practical to adopt factorial RCDs in incomplete rows and columns. A series of symmetric factorial RCDs with incomplete rows and columns has been obtained here and the effects confounded in rows and columns have been identified.

Keywords: Designs with two-dimensional blocking; Row-column designs; Symmetric factorial experiments; Confounding.

1. INTRODUCTION

When heterogeneity in the experimental material is due to two cross-classified sources, then twodimensional blocking or double blocking of the experimental units is recommended for control or reduction of experimental error. For example, in an agronomic or forestry trial it is not uncommon to compare a large number of hybrid varieties, which leads to a large experimental area, and it may be important to account for or eliminate the effects of fertility trends in the land in two perpendicular directions. The two blocking systems are referred to generally as row blocking and column blocking, and the resulting designs are referred to as designs under two-dimensional blocking or designs for two-way elimination of heterogeneity or simply, row-column designs (RCDs). These designs are also suitable for animal experiments, greenhouse experiments, irrigation trials, laboratory trials, etc. wherever two blocking systems are present.

If an experimenter wants to study the effect of two or more factors simultaneously, adopting an appropriate factorial RCD is statistically more informative and efficient than running separate experiments for each factor. Such situations arise commonly in agricultural, horticultural and forestry field trials.

When *v* treatment combinations in a factorial experiment are set out in a RCD with *v* rows and *v* columns, all treatment comparisons (main effects and interactions) are estimated independently of row and column parameters. However, such fully orthogonal designs are impractical with large *v*. Thus, RCDs with incomplete rows and columns is required by totally or partially confounding certain higher order interactions with rows as well as columns.

The earliest type of row-column designs for factorial experiments were the Quasi-Latin or Lattice squares introduced by Yates (1937). Rao (1946) gave a method for constructing partially confounded square RCDs. In a square row-column design, the number of rows is equal to the number of columns. Cochran and Cox (1957) listed some RCDs for factorial experiments. John and Lewis (1983) obtained factorial RCDs

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using a generalized cyclic method of construction by amalgamating row and column component designs and gave some guidelines for choosing the row and column component designs appropriately. Williams and John (1996) defined a new objective function that gives varying degrees of emphasis to the main effects in the factorial situation. A computer program was used to generate such designs.

Bailey and Patterson (1991) showed that tworeplicate resolvable RCDs are combinatorially equivalent to a single replicate RCD for two factors. They showed that useful designs for variety trials with two replicates, each arranged in a compact array of plots, can be derived from existing RCDs for two non-interacting sets of treatments. RCDs with the column component design resolvable in 2 replicates were constructed by Jarrett et al. (1997) and used it for factorial set up. Wright et al. (2005) extended the result given by Bailey and Patterson (1991) for resolvable RCDs with more than two replications. Choi and Gupta (2008) considered confounded RCDs for symmetric factorial experiments and gave some methods to construct these designs by confounding appropriate interactions over rows and columns.

Bose and Dey (2009) have shown the correspondence of a crossover design balanced for first residuals to a v^2 factorial experiment arranged in a RCD assuming that the first has only the main effects of one factor and there are no interactions in the first row. The direct effects of treatments are considered as levels of the first factor and the residual effects are considered as the levels of the second factor.

Dash *et al.* (2013) developed a method of construction of RCDs for estimation of main effects and two factor interaction effects in 2^n factorial 2-colour microarray experiments. RCDs with unequal replications of different treatment combinations were also presented. Subsequently, Godolphin (2018) suggested some single replicate constructions that enable estimation of all main effects and maximize the number of estimable two-factor interactions.

The objective of the present study is to obtain a general method for constructing symmetric factorial RCDs with incomplete rows and columns. The construction method has been made more easily understandable using an appropriate example.

2. SYMMETRIC FACTORIAL RCDS

A factorial RCD with 3 factors (A, B and C) each at v (v > 2) levels (denoted as 1, 2, 3,...,v) arranged in 3v rows and v^2 columns can be obtained using the four steps given below:

Step 1: Consider an array of size $3 \times v$ of the following form:

1, <i>u</i> + <i>v</i> -1, <i>u</i> + <i>v</i> -1	2, <i>u</i> , <i>u</i> + <i>v</i> -1	 v-1, u+v-3, u+v-1	v, u+v-2, u+v-1
<i>u</i> , 1, <i>u</i> + <i>v</i> -1	u, 2, u	 <i>u</i> , <i>v</i> -1, <i>u</i> + <i>v</i> -3	u, v, u+v-2
u, u, 1	<i>u</i> +1, <i>u</i> , 2	 <i>u</i> + <i>v</i> -2, <i>u</i> , <i>v</i> -1	<i>u</i> + <i>v</i> -1, <i>u</i> , <i>v</i>

Step 2: Obtain v arrays of size $3 \times v$ each by developing the above array for u = 1, 2, ..., v.

Step 3: Juxtapose the v arrays obtained in Step 2, horizontally one after another, to yield an initial array (A) of size $3 \times v^2$.

Step 4: Generate each row of the initial array \mathcal{A} by adding 1,2,...,v-1 successively (mod v) resulting in 3 sets each of v rows and v^2 columns.

The final arrangement obtained after Step 4 results in a v^3 factorial arranged in 3v rows and v^2 columns. Each set constitutes a complete replication.

It can be seen that the RCD so obtained is a v^3 factorial in v^2 units per row partially confounding one effect in each set. It is interesting to note that the highest order interaction is confounded in all three sets. In Set I, the effect confounded is $AB^{v-1}C \equiv A^{v-1}BC^{v-1}$. In Sets II and III, the effect confounded is $ABC^{v-1} \equiv A^{v-1}B^{v-1}C$ and $A^{v-1}BC \equiv AB^{v-1}C^{v-1}$, respectively. Within each set, considering columns, the design is a v^3 factorial with v units per column totally confounding (v + 1) effects, with two independent effects viz., ABC^{v-2} (or $AB^{v-2}C$ or $A^{v-2}BC$) and AC^{v-1} (or AB^{v-1} or $A^{v-1}B$ or $A^{v-1}C$ or BC^{v-1} or $A^{v-1}C$) and remaining as generalized effects.

Example 2.1: For v = 3, a 3×9 initial array \mathcal{A} is obtained by taking u = 1, 2, 3 and juxtaposing the 3 arrays each of size 3×3 side by side (following steps 1-3) as follows:

u = 1				u = 2		u = 3			
1,3,3	1,3,3 2,1,3 3,2,3		1,1,1 2,2,1		3,3,1	1,2,2	2,3,2	3,1,2	
1,1,3	1,2,1	1,3,2	2,1,1	2,2,2	2,3,3	3,1,2	3,2,3	3,3,1	
1,1,1	2,1,2	3,1,3	2,2,1	3,2,2	1,2,3	3,3,1	1,3,2	2,3,3	

Now, first row of above array is developed cyclically by adding 1 and 2 successively to it (mod 3) to yield Set I of size 3×9 given below.

Set I

133	213	323	111	221	331	122	232	312
211	321	131	222	332	112	233	313	123
322	132	212	333	113	223	311	121	231

Similarly, two more sets each of dimension 3×9 (Sets II and III) can be obtained by developing second and third rows respectively of **A**.

Set II

	113	121	132	211	222	233	312	323	331
	221	232	213	322	333	311	123	131	112
ſ	332	313	321	133	111	122	231	212	223

Set III

111	212	313	221	322	123	331	132	233
222	323	121	332	133	231	112	213	311
333	131	232	113	211	312	223	321	122

Appending the three sets one below the other results in a 3³ RCD in 9 rows and 9 columns as follows:

	Rows				C	olumi	18			
	Rows	i	ii	iii	iv	v	vi	vii	viii	ix
I	i	133	213	323	111	221	331	122	232	312
	ii	211	321	131	222	332	112	233	313	123
	iii	322	132	212	333	113	223	311	121	231
II	iv	113	121	132	211	222	233	312	323	331
	v	221	232	213	322	333	311	123	131	112
	vi	332	313	321	133	111	122	231	212	223
III	vii	111	212	313	221	322	123	331	132	233
	viii	222	323	121	332	133	231	112	213	311
	ix	333	131	232	113	211	312	223	321	122

The above RCD is a 3^3 factorial in 3^2 units per row with three replications or sets partially confounding one effect in each set. Considering set I, the effect confounded is $AB^2C \equiv A^2BC^2$. In sets II and III, the effect confounded is $ABC^2 \equiv A^2B^2C$ and $A^2BC \equiv AB^2C^2$ respectively. Within each set, considering columns the design is a 3^3 factorial in 3 units per column with ABC and AC^2 getting confounded in all the three sets.

The average variance of estimated elementary contrasts pertaining to various treatment comparisons (considering factorial combinations as treatments) have been computed using a code (given in Appendix) written in PROC IML of SAS software. For this design, the elementary contrasts pertaining to treatment combinations are estimated with an average variance,

 $V = 0.692\sigma^2$ (A snapshot of the SAS output indicating the same is also given in Appendix).

3. CONCLUDING REMARKS

A general method of constructing symmetric RCDs with incomplete rows and columns has been developed for three factors confounding highest order effects in rows and columns. The added advantage is that when considered row-wise, the design is resolvable in 3 sets.

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APPENDIX

SAS code for computing information matrix and average variance of estimated elementary contrasts pertaining to various treatment comparisons (considering factorial combinations as treatments) of factorial RCDs

```
proc iml;
```

```
/*
a original={
133
    213 323
                 111
                      221
                            331
                                  122
                                      232
                                             312,
                                  233
211
     321
           131
                 222
                      332
                            112
                                        313
                                             123,
322
                 333
     132 212
                      113
                            223
                                  311
                                        121
                                             231.
     121
          132
                211
                      222
                            233
                                       323
113
                                  312
                                             331,
     232
                 322
221
           213
                      333
                            311
                                  123
                                        131
                                             112,
332
     313
           321
                 133
                      111
                            122
                                  231
                                       212
                                             223,
     212
           313
                 221
                      322
                            123
                                  331
                                        132
                                             233,
111
222
     323
          121
                 332
                      133
                            231
                                  112
                                       213
                                             311,
                                      321 122};
333
     131 232
                113 211 312 223
actual treatment combinations in the row-column set
up*/
a={
9
     12
           24
                            25
                                  5
                                        17
                                             20,
                 1
                      13
     22
           7
                      26
                            2
                                  18
                                        21
                                             6,
10
                 14
     8
           11
                 27
                      3
                            15
23
                                  19
                                        4
                                             16,
                 10
3
     4
           8
                      14
                            18
                                  20
                                        24
                                             25,
13
     17
           12
                 23
                      27
                            19
                                  6
                                        7
                                             2,
                            5
26
     21
           22
                 9
                      1
                                  16
                                        11
                                             15,
     11
           21
                      23
                            6
                                  25
                                        8
                                             18,
1
                 13
                                  2
14
     24
           4
                 26
                      9
                            16
                                        12
                                             19,
27
     7
           17
                 3
                                  15
                                        22
                                             5};
                       10
                            20
       Renumbered
                        treatment
                                      combinations
111,112,...,332,333 in a original array as 1, 2,...,26,
m=j(nrow(a)*ncol(a),1,1);/*mean vector*/
/*print m;*/
trt=i(nrow(a)*ncol(a),max(a),0);/*design
                                        matrix
observation VS treatment*/
k=1;
```

```
do i=1 to nrow(a);
do j=1 to ncol(a);
if a[i,j] > 0
then trt[k,a[i,j]]=1;
k=k+1;
end;
end;
/*print trt;*/
r=j(nrow(a)*ncol(a),nrow(a),0);/*design
                                            matrix
observation VS row*/
k=1:
do i=1 to nrow(a);
do j=1 to ncol(a);
if a[i,j] > 0
then r[k,i]=1;
k=k+1;
end;
end;
/*print r;*/
c=j(nrow(a)*ncol(a),ncol(a),0);/*design
                                            matrix
observation VS column*/
k=1;
do i=1 to nrow(a);
do j=1 to ncol(a);
if a[i,j] > 0
then c[k,j]=1;
k=k+1;
end;
end;
/*print c;*/
x=m||trt||r||c;/*combined design matrix*/
/*print x[format=3.0];*/
x1=trt;
x2=m||r||c;
c mat=(x1)*x1-(x1)*x2*(ginv(x2)*x2))*x2)*x1/*In
formation (C) matrix*/;
```

```
print c_mat;
/*Elementary contrasts matrix*/
b1=comb(max(a),2);
co=j(b1,max(a),0);
k=1;
do i=1 to max(a);
do j=i+1 to max(a);
co[k,i]=1;
co[k,j]=-1;
k=k+1;
end;
end;
/*print co;*/
```

```
cov=co*ginv(c_mat)*co`;/*variance-covariance
matrix*/
var1=diag(cov);
one=j(b1,1,1);
var2=var1*one;
av_var=sum(var2)/nrow(var2); /*average variance*/
print av_var;
quit;
```

Screenshot (two pages merged together) of the output consisting of information matrix (C) and average variance of estimated elementary contrasts pertaining to various treatment comparisons (considering factorial combinations as treatments) obtained from SAS is given below:



