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JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 75(1) 2021 9–12

Fitting of Size Biased Generalized Negative Binomial and Poisson Distributions on Crop Pests Data

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Received 15 February 2020; Revised 24 November 2020; Accepted 14 December 2020

SUMMARY

This paper is concerned with the fitting of size biased generalized negative binomial (SBGNBD) and size biased generalized Poisson distributions (SBGPD) on crop pest's data. In these distributions, the recorded observations cannot be considered as a random sample from the original distributions due to the observations fall in the non-experimental, non-random and non-replicated categories where the truncated distributions which lie above or below a given threshold level of the distributions. The parameters involved in SBGNBD and SBGPD have been estimated by method of proportion of one'th cell (MPOC) and method of moments (MM). The distributions describe the data satisfactorily well.

Keywords: Method of moments, MPOC, Parameter, χ^2 test of goodness of fit.

1. INTRODUCTION

Size-biased distributions provide an approach to dealing with model specification and data interpretation problems. Fisher (1934) and Rao (1965) introduced and unified the concept of weighted distribution. Fisher (1934) studied on how methods of ascertainment can influence the form of distribution of recorded observations and then Rao (1965) introduced and formulated it in general terms in connection with modeling statistical data where the usual practice of using standard distributions for the purpose was not found to be appropriate.

Rao (1965) identified the various situations that can be modeled by size biased distributions. These situations refer to instances where the recorded observations cannot be considered as a random sample from the original distributions.

Size biased distributions occur frequently in research in relation to the data where the units are selected randomly with unequal probability such as in bio-medicinal, ecological and branching processes (Patil and Rao 1978; Gupta and Kirmani 1990; Gupta and Keating 1985; Oluyede 1999; Das and Roy 2011). Section deals with probability distributions while Section 3 provides the estimation procedures. Results and discussions are given in Section 4. In the last, conclusions have been added.

2. PROBABILITY DISTRIBUTIONS

2.1 Generalized Negative Binomial distribution (GNBD)

Jain and Consul (1971) first defined Generalized Negative Binomial distribution as

$$\mathrm{P}(\mathrm{X}=\mathrm{x}) = \frac{\mathrm{m}}{\mathrm{m}+\beta\mathrm{X}}[\binom{m+\beta X}{x}]\,\alpha^{x}\left(1-\alpha\right)^{\mathrm{m}+\beta\mathrm{X}-\mathrm{X}} \quad (2.1)$$

x = 0, 1, 2 ...,

Where, $0< \alpha<1$, $m>0$ and $|\alpha\beta|<1$

The probability model (2.1) reduces to the binomial distribution when $\beta = 0$, and to the negative binomial distribution when $\beta = 1$. It also resembles the Poisson distribution at $\beta=½$ because, for this value of β the mean and variance are approximately equal. The moments about the origin of the model (2.1) are given as:

$$\mu_1' = \frac{\mathrm{m}\alpha}{(1-\alpha\beta)} \quad (2.2)$$

$$\mu_2' = \frac{(\mathrm{m}\alpha)^2}{(1-\alpha\beta)^2} + \frac{m\alpha(1-\alpha)}{(1-\alpha\beta)^3} \quad (2.3)$$

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and Variance = $\mu_2 = \dfrac{m\alpha(1-\alpha)}{(1-\alpha\beta)^3}$ (2.4)

$$\mu_3' = \frac{(m\alpha)^3}{(1-\alpha\beta)^3} + \frac{3(m\alpha)^2(1-\alpha)}{(1-\alpha\beta)^4} + \frac{m\alpha(1-\alpha)}{(1-\alpha\beta)^5}\left[1-2\alpha+\alpha\beta(2-\alpha)\right] \quad (2.5)$$

Size-biased generalized negative binomial distribution (SBGNBD)

The following gives the size-biased generalized negative binomial distribution (SBGNBD) as

$$P(X=x) = (1-\alpha\beta)\binom{m+\beta X-1}{x-1}\alpha^{x-1}(1-\alpha)^{m+\beta X-X}$$

x=1, 2…, where 0<α<1, m>0, |αβ|<1 (2.6)

Putting β=0 and β=1, results in size-biased binomial (SBB) and size-biased negative binomial (SBNB) distributions.

2.2 Generalized Poisson distribution (GPD)

Consul and Jain (1973) first defined Generalization of Poisson distribution as

$$P(X=x) = \frac{\theta(\theta+x\lambda)^{x-1}e^{-(\theta_1+x\lambda)}}{x!},$$

$$\theta > 0, |\lambda| < 1 \; for \; x = 0,1,2\ldots, \quad (2.7)$$

Mean $= \mu_1' = \dfrac{\theta}{1-\lambda}$ (2.8)

Variance = $\dfrac{\theta}{(1-\lambda)^3}$ (2.9)

Size Biased Generalized Poisson distribution (SBGPD)

$$P(X=x) = \frac{(1-\lambda)(\theta+x\lambda_2)^{x-1e-(\theta+x\lambda)}}{x-1!} \quad x = 1, 2, 3, \ldots, \quad (3.1)$$

x > m

Mean = $\mu_1' = \dfrac{\theta(1-\lambda)}{(1-\lambda)^2}$ (3.2)

Variance = $\mu_2 = \dfrac{\theta(1-\lambda)+2\lambda}{(1-\lambda)^4}$ (3.3)

3. ESTIMATION OF PARAMETERS

3.1 Size-biased generalized negative binomial distribution (SBGNBD)

(i) Proportion of one`th cell (MPOC)

This distribution consists of three parameters which are estimated by equating the observed proportion of one’th cell, mean and variance to their theoretical values. These are given below:

$P_1 = \dfrac{N_1}{N} = (1-\alpha\beta)(1-\alpha)^{m+\beta-1}$ Where P_1 is the proportion of one’th cell, N_1 is the first cell frequency and N is the total frequency.

Mean = $\mu_1' = \dfrac{m\alpha}{1-\alpha\beta} + \dfrac{1-\alpha}{(1-\alpha\beta)^2}$

variance = $\mu_2 = \dfrac{m\alpha(1-\alpha)}{(1-\alpha\beta)^3} + \dfrac{\alpha(1-\alpha)}{(1-\alpha\beta)^4}\left[\beta(2-\alpha)-1\right]$

By trial and error method, the parameters can be estimated.

(ii) Method of Moments (MM)

Mean = $\mu_1' = \dfrac{m\alpha}{1-\alpha\beta} = \dfrac{m\alpha(1-\alpha)}{(1-\alpha\beta)^3} + \dfrac{\alpha(1-\alpha)}{(1-\alpha\beta)^4}\left[\beta(2-\alpha)-1\right]$

$$\mu_3 = \frac{(m\alpha)^3}{(1-\alpha\beta)^3} + \frac{3(m\alpha)^2(1-\alpha)}{(1-\alpha\beta)^4} + \frac{m\alpha(1-\alpha)}{(1-\alpha\beta)^5}\left[1-2\alpha+\alpha\beta(2-\alpha)\right]$$

By trial and error method, the parameters α, m and β can be estimated.

3.2 Size-biased generalized Poisson distribution (SBGPD)

(i) Proportion of one`th cell (MPOC)

This distribution consists of two parameters which are estimated by equating the observed proportion of one’th cell, and mean to their theoretical values. These are given below

$P_1 = \dfrac{N_1}{N} = (1-\lambda)e^{-(\theta+\lambda)}$ Where P_1 is the proportion of one’th cell, N_1 is the first cell frequency and N is the total frequency.

Mean = $\mu_1' = \dfrac{\theta(1-\lambda)}{(1-\lambda)^2}$

By trial and error method, the parameters $\hat{\lambda}$ and $\hat{\theta}$ can be estimated.

(ii) Method of Moments (MM)

This distribution consists of two parameters which are estimated by equating the theoretical expression of proportion of mean and variance to their observed values. By trial and error method, the parameters $\hat{\lambda}$ and $\hat{\theta}$ can be estimated. These are given below:

$$\text{Mean} = \mu_1' = \frac{\theta(1-\lambda)}{(1-\lambda)^2}$$

$$\text{Variance} = \mu_2 = \frac{\theta(1-\lambda)+2\lambda}{(1-\lambda)^4}$$

4. RESULTS AND DISCUSSION

In present investigation, attempts have been made to advocate the suitable probability distributions which are applied to the data of insect pest on mango and moong crop which are taken from M.Sc. (Agril. Stat) Thesis of Bhodriya (2009), Jawaharlal Nehru Krishi Vishwavidyalaya, Jabalpur, Madhya Pradesh.

For this purpose, the size biased generalized negative binomial and the size biased generalized Poisson distribution are considered.

Sized-Biased Generalized Negative Binomial and Poisson distribution

Table 1, 2 and 3 show the observed and expected number of Mango and Moog plant according to number of mango hoppers, jassids and whiteflies. SBGNBD and SBGPD were fitted by two methods, described earlier. In these distributions, the values of $\hat{\alpha}$, $\hat{m}$ and $\hat{\beta}$ in two methods are found to be 0.48, 0.45, 1.00 and 0.41, 0.79, .98 for Mango hoppers, whereas the value of $\hat{\lambda}$ and $\hat{\theta}$ are 0.11, 0.55 and 0.12, 0.57, for Jassidson Moog and 0.06, 0.33 and 0.06, 0.32 for whiteflies on Moog respectively. It shows that all the Mango and Moong plant are exposed to the incidence of Mango hoppers and jassids. It shows that the MPOC and MM provide good fitting of SBGNBD and SBGPD. For applying a χ^2 test, some last cells are grouped together. The values of χ^2 for SBGNBD are 2.32, 2.96 and for SBGPD are 6.08, 6.24 and 0.94, 1.01 respectively under method I, and II. Since the calculated value of is found to be less than tabulated value at 5% level of significance therefore data is well fitted. For visual displayed, the graphical representation of size –biased generalized negative binomial and Poisson distribution using two method of fitting are exhibited in Figs. 1, 2 and 3.

Table 1. Distribution of observed and expected number of mango plants according to number of mango hopper

No of mango Hoppers	Observed frequency	Size –Biased Generalized Negative Binomial Distribution	
		MM	MPOC
1	74	73.92	73.99
2	46	51.50	54.52
3	35	30.30	31.26
4	17	16	16.22
5	11	8.94	7.97
6	4	10.34	7.04
7	3		
8	1		
Total	191	191	191
Estimates of parameters		$\hat{\alpha}=0.48$ $\hat{m}=0.4516$ $\hat{\beta}=1.00$	$\hat{\alpha}=0.41$ $\hat{m}=0.7971$ $\hat{\beta}=.98$
	χ^2 d.f.	2.3229 3	2.9679 3

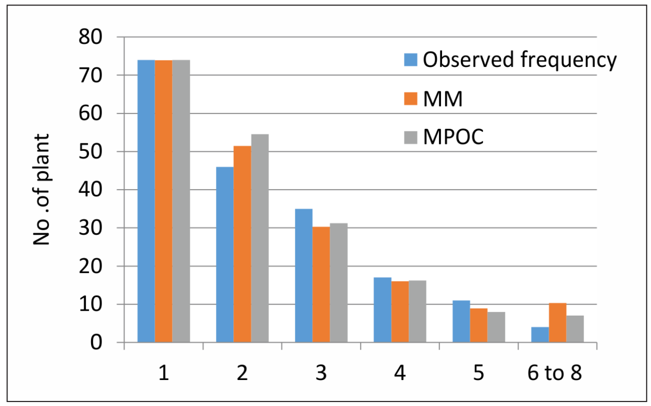

Fig. 1. Distribution of observed and expected number of mango plants according to number of hoppers

Table 2. Distribution of observed and expected number of moong plants according to number of jassid

No of jassids	Observed frequency	Size Biased Generalized Poisson Distribution	
		MM	MPOC
1	115	114.7	115
2	84	80.9	81.8
3	29	37.8	37.7
4	22	14.8	14.4
5	3	7.8	7.1
6	2		
7	1		
Total	256	256	256
Estimates of parameters		$\hat{\lambda}=0.11$ $\hat{\theta}=0.55$	$\hat{\lambda}=0.12$ $\hat{\theta}=0.57$
	χ^2 d.f.	6.086 2	6.248 2

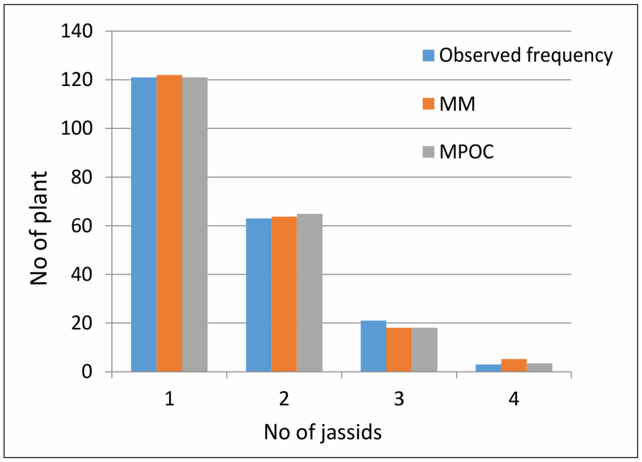

Fig. 2. Distribution of observed and expected number of moong plants according to number of jassid

Table 3. Distribution of observed and expected number of moong plant according to number of whiteflies

No of whiteflies	Observed frequency	Size –Biased Generalized Poisson Distribution	
		MM	MPOC
1	103	102.41	103.28
2	42	44	43.59
3	14	12.12	11.82
4	2	2.73	2.77
5	1	0.55	0.56
Total	162	162	162
Estimates of parameters		$\hat{\lambda} = 0.06$ $\hat{\theta} = 0.3367$	$\hat{\lambda} = 0.06$ $\hat{\theta} = 0.3282$
	χ^2 d.f.	0.9479 1	1.0188 1

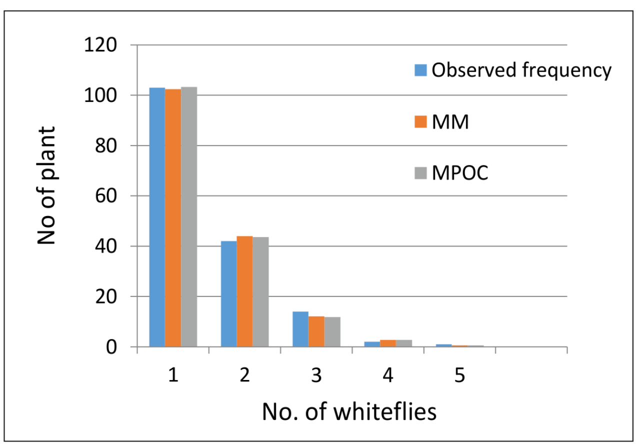

Fig. 3. Distribution of observed and expected number of moong plant according to number of whiteflies

5. CONCLUSIONS

1. The size biased generalized negative binomial distribution and size biased generalized Poisson were found to be adequate for describing the inherent variation ofmango hoppers, jassids and whiteflies population on moong and mango crops.
2. The method of proportion of one'th cell (MPOC) and method of moments (MM) both methods have been found suitable for estimation of parameters.

ACKNOWLEDGEMENTS

The authors are grateful to the editor and referee's for giving their cogent comments on the paper that have really changed the shape of the paper.

REFERENCES

Bhodriya, S. (2009). 'Inflated Modified Power Series Distribution and Path Analysis Application to Agricultural Data, unpublished M.Sc. thesis, J.N.K.V.V., Jabalpur.

Consul, P.C. and Jain, G.C. (1973). A generalization of Poisson distribution. *Technometrics,* **15(4)**, 791-799.

Das, K.K. and Roy, T.D. (2011). On some length-biased weighted Weibull distribution. *Adv. Applied Sci. Res.,* **2**, 465-475.

Fisher, R.A. (1934). The effects of methods of ascertainment upon the estimation of frequencies. *Annals of Eugenics* **6**, 13-25.

Gupta, R.C. and Keating, J. P. (1985). Relations for reliability measures under length biased sampling. *Scan. J. Statist.,* **13**, 49-56.

Gupta, R.C. and Kirmani, S.N.U.A. (1990). The role of weighted distributions in stochastic modeling. *Commun. Statist.,* **19(9)**, 3147-3162.

Jain, G.C., and Consul, P.C. (1971). A generalized negative binomial distribution. *SIAM Journal of Applied Mathematics,* **21(4)**, 501-513.

Oluyede, B.O. (1999). On inequalities and selection of experiments for length-biased distributions. *Probability in the Engineering and Informational Sciences,* **13**, 169-185.

Patil, G.P. and Rao, C.R. (1978). Weighted distributions and size-biased sampling with applications to wild life populations and human families, *Biometrics,* **34**, 179-189.

Rao, C.R. (1965). On discrete distributions arising out of methods on ascertainment, Classical and Contagious Discrete Distribution, Patil, G.P. (Ed.), *Statistical Publishing Society, Calcutta,* 320-332.