https://doi.org/10.56093/JISAS.V78I2.7
Available online at http://isas.org.in/isa/jisas
JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 78(2) 2024 135–141

Improved Ratio-type Exponential Estimator for Estimating Population Mean in Ranked Set Sampling

Nirupama Sahoo and Sananda Kumar Jhankar

Gangadhar Meher University, Sambalpur

Received 03 February 2024; Revised 25 July 2024; Accepted 03 August 2024

SUMMARY

In this paper we propose a ratio-type exponential estimator for estimating population mean of the study variable under Ranked set sampling when auxiliary information is known. The bias and Mean square error of the proposed estimator has been derived up to the first degree of approximation. A simulation study has been carried out to judge the performance of the newly proposed estimator along with existing estimators. It is obtained that the proposed estimator is more efficient as compared to the competing estimators.

Keywords: Ranked Set Sampling; Bias; Mean square error; Relative efficiency; Simulation.

1. INTRODUCTION

In sample surveys, survey specialist always magnifies the efficiency of the proposed estimators using different sampling techniques. In some circumstances pragmatic interest, primly in environmental and ecological studies, the study variable Y, is not easily available or we can say that measurement may be lavish, time consuming and nosy. Though data collection may be convoluted, ranking the sample units regarding auxiliary variable in a small sets recurrently easy and inexpensive. The concept of Ranked set sampling (RSS) was given by Mclntyre (1952) in order to estimate the population mean of pasture and forage yields. He claimed that Ranked set sampling was more accurate than the Simple Random Sampling and an alternative method to SRS in situations where the units can be ranked easily. The concept of RSS is reviewed by Dell, T. and Clutter, J. (1972) with particular consideration of error in judgment ordering. When population mean of the auxiliary variable is known, Khan, L. and Shabbir, J. (2015) and Khan, L and Shabbir, J (2016) suggested an unbiased estimator for estimating the finite population mean of the study variable. Under SRSWOR, Khoshnevisan, M., Singh, R., Chauhan, P., Sawan, N. and Smarandache, F. (2007) proposed a general family of estimators for estimating the population mean of the study variable. Lohr, S. (1999) concentrated on the statistical aspect of taking and analyzing a sample. He focused for a class of statistics majors, or for a class of students from business, sociology, psychology, or biology who want to learn about designing and analyzing data from sample surveys. In RSS the perfect ranking of element was considered by Takahasi, K. and Wakimoto, K. (1968) for estimating population mean of the study variable and Bouza *et al.* (2018) provided a review of RSS, its modification, and its application. Bhusan *et al.* (2022) proposed some efficient combined and separate classes of estimators of the population mean in the presence of bivariate auxiliary information under stratified ranked set sampling. In the context of Ranked Set Sampling (RSS), Singh, R. and Kumari, A. (2023) proposed some novel classes of estimators using RSS to evaluate the population mean utilizing additional information on an auxiliary variable. Another paper also suggested by Singh, R. and Kumari, A. (2023) in which some improved estimators of population mean using auxiliary variables developed in RSS.

Corresponding author: Nirupama Sahoo
E-mail address: nirustatistics@gmail.com

2. NOTATIONS

We use the following notations to obtain the Bias and Mean Square Error of estimators

$$\bar{y}_{[rss]} = \bar{Y}(1+e_0), \quad \bar{x}_{(rss)} = \bar{X}(1+e_1)$$

such that

$$E(e_0) = E(e_1) = 0$$

and

$$E(e_0^2) = \gamma C_y^2 - W_y^2, \qquad E(e_1^2) = \gamma C_x^2 - W_x^2,$$

$$E(e_0 e_1) = \gamma\rho C_y C_x - W_{yx}$$

where

$$W_{yx} = \frac{1}{m^2 r\bar{X}\bar{Y}}\sum_{i=1}^{m}\tau_{yx(i)}, \; W_x^2 = \frac{1}{m^2 r\bar{X}^2}\sum_{i=1}^{m}\tau_{x(i)}^2,$$

$$W_y^2 = \frac{1}{m^2 r\bar{Y}^2}\sum_{i=1}^{m}\tau_{y[i]}^2 \quad \tau_{x(i)} = \left(\mu_{x(i)} - \bar{X}\right),$$

$$\tau_{y[i]} = \left(\mu_{y[i]} - \bar{Y}\right), \; \tau_{yx(i)} = \left(\mu_{y[i]} - \bar{Y}\right)\left(\mu_{x(i)} - \bar{X}\right),$$

$$C_{yx} = \rho C_y C_x .$$

C_y and C_x are coefficient of variations of Y and X respectively.

3. LIST OF EXISTING ESTIMATORS

Let $\left(y_{[i]}, x_{(i)}\right)$ be the i th judgment ordering in the i th set for the study variable Y, based on the i th order statistics of the i th set of the auxiliary variable X at the j th cycle. Based on RSS, the sample mean estimator $\bar{y}_{RSS}$ of the population mean $\left(\bar{Y}\right)$ is given by

$$\bar{y}_{RSS} = \bar{y}_{[rss]}, \tag{1}$$

where

$$\bar{y}_{[rss]} = \left(\frac{1}{mr}\right)\sum_{j=1}^{r}\sum_{i=1}^{m} y_{[i]j}$$

The variance of $\bar{y}_{RSS}$ under RSS scheme is given by

$$Var(\bar{y}_{RSS}) = \bar{Y}^2\left(\gamma C_y^2 - W_y^2\right). \tag{2}$$

Samawi, H. M. and Muttlak, M.A (1996) proposed an estimator of the population ratio $R = \frac{\bar{Y}}{\bar{X}}$ under RSS as

$$\hat{R}_{RSS} = \frac{\bar{y}_{[rss]}}{\bar{x}_{(rss)}} \tag{3}$$

When population mean $\left(\bar{X}\right)$ of the auxiliary variable (X) is known and the variables Y and X are positively correlated, Kadilar, C., Unyazici, Y. and Cingi, H (2009) suggested the ratio estimator for population mean $\left(\bar{Y}\right)$ based on RSS as

$$\bar{y}_{rRSS} = \frac{\bar{y}_{[rss]}}{\bar{x}_{(rss)}}\bar{X} \tag{4}$$

The bias and MSE of $\bar{y}_{rRSS}$ up to the first order of approximation are given by

$$Bias(\bar{y}_{rRSS}) = \bar{Y}\left[\gamma\left(C_x^2 - \rho C_y C_x\right) - \left(W_x^2 - W_{yx}\right)\right], \tag{5}$$

and

$$MSE(\bar{y}_{rRSS}) = \bar{Y}^2\left[\gamma\left(C_y^2 + C_x^2 - 2\rho C_y C_x\right) - \left(W_y^2 + W_x^2 - 2W_{yx}\right)\right]. \tag{6}$$

When population mean $\left(\bar{X}\right)$ of the auxiliary variable (X) is known and the variables Y and X are negatively correlated, the product estimator based on RSS is defined as

$$\bar{y}_{pRSS} = \bar{y}_{[rss]}\frac{\bar{x}_{(rss)}}{\bar{X}} \tag{7}$$

The bias and MSE of $\bar{y}_{pRSS}$ up to the first degree of approximation are given by

$$Bias(\bar{y}_{pRSS}) = \bar{Y}\left(\gamma\rho C_y C_x - W_{yx}\right), \tag{8}$$

and

$$MSE(\bar{y}_{pRSS}) = \bar{Y}^2\left[\gamma\left(C_y^2 + C_x^2 + 2\rho C_y C_x\right) - \left(W_y^2 + W_x^2 + 2W_{yx}\right)\right]. \tag{9}$$

Searls, D.T. (1964) suggested an estimator under RSS as

$$\bar{y}_{sRSS} = \lambda\bar{y}_{[rss]}, \tag{10}$$

where λ is suitably chosen constant.

$$\lambda_{(opt)} = \frac{1}{\left(1 + C_y^2 - W_y^2\right)}$$

The minimum Bias and MSE of $\bar{y}_{sRSS}$ at optimum value of λ are given by

$$Bias\left(\bar{y}_{sRSS}\right)_{min} = -\frac{\bar{Y}\left(\gamma C_y^2 - W_y^2\right)}{\left(1+\gamma C_y^2 - W_y^2\right)} \tag{11}$$

$$MSE\left(\bar{y}_{sRSS}\right)_{min} \cong \frac{\bar{Y}^2\left(\gamma C_y^2 - W_y^2\right)}{\left(1+\gamma C_y^2 - W_y^2\right)} \tag{12}$$

The difference-type of estimator for population mean ($\bar{Y}$) based on RSS is given by

$$\bar{y}_{d(RSS)} = \bar{y}_{[rss]} + d\left(\bar{X} - \bar{x}_{(rss)}\right), \tag{13}$$

where d is a constant and

$$d_{opt} = \frac{R\left(\gamma C_{yx} - W_{yx}\right)}{\left(\gamma C_x^2 - W_x^2\right)}$$

The minimum variance of $\bar{y}_{d(RSS)}$ at optimum value of d is given as

$$Var\left(\bar{y}_{dRSS}\right)_{min} = \bar{Y}^2\left[\gamma C_y^2 - W_y^2 - \frac{\left(\gamma C_{yx} - W_{yx}\right)^2}{\left(\gamma C_x^2 - W_x^2\right)}\right]. \tag{14}$$

Following Stokes, S. L. (1977), Singh, H. P., Tailor, R. and Singh, S.(2014) suggested a class of estimator of the population mean $\bar{Y}$ based on RSS as

$$\bar{y}_{s(RSS)} = \lambda_1\bar{y}_{[rss]} + \lambda_2\bar{y}_{[rss]}\left[\frac{a\bar{X}+b}{\alpha\left(a\bar{x}_{(rss)}+b\right)\left(1-\alpha\right)\left(a\bar{x}+b\right)}\right]^g, \tag{15}$$

where α is a suitably chosen constant, a and b are either real numbers or function of known parameters of the auxiliary variable X, g is a scalar which takes value of 1 (for generating ratio-type estimators) and -1 (for generating product-type estimators) and (λ_1, λ_2) are constants whose sums need to be unity.

$$Bias(\bar{y}_{s(RSS)}) = \bar{Y}\left[(\lambda_1+\lambda_2-1) + \lambda_2 g\alpha^2\theta^2\left(\frac{g+1}{2}\right)\left(\gamma C_x^2 - W_x^2\right) - \lambda_2 g\alpha\theta\left(\gamma\rho C_y C_x - W_{yx}\right)\right] \tag{16}$$

The MSE of $\bar{y}_{s(RSS)}$ to the first degree of approximation is given as

$$MSE\left(\bar{y}_{s(RSS)}\right) \cong \bar{Y}^2\left[1+\lambda_1^2\left(A_s - A_w\right) + \lambda_2^2\left(B_s - B_w\right) + 2\lambda_1\lambda_2\left(C_s - C_w\right) - 2\lambda_1 - 2\lambda_2\left(D_s - D_w\right)\right], \tag{17}$$

where

$A_s = \left(1+\gamma C_y^2\right)$, $A_w = W_y^2$,

$B_s = 1+\gamma\left\{\left(C_y^2 + g\left(2g+1\right)\theta^2\alpha^2 C_x^2 - 4g\alpha\theta C_{yx}\right)\right\}$,

$B_w = W_y^2 + g\left(2g+1\right)\theta^2\alpha^2 W_x^2 - 4g\alpha\theta W_{yx}$,

$C_s = 1+\gamma\left(C_y^2 - 2g\alpha\theta C_{yx} + \frac{g\left(g+1\right)}{2}\theta^2\alpha^2 C_x^2\right)$,

$C_w = W_y^2 - 2g\theta\alpha C_{yx} + \frac{g\left(g+1\right)}{2}\theta^2\alpha^2 W_x^2$,

$D_s = 1+\gamma\left\{\frac{g\left(g+1\right)}{2}\theta^2\alpha^2 C_x^2 - g\theta\alpha C_{yx}\right\}$,

$D_w = \frac{g\left(g+1\right)}{2}\theta^2\alpha^2 W_x^2 - g\theta\alpha W_{yx}$,

Here we discuss two cases:

Case-1

Sum of weights is unity (i.e, $\lambda_1 + \lambda_2 = 1$)

Solving (17), the optimum value of λ_1 is

$$\lambda_{1(opt)} = \frac{\left[1+\left(B_s - B_w\right) - \left(C_s - C_w\right) - \left(D_s - D_w\right)\right]}{\left[\left(A_s - A_w\right) + \left(B_s - B_w\right) - 2\left(C_s - C_w\right)\right]}.$$

Substituting the value of $\lambda_{1(opt)}$ in (17), we get the minimum MSE of $\bar{y}_{s(RSS)}$ as

$$MSE\left(\bar{y}_{s(RSS)1}\right)_{min} \cong \bar{Y}^2\left[1+\left(B_s - B_w\right) - 2\left(D_s - D_w\right) - \frac{\left(1+B_s - B_w - C_s + C_w - D_s + D_w\right)^2}{\left(A_s - A_w\right)\left(B_s - B_w\right) - 2\left(C_s - C_w\right)^2}\right]. \tag{18}$$

Case-2

Sum of weights is flexible (i.e., $\lambda_1 + \lambda_2 \neq 1$)

Solving (17), the optimum values of λ_1 and λ_2 are given by

$$\lambda_{1(opt)} = \frac{\left[\left(B_s - B_w\right) - \left(C_s - C_w\right)\left(D_s - D_w\right)\right]}{\left[\left(A_s - A_w\right)\left(B_s - B_w\right) - \left(C_s - C_w\right)^2\right]}$$

and

$$\lambda_{2(opt)} = \frac{\left[\left(A_s - A_w\right)\left(D_s - D_w\right) - \left(C_s - C_w\right)\right]}{\left[\left(A_s - A_w\right)\left(B_s - B_w\right) - \left(C_s - C_w\right)^2\right]}.$$

By substituting the optimum values of λ_1 and λ_2 in equation (17), we get

$$MSE\left(\bar{y}_{s(RSS)2}\right)_{min} \cong \bar{Y}^2\left[1 - \frac{\left\{\begin{array}{c}\left(B_s - B_w\right) - 2\left(C_s - C_w\right)\left(D_s - D_w\right) + \\ \left(A_s - A_w\right)\left(D_s - D_w\right)^2\end{array}\right\}}{\left\{\begin{array}{c}\left(A_s - A_w\right)\left(B_s - B_w\right) - \\ \left(C_s - C_w\right)^2\end{array}\right\}}\right]. \quad (19)$$

Khan. L and Shabbir. J (2016) proposed a class of estimators of the population mean $\left(\bar{Y}\right)$ under RSS as

$$\bar{y}_{L(RSS)} = \left[k_1\bar{y}_{[rss]} + k_2\left(\bar{X} - \bar{x}_{(rss)}\right)\right] \left[\alpha\left\{exp\frac{\left(a\bar{x} + b\right) - \left(a\bar{x}_{(rss)} + b\right)}{\left(a\bar{x} + b\right) + \left(a\bar{x}_{(rss)} + b\right)}\right\} + \left(1 - \alpha\right)\frac{\left(a\bar{x} + b\right)}{\left(a\bar{x}_{(rss)} + b\right)}\right]$$

The Bias of $\bar{y}_{L(RSS)}$ is given as below

$$Bias\left(\bar{y}_{L(RSS)}\right) = \bar{Y}\left(k_1 - 1\right) + k_1\bar{Y}\theta^2\left(1 - \frac{5\alpha}{8}\right)\left(\gamma C_x^2 - W_x^2\right) - k_1\bar{Y}\theta\left(1 - \frac{\alpha}{2}\right)\left(\gamma C_{yx} - W_{yx}\right) + k_2\bar{X}\theta\left(1 - \frac{\alpha}{2}\right)\left(\gamma C_x^2 - W_x^2\right), \quad (20)$$

and

$$MSE\left(\bar{y}_{L(RSS)}\right) = \bar{Y}^2\left(k_1 - 1\right)^2 + k_1^{\,2}\left(E_S - E_W\right) + k_2^{\,2}\left(F_S - F_W\right) + 2k_1\left(k_1 - 1\right)\left(G_S - G_W\right) + 2k_2\left(k_1 - 1\right)\left(H_S - H_W\right) + 2k_1k_2\left(I_S - I_W\right) \quad (21)$$

Here we have considered one case only.

Case-1

Sum of weights is unity (i.e, $k_1 + k_2 = 1$)

The optimum value of k_1 is given by

$$k_{1(opt)} = \frac{\left[\begin{array}{c}\bar{Y}^2 + \left(F_s - F_w\right) + \left(G_s - G_w\right) - \\ 2\left(H_s - H_w\right) - \left(I_s - I_w\right)\end{array}\right]}{\left[\begin{array}{c}\bar{Y}^2 + \left(E_s - E_w\right) + \left(F_s - F_w\right) + 2\left(G_s - G_w\right) \\ -2\left(H_s - H_w\right) - 2\left(I_s - I_w\right)\end{array}\right]}.$$

So the minimum MSE of $\bar{y}_{L(RSS)}$ is given by

$$\left(\bar{y}_{L(RSS)1}\right)_{min} \cong \frac{\begin{array}{c}\left(E_s - E_w\right)\left\{\bar{Y}^2 - 2\left(H_s - H_w\right) + \left(F_s - F_w\right)\right\} - \\ \left\{\left(I_s - I_w\right) - \left(G_s - G_w\right)\right\}^2\end{array}}{\left[\begin{array}{c}\bar{Y}^2 + \left(E_s - E_w\right) + \left(F_s - F_w\right) + \\ 2\left(G_s - G_w\right) - 2\left(H_s - H_w\right) - 2\left(I_s - I_w\right)\end{array}\right]}. \quad (22)$$

4. PROPOSED ESTIMATOR

We have developed a ratio-type exponential estimator under ranked set sampling by using the estimators mentioned before, which is provided as

$$\bar{y}_{RSS}^{Re} = \left[k_1\bar{y}_{[rss]} + k_2\left(\bar{X} - \bar{x}_{(rss)}\right)\right]\left[\alpha\left(\frac{\left(a\bar{X} + b\right)}{\left(a\bar{x}_{(rss)} + b\right)}\right) + \left(1 - \alpha\right)exp\left(\frac{a\left(\bar{X} - \bar{x}_{(rss)}\right)}{a\left(\bar{X} + \bar{x}_{(rss)}\right) + 2b}\right)\right], \quad (23)$$

where α is a suitably chosen constant, a and b are either real number or function of known parameters of the auxiliary variables X. The proposed estimator $\bar{y}_{RSS}^{Re}$ can be written in terms of e_0 and e_1 as

$$\bar{y}_{RSS}^{Re} = \left[k_1\bar{Y}\left(1 + e_0\right) - k_2\bar{X}e_1\right]\left[\alpha\left(\frac{1}{1 + \theta e_1}\right) + \left(1 - \alpha\right)exp\left(\frac{-\theta e_1}{2 + \theta e_1}\right)\right],$$

where $\theta = \dfrac{a\bar{X}}{a\bar{X} + b}$

The bias of the proposed estimator $\bar{y}_{RSS}^{Re}$ is given by

$$Bias\left(\bar{y}_{RSS}^{Re}\right) = \bar{Y}\left(K_1 - 1\right) - \frac{1}{2}k_1\bar{Y}\left(1 + \alpha\right)\theta\left(\gamma\rho C_y C_x - W_{yx}\right) + k_1\bar{Y}\left(\frac{3}{8} + \frac{5}{8}\alpha\right)\theta^2\left(\gamma C_x^2 - W_x^2\right)\frac{1}{2}k_2\bar{X}\left(1 + \alpha\right)\theta\left(\gamma C_x^2 - W_x^2\right). \quad (24)$$

The Mean square error of the proposed estimator $\bar{y}_{RSS}^{Re}$ is given by

$$MSE\left(\bar{y}_{RSS}^{Re}\right) = \bar{Y}^2\left(k_1-1\right)^2 + k_1^2\left(E_A-E_B\right) + k_2^2\left(F_A-F_B\right) + k_1\left(k_1-1\right)\left(G_A-G_B\right) + k_2\left(k_1-1\right)\left(H_A-H_B\right) + 2k_1k_2\left(I_A-I_B\right), \tag{25}$$

where

$$E_A = \bar{Y}^2\left[\gamma\left\{C_y^2 + \frac{1}{4}(1+\alpha)^2\theta^2C_x^2 - (1+\alpha)\theta\rho C_yC_x\right\}\right],$$

$$E_B = \bar{Y}^2\left[W_y^2 + \frac{1}{4}(1+\alpha)^2\theta^2W_x^2 - (1+\alpha)\theta W_{yx}\right],$$

$$F_A = \gamma C_x^2\bar{X}^2,\ F_B = W_x^2\bar{X}^2,$$

$$G_A = \bar{Y}^2\left[\gamma\left\{\left(\frac{3}{4}+\frac{5}{4}\alpha\right)\theta^2C_x^2 - (1+\alpha)\theta\rho C_yC_x\right\}\right],$$

$$G_B = \bar{Y}^2\left[\left\{\left(\frac{3}{4}+\frac{5}{4}\alpha\right)\theta^2W_x^2 - (1+\alpha)\theta W_{yx}\right\}\right],$$

$$H_A = \overline{YX}\left[(1+\alpha)\theta\gamma C_x^2\right],\ H_A = \overline{YX}\left[(1+\alpha)\theta W_x^2\right],$$

$$I_A = \overline{YX}\left[\gamma\left\{\frac{1}{2}(1+\alpha)\theta C_x^2 - \rho C_yC_x\right\}\right],$$

$$I_B = \overline{YX}\left[\frac{1}{2}(1+\alpha)\theta W_x^2 - W_{yx}\right].$$

Here, we have considered two cases. These are given below.

Case-1

Sum of weights is unity (i.e., $k_1 + k_1 = 1$)

The optimum value of k_1 is given by

$$k_{1(opt)} = \frac{\left[\begin{array}{c}2\left\{\bar{Y}^2 + \left(F_A-F_B\right) - \left(I_A-I_B\right)\right\} + \\ \left(G_A-G_B\right)\left(H_A-H_B\right)\end{array}\right]}{2\left[\begin{array}{c}\bar{Y}^2 + \left(E_A-E_B\right) + \left(F_A-F_B\right) + \\ \left(G_A-G_B\right) - \left(H_A-H_B\right) - 2\left(I_A-I_B\right)\end{array}\right]},$$

So the minimum MSE of $\bar{y}_{RSS}^{Re}$ is given by

$$MSE\left(\bar{y}_{RSS1}^{Re}\right)_{min} = \bar{Y}^2\left(k_{1(opt)}-1\right)^2 + k_{1(opt)}^2\left(E_A-E_B\right) + \left(1-k_{1(opt)}\right)^2\left(F_A-F_B\right) + k_{1(opt)}\left(k_{1(opt)}-1\right)\left(G_A-G_B\right) + \left(1-k_{1(opt)}\right)\left(k_{1(opt)}-1\right)\left(H_A-H_B\right) + 2k_{1(opt)}\left(1-k_{1(opt)}\right)\left(I_A-I_B\right). \tag{26}$$

Case-2

For $k_1 + k_2 \neq 1$

$$k_{1(opt)} = \frac{\begin{array}{c}2\left(F_A-F_B\right)\left\{2\bar{Y}^2 + \left(G_A-G_B\right)\right\} - \\ \left(H_A-H_B\right)^2 - \left(H_A-H_B\right)\left(I_A-I_B\right)\end{array}}{\begin{array}{c}4\left(F_A-F_B\right)\left\{\bar{Y}^2 + \left(E_A-E_B\right) + \left(G_A-G_B\right)\right\} - \\ \left\{\left(H_A-H_B\right)^2 + 4\left(H_A-H_B\right)\left(I_A-I_B\right) + 2\left(I_A-I_B\right)^2\right\}\end{array}},$$

$$k_{2(opt)} = \frac{\begin{array}{c}\left(H_A-H_B\right)\left[2\left\{\bar{Y}^2 + \left(E_A-E_B\right) + \left(G_A-G_B\right)\right\}\right] - \\ \left\{2\bar{Y}^2 + \left(G_A-G_B\right)\right\}\left\{\left(H_A-H_B\right) + 2\left(I_A-I_B\right)\right\}\end{array}}{\begin{array}{c}4\left(F_A-F_B\right)\left\{\bar{Y}^2 + \left(E_A-E_B\right) + \left(G_A-G_B\right)\right\} - \\ \left\{\left(H_A-H_B\right)^2 + 4\left(H_A-H_B\right)\left(I_A-I_B\right) + 4\left(I_A-I_B\right)^2\right\}\end{array}}.$$

Substituting the optimum values of k_1 and k_2 in equation (25), we get

$$MSE\left(\bar{y}_{RSS2}^{Re}\right)_{min} = \bar{Y}^2\left(k_{1(opt)}-1\right)^2 + k_{1(opt)}^2\left(E_A-E_B\right) + k_{2(opt)}^2\left(F_A-F_B\right) + k_{1(opt)}\left(k_{1(opt)}-1\right)\left(G_A-G_B\right) + k_{2(opt)}\left(k_{1(opt)}-1\right)\left(H_A-H_B\right) + 2k_{1(opt)}k_{2(opt)}\left(I_A-I_B\right). \tag{27}$$

5. SIMULATION STUDY

To investigate the performances of the estimators a simulation study has been carried out.

Population (Source: [Lohr, S.(1999)])

$Y =$ Number of acres devoted to farms during 1992 (ACRES92)

$X =$ Number of large farms during 1992 (LARGEF92)

$N = 3059, \rho_{yx} = 0.677428$,

$\bar{Y} = 308582.4,\ \bar{X} = 56.5$, $S_y = 425312.8, S_x = 72.3$

We set $r = 10$ and $m = 5$ to select a sample of $n = mr = 50$ units from the population of size $N = 3059$. In this study we have considered the value of W_y^2, W_x^2 and W_{yx} which are simulated by [Kha, L. and Shabbir, J. (2016)] using an appropriate simulation methodology with the help of R software.

The following graph represents the PRE of proposed as well as existing estimators.

Table 1. PRE of proposed estimator through Simulation Study

a	b	g	$\propto$	R(0, 1)	R(0, 2)	R(0, 3)	R(0, 4)	R(0, 5)	R(0, 6)	R(0, 7)	$w^2_{x(i)}$	$w^2_{y(i)}$	$w^2_{xy(i)}$
-1.5	-1.5	-1	0.1	140.6	103.2	160.9	161.4	153.2	160.9	**165.4**	0.00573	0.00574	0.00573
-1.5	-1.5	-1	0.5	139.3	103.2	159.9	160.5	163.8	163.7	**164.4**	0.00590	0.00604	0.00596
-1.5	-1.5	-1	0.9	148.1	103.4	167.1	167.5	165.5	161.0	**170.8**	0.00462	0.00404	0.00431
-1.5	-1.5	1	0.1	144.5	103.3	164.1	164.8	157.3	163.3	**168.7**	0.00516	0.00485	0.00499
-1.5	-1.5	1	0.5	132.4	103	154.5	156.8	157.5	157.5	**158.7**	0.00689	0.00764	0.00725
-1.5	-1.5	1	0.9	144.6	103.3	164.2	168.6	163.4	157.3	**167.9**	0.00514	0.00482	0.00497
-1.5	0	-1	0.1	136.9	103.1	158	158.6	148	159.3	**162.4**	0.00625	0.00658	0.00641
-1.5	0	-1	0.5	137.6	103.1	158.6	159.2	162	161.9	**163.0**	0.00615	0.00642	0.06280
-1.5	0	-1	0.9	142.5	103.3	162.4	162.9	162.7	153.7	**166.0**	0.00546	0.00530	0.00538
-1.5	0	1	0.1	130.1	103	152.8	153.7	141	150.8	**151.4**	0.00520	0.00816	0.00766
-1.5	0	1	0.5	140.9	103.2	161.2	163.5	164.9	164.9	**165.6**	0.00568	0.00567	0.00567
-1.5	0	1	0.9	137.2	103.1	158.3	162.7	159.5	148.3	**161.8**	0.00620	0.00651	0.00635
-1.5	1.5	-1	0.1	140.8	103.2	161.1	161.6	158.5	162.2	**165.6**	0.00569	0.00570	0.00569
-1.5	1.5	-1	0.5	140.3	103.2	160.7	161.2	164.1	164.0	**165.3**	0.00576	0.00582	0.00578
-1.5	1.5	-1	0.9	135.2	103.1	156.7	157.3	158.7	144.6	**160.1**	0.00649	0.00697	0.00673
-1.5	1.5	1	0.1	138.2	103.2	159.1	159.9	147.8	160.7	**163.6**	0.00605	0.00629	0.00616
-1.5	1.5	1	0.5	139.2	103.2	159.8	162.2	163	162.9	**164.3**	0.00592	0.00602	0.00598
-1.5	1.5	1	0.9	143.3	103.3	163.1	168	163.9	153.0	**166.8**	0.00533	0.00513	0.00522
1.5	-1.5	-1	0.1	133.4	103.1	155.4	156	142.9	157.6	**159.6**	0.00672	0.00743	0.00706
1.5	-1.5	-1	0.5	140.8	103.2	161.1	161.6	164.5	163.9	**165.0**	0.00569	0.00578	0.00576
1.5	-1.5	-1	0.9	140.3	103.2	160.8	161.3	162	150.1	**164.4**	0.00575	0.00578	0.00576
1.5	-1.5	1	0.1	142.3	103.2	162.4	163.1	152.1	163.3	**167.0**	0.00546	0.00540	0.00541
1.5	-1.5	1	0.5	145.3	103.3	164.7	167.1	168.7	168.7	**169.4**	0.00504	0.00467	0.00484
1.5	-1.5	1	0.9	139.1	103.2	159.9	164.3	161.1	148.8	**163.4**	0.00592	0.00605	0.00598
1.5	0	-1	0.1	133.4	103	155.4	156	144.4	161.2	**164.7**	0.00672	0.00743	0.00658
1.5	0	-1	0.5	140.8	103.2	161.1	161.6	164.8	165.1	**165.8**	0.00569	0.00568	0.00566
1.5	0	-1	0.9	145.9	103.3	165.2	165.6	164.8	157.3	**169.0**	0.00496	0.00453	0.00473
1.5	0	1	0.1	142.3	103.3	162.4	163.1	153.6	162.2	**167.0**	0.00545	0.00540	0.00540
1.5	0	1	0.5	141.6	103.2	161.8	164.1	165.6	165.6	**166.3**	0.00557	0.00551	0.00553
1.5	0	1	0.9	140.3	103.2	160.7	165.1	161.4	151.5	**164.3**	0.00576	0.00582	0.00578
1.5	1.5	-1	0.1	139.2	103.2	159.9	160.4	151.8	163.7	**164.3**	0.00591	0.00605	0.00597
1.5	1.5	-1	0.5	133	103	155.1	155.7	158.1	158.0	**159.1**	0.00679	0.00749	0.00713
1.5	1.5	-1	0.9	137.3	103.1	158.4	158.9	159	149.8	**161.8**	0.00619	0.00650	0.00634
1.5	1.5	1	0.1	141.7	103.2	161.9	162.4	154.4	164.3	**169.9**	0.00555	0.00551	0.00520
1.5	1.5	1	0.5	142.3	103.3	162.3	164.6	166.4	166.4	**166.8**	0.00548	0.00534	0.00540
1.5	1.5	1	0.9	135.2	103.1	156.8	161	157.8	147.7	**160.2**	0.00648	0.00701	0.00672

A comparison has been made between the percentage relative efficiency of the proposed and existing estimators. In this case, $\bar{y}_{rRSS} = E_1$, the searls estimator $\bar{y}_{sRSS} = E_2$, the difference estimator $\bar{y}_{dRSS} = E_3$, $\bar{y}_{S(RSS)} = E_4$ when $(\lambda_1 + \lambda_2 \neq 1)$ and $\bar{y}_{L(RSS)1} = E_5$ when $(k_1 + k_1 = 1)$ have been taken into consideration in relation to the standard unbiased estimator $\bar{y}_{(RSS)} = E_0$.

The percent relative efficiency of proposed class of estimators $\bar{y}^{Re}_{RSS1} = E_6$ and $\bar{y}^{Re}_{RSS2} = E_7$ has also been calculated when $(k_1 + k_1 = 1)$ and $(k_1 + k_2 \neq 1)$, respectively, with respect to $\bar{y}_{RSS} = E_0$.

The percent relative efficiency of proposed as well as traditional estimators with respect to conventional estimator is defined as

$$PRE\left(E_0, E_i\right) = \frac{MSE\left(E_0\right)}{MSE\left(E_i\right)} \times 100,\ i = 1,2,3,4,5,6,7 \quad (28)$$

The value of PRE (0, 1) increases from 140.6 to 148.1 as the value of α changes from 0.1 to 0.9, and abate slightly when α close to 0.5. The decorations of the PRE value remain the same in the results obtained by R programming. It is demonstrated that the suggested estimator is efficient when α is near to 1.

6. CONCLUSION

To assess the population mean, this work presents a ratio-type exponential estimator under RSS. It is evident from Table 1 that the suggested estimator outperforms the current estimators in terms of efficiency. It is demonstrated that as α value shifts from 0.1 to 0.9, percent relative efficiency rises to 170.8, and when α value approaches 0.5, it significantly falls. The second scenario ($k_1 + k_2 \neq 1$) of our suggested estimator is more efficient than the first instance ($k_1 + k_1 = 1$). Considering this, we recommend surveying expounder for population mean estimate in applications using ranked set sampling with the proposed ratio-type exponential estimator.

REFERENCES

Bouza, C.N., Singh, P., and Singh, R. (2018). Ranked set sampling and optional scrambling randomized response modeling. *Revista Investigacion Operacional*. **39**(1), 100-107.

Bhusan, S., Kumar, A., Shahab, S., Lone, S. A. and Akhtar, T. (2022). On Efficient

Estimation of the Population Mean under Stratified Ranked Set Sampling, Hindawi Journal of Mathematics, 01-20.

Dell, T. and Clutter, J. (1972). Ranked Set Sampling Theory with Order Statistics Background. *Biometrics*, 545-555.

Kadilar, C., Unyazici, Y. and Cingi, H. (2009). Ratio Estimator for the Population Mean Using Ranked Set Sampling. *Statistical Papers*, 50, 301-309.

Khan, L. and Shabbir, J. (2016). An efficient Class of Estimators for the finite population mean in Ranked set Sampling. *Open Journal of Statistics*, 6, 426-435

Khan, L. and Shabbir, J. (2015). A Class of Hartley-Ross Type Unbiased Estimators for Population Mean Using Ranked Set Sampling. *Hacettepe Journal of Mathematics and Statistics*.

Khan, L. and Shabbir, J. (2015). A Class of Hartley-Ross Type Unbiased Estimators for Population Mean Using Ranked Set Sampling. Hacettepe Journal of Mathematics and Statistics.

Khan, L. and Shabbir, J. (2016). Hartley-Ross Type Unbiased Estimators Using Ranked Set Sampling and Stratified Ranked Set Sampling. *North Carolina Journal of Mathematics and Statistics*, 2, 10-22.

Khan, L. and Shabbir, J. (2016). An efficient class of estimators for the finite population Mean in Ranked Set Sampling. Open *Journal of Statistics*, 6, 426-435.

Khoshnevisan, M., Singh, R., Chauhan, P., Sawan, N. and Smarandache, F. (2007). A General Family of Estimators for Estimating Population Mean Using Known Value of Some Population Parameter(s). *Far East Journal of Theoretical Statistics*, 22, 181-191.

Lohr, S. (1999): Sampling Design and Analysis. Duxbury Press, Boston.

Mclntyre, G. (1952). A Method for Unbiased Selective Sampling, Using Ranked Sets. Crop and Pasture Science, 3, 385-390.

Samawi, H.M. and Muttlak, M.A. (1996). Estimation of Ratio Using Ranked Set Sampling. Biometrical Journal, 38, 753-764.

Searls, D.T. (1964). The Utilization of a Known Coefficient of Variation in the Estimation Procedure. *Journal of the American Statistical Association*, 59, .

Singh, H.P., Tailor, R. and Singh, S. (2014):. General Procedure for Estimating the Population Mean Using Ranked Set Sampling. *Journal of Statistical Computation and Simulation*, 84, 931-945

Singh, R. and Kumari, A. (2023). Improved Estimators of Population Mean Using Auxiliary Variables in Ranked Set Sampling. *Revista Investigacion Operacional*, **44(2)**, 271-280.

Singh, R. and Kumari, A. (2023). Generalized Class of Some Novel Estimators under Ranked Set Sampling. *Journal of the Indian Society of Agricultural Statistics*, **77(1)**, 45-53.

Stokes, S.L. (1977). Ranked Set Sampling with Concomitant Variables. *Communication in Statistics*: Theory and Methods, 6, 1207-1211.

Takahasi, K. and Wakimoto, K. (1968). On Unbiased Estimates of the Population Mean Based on the Sample Stratified by Means of Ordering. Annals of the Institute of Statistical Mathematics, 20, 1-31.