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Generalized-Type Calibration Estimator of Population Mean in Stratified Random Sampling

Manoj Kumar Chaudhary, Basant Kumar Ray and G.K. Vishwakarma
Banaras Hindu University, Varanasi

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SUMMARY

In the present paper, we have suggested a generalized-type calibration estimator for estimating the population mean in stratified random sampling. We have pioneered out the generalized-type calibration estimator using chi-square type distance function subject to the calibration constraints based on a single auxiliary variable. The new set of stratum weights is chosen such that the chi-square type distance would be minimized under the given calibration constraints. One can derive a number of existing calibration estimators and some new calibration estimators of the mean of a stratified population using the suggested generalized-type calibration estimator. Some special cases of the suggested generalized-type calibration estimator have been discussed in detail. A simulation study has also been carried out to strengthen the performance of the suggested generalized-type calibration estimator

Keywords: Generalized-type calibration estimator; Population mean; Stratified random sampling, Auxiliary information; Chi-square distance.

1. INTRODUCTION

An estimation method can be improved by the use of suitable auxiliary information. There are some well-known methods of incorporating the auxiliary information, viz., ratio method of estimation, product method of estimation, regression method of estimation, combined ratio-type of estimation, unbiased ratio-type of estimation, etc. The calibration approach also incorporates the auxiliary information in estimating the population parameters and improves the estimation method. A calibration estimator is derived using some distance functions under different constraints. The distance between improved calibrated weights and design weights must be least as possible. Deville and Särndal (1992) were the first who use the calibration approach to estimate the population total. There are some other researchers such as Singh (1998), Särndal and Estevao (2000), Wu and Sitter (2001), Tracy *et al.* (2003), Särndal (2007), Kim *et al.* (2007), Kim and Park (2010), Koyuncu and Kadilar (2014), Clement (2017), Nidhi *et al.* (2017) and Özgül (2018) who have used the calibration approach in estimating the population parameters at the estimation stage.

The objective of the present study is to estimate the population mean under stratified random sampling using the calibration approach. We have tried to find out a generalized-type calibration estimator of the mean of a stratified population. To obtain the new stratum weights, we have used chi-square type distance which is minimized subject to the calibration constraints based on a single auxiliary variable. We have derived some existing calibration estimators from the suggested generalized-type calibration estimator and it has been verified that the suggested generalized-type calibration estimator works well. We have performed a simulation study to strengthen the theoretical outputs. An extensive effort has also been made where the information on some other characteristics is available.

Corresponding author: Manoj Kumar Chaudhary
E-mail address: mk15@bhu.ac.in

2. SAMPLING PROCEDURE, NOTATIONS AND DEFINITIONS

Let $U=(U_1,U_2,...,U_N)$ be a finite population of N distinct and identifiable units. Let us suppose that the population can be stratified into L strata and h^{th} stratum consists of N_h units such that $\sum_{h=1}^{L} N_h = N \; ;(h=1,2,...,L)$. Let Y and X be the characteristics under study and auxiliary respectively. Let y_{hi} and x_{hi} be the observations on the i^{th} unit in the h^{th} stratum for the study and auxiliary variables respectively $;(i=1,2,...,N_h)$. Let us draw a random sample of n_h units from the h^{th} strata using simple random sampling without replacement (SRSWOR) scheme such that $\sum_{h=1}^{L} n_h = n$. Let us now consider some usual notations and terminologies:

$\overline{Y} = \dfrac{\sum_{h=1}^{L} N_h \overline{Y}_h}{N}$: Population mean under study variable

$\overline{X} = \dfrac{\sum_{h=1}^{L} N_h \overline{X}_h}{N}$: Population mean under auxiliary variable

$\overline{Y}_h = \dfrac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$: Population mean for the h^{th} stratum under study variable

$\overline{X}_h = \dfrac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: Population mean for the h^{th} stratum under auxiliary variable

$\overline{y}_h = \dfrac{1}{n_h}\sum_{i}^{n_h} y_{hi}$: Sample mean for the h^{th} stratum under study variable

$\overline{x}_h = \dfrac{1}{n_h}\sum_{i}^{n_h} x_{hi}$: Sample mean for the h^{th} stratum under auxiliary variable

$S_{hY}^2 = \dfrac{1}{N_h-1}\sum_{i=1}^{N_h}\left(y_{hi}-\overline{Y}_h\right)^2$: Population mean square for the h^{th} stratum under study variable

$S_{hX}^2 = \dfrac{1}{N_h-1}\sum_{i=1}^{N_h}\left(x_{hi}-\overline{X}_h\right)^2$: Population mean square for the h^{th} stratum under auxiliary variable

$s_{hy}^2 = \dfrac{1}{n_h-1}\sum_{i}^{n_h}\left(y_{hi}-\overline{y}_h\right)^2$: Sample mean square for the h^{th} stratum under study variable

$s_{hx}^2 = \dfrac{1}{n_h-1}\sum_{i}^{n_h}\left(x_{hi}-\overline{x}_h\right)^2$: Sample mean square for the h^{th} stratum under auxiliary variable

3. LITERATURE REVIEW

The objective of the present research is to estimate the population mean $\overline{Y}$. The usual unbiased estimator of the population mean $\overline{Y}$ in stratified random sampling is given by

$$\overline{y}_{st} = \sum_{h=1}^{L} w_h \overline{y}_h \tag{1}$$

where $w_h = \dfrac{N_h}{N}$

In sequence of improving the estimation method, a number of estimators have been suggested for estimating the population mean $\overline{Y}$ under calibration approach. Some of the well known calibration estimators have been discussed in the subsequent subsections.

3.1 Singh *et al.* (1998) Estimator

A new calibration estimator of the population mean $\overline{Y}$, proposed by Singh *et al.* (1998) in stratified random sampling is given as

$$\overline{y}_{c,st} = \sum_{h=1}^{L} \delta_h \overline{y}_h \tag{2}$$

Here, the calibrated weight δ_h is chosen such that the chi-square type distance

$$\Delta = \sum_{h=1}^{L} \frac{\left(\delta_h - w_h\right)^2}{w_h q_h} \tag{3}$$

is minimum subject to the condition

$$\sum_{h=1}^{L} \delta_h \overline{x}_h = \overline{X} \tag{4}$$

The optimum calibrated weight δ_h under the above assumptions is represented as

$$\delta_h = w_h + \left(\frac{w_h q_h \bar{x}_h}{\sum_{h=1}^{L} w_h q_h \bar{x}_h^2} \right) \left\{ \bar{X} - \sum_{h=1}^{L} w_h \bar{x}_h \right\}; \qquad h = 1, 2, ..., L \tag{5}$$

Thus, the calibration estimator becomes

$$\bar{y}_{c,st} = \sum_{h=1}^{L} w_h \bar{y}_h + \left(\frac{\sum_{h=1}^{L} w_h q_h \bar{x}_h \bar{y}_h}{\sum_{h=1}^{L} w_h q_h \bar{x}_h^2} \right) \left\{ \bar{X} - \sum_{h=1}^{L} w_h \bar{x}_h \right\} \tag{6}$$

3.2 Nidhi *et al.* (2017) Estimator

Another calibration estimator of the population mean $\bar{Y}$, proposed by Nidhi *et al.* (2017) in stratified random sampling is given by

$$\bar{y}_{c,st}(n) = \sum_{h=1}^{L} \delta'_h \bar{y}_h \tag{7}$$

The calibrated weight δ'_h is chosen such that the chi-square type distance

$$\Delta' = \sum_{h=1}^{L} \frac{(\delta'_h - w_h)^2}{w_h q_h} \tag{8}$$

is minimum subject to the constraints

$$\sum_{h=1}^{L} \delta'_h \bar{x}_h = \bar{X} \tag{9}$$

$$\sum_{h=1}^{L} \delta'_h = 1 \tag{10}$$

The Lagrange function is defined as

$$\mu = \sum_{h=1}^{L} \frac{(\delta'_h - w_h)^2}{q_h w_h} - 2\phi_1 \left(\sum_{h=1}^{L} \delta'_h \bar{x}_h - \sum_{h=1}^{L} w_h \bar{X}_h \right) - 2\phi_2 \left(\sum_{h=1}^{L} \delta'_h - 1 \right) \tag{11}$$

where ϕ_1 and ϕ_2 are the Lagrange multipliers.

The Lagrange function given in equation (11) leads to the optimum calibrated weight δ'_h as

$$\delta'_h = w_h + w_h q_h \bar{x}_h \left(\frac{\sum_{h=1}^{L} w_h (\bar{X}_h - \bar{x}_h) \left(\sum_{h=1}^{L} w_h q_h \right)}{\left(\sum_{h=1}^{L} w_h q_h \bar{x}_h^2 \right) \left(\sum_{h=1}^{L} w_h q_h \right) - \left(\sum_{h=1}^{L} w_h q_h \bar{x}_h \right)^2} \right)$$

$$+ w_h q_h \left(\frac{-\sum_{h=1}^{L} w_h (\bar{X}_h - \bar{x}_h) \left(\sum_{h=1}^{L} w_h q_h \bar{x}_h \right)}{\left(\sum_{h=1}^{L} w_h q_h \bar{x}_h^2 \right) \left(\sum_{h=1}^{L} w_h q_h \right) - \left(\sum_{h=1}^{L} w_h q_h \bar{x}_h \right)^2} \right); \qquad h = 1, 2, ..., L$$

Thus, the final form of the calibration estimator $\bar{y}_{c,st}(n)$ is given as

$$\bar{y}_{c,st}(n) = \sum_{h=1}^{L} w_h \bar{y}_h + \frac{\left(\sum_{h=1}^{L} w_h q_h \bar{x}_h \bar{y}_h \right) \left(\sum_{h=1}^{L} w_h q_h \right) - \left(\sum_{h=1}^{L} w_h q_h \bar{y}_h \right) \left(\sum_{h=1}^{L} w_h q_h \bar{x}_h \right)}{\left(\sum_{h=1}^{L} w_h q_h \bar{x}_h^2 \right) \left(\sum_{h=1}^{L} w_h q_h \right) - \left(\sum_{h=1}^{L} w_h q_h \bar{x}_h \right)^2} \left\{ \bar{X} - \sum_{h=1}^{L} w_h \bar{x}_h \right\} \tag{12}$$

4. PROPOSED GENERALIZED-TYPE CALIBRATION ESTIMATOR

Let us now define a generalized-type calibration estimator of the population mean $\bar{Y}$ in stratified random sampling as follows:

$$\bar{y}_{st,c}(g) = \sum_{h=1}^{L} \delta^*_h \bar{y}_h \tag{13}$$

where δ^*_h is a new calibrated weight for the h^{th} stratum. The new calibrated weight δ^*_h is chosen such that the chi-square type distance

$$\Delta^* = \sum_{h=1}^{L} \frac{(\delta^*_h - w_h)^2}{w_h q_h} \tag{14}$$

is minimum subject to the calibration constraints

$$\sum_{h=1}^{L} \delta^*_h f_{1h}(x_1, x_2, ..., x_{n_h}) = \sum_{h=1}^{L} w_h g_{1h}(X_1, X_2, ..., X_{N_h}) \quad (I_1)$$

$$\sum_{h=1}^{L} \delta^*_h f_{2h}(x_1, x_2, ..., x_{n_h}) = \sum_{h=1}^{L} w_h g_{2h}(X_1, X_2, ..., X_{N_h}) \quad (I_2)$$

$$\sum_{h=1}^{L} \delta^*_h f_{kh}(x_1, x_2, ..., x_{n_h}) = \sum_{h=1}^{L} w_h g_{kh}(X_1, X_2, ..., X_{N_h}) \quad (I_k)$$

where $f_{lh}(x_1, x_2, ..., x_{n_h}); l = 1, ..., k$ are the functions of the auxiliary variable for the sample drawn from the h^{th} stratum such as mean, standard deviation, coefficient of variation, etc. The functions $g_{lh}(X_1, X_2, ..., X_{N_h}); l = 1, ..., k$ are the functions of the auxiliary variable for the population in the h^{th} stratum such as mean, standard deviation, coefficient of variation, etc.

Let us consider the Lagrange function

$$\mu^* = \sum_{h=1}^{L}\frac{\left(\delta_h^* - w_h\right)^2}{w_h q_h} - 2\sum_{l=1}^{k}\phi_l\left(\sum_{h=1}^{L}\delta_h^* f_{lh}\left(x_1, x_2, ..., x_{n_h}\right) - \sum_{h=1}^{L} w_h g_{lh}\left(X_1, X_2, ..., X_{N_h}\right)\right) \quad (15)$$

where ϕ_l's; $l = 1,...,k$ are the Lagrange multipliers.

Here, we use the mathematical theory of maxima and minima to find the optimum value of the new calibrated weight δ_h^* for which the Lagrange function given in equation (15) would become minimum. Differentiating the equation (15) with respect to δ_h^* and equating the derivative to zero, we get

$$\frac{\partial\mu^*}{\partial\delta_h^*} = \frac{\partial}{\partial\delta_h^*}\sum_{h=1}^{L}\frac{\left(\delta_h^* - w_h\right)^2}{w_h q_h} - 2\frac{\partial}{\partial\delta_h^*}\sum_{i=1}^{k}\phi_i\left(\sum_{h=1}^{L}\delta_h^* f_{ih}\left(x_1, x_2,\right) - \sum_{h=1}^{L} w_h g_{ih}\left(X_1, X_2, ..., X_{N_h}\right)\right) = 0$$

$$\Rightarrow 2\frac{\left(\delta_h^* - w_h\right)}{w_h q_h} - 2\sum_{l=1}^{k}\phi_l f_{lh}\left(x_1, x_2, ..., x_{n_h}\right) = 0$$

$$\Rightarrow \delta_h^* = w_h + w_h q_h \sum_{l=1}^{k}\phi_l f_{lh}\left(x_1, x_2, ..., x_{n_h}\right) \quad (16)$$

Let us assume $\underline{x}_h = \left(x_1, x_2, ..., x_{n_h}\right)$ and $\underline{X}_h = \left(X_1, X_2, ..., X_{N_h}\right)$. Thus, we have

$$\delta_h^* = w_h + w_h q_h \sum_{l=1}^{k}\phi_l f_{lh}\left(\underline{x}_h\right)$$

$$\Rightarrow \delta_h^* = w_h + w_h q_h\left(\phi_1 f_{1h}\left(\underline{x}_h\right) + \cdots + \phi_k f_{kh}\left(\underline{x}_h\right)\right)$$

$$\Rightarrow \delta_h^* = w_h + w_h q_h\begin{bmatrix}\phi_1 & \cdots & \phi_k\end{bmatrix}\begin{bmatrix} f_{1h}\left(\underline{x}_h\right) \\ \vdots \\ f_{kh}\left(\underline{x}_h\right)\end{bmatrix}$$

$$\Rightarrow \delta_h^* = w_h + w_h q_h \phi^T F \quad (17)$$

where $\left(\phi = \begin{bmatrix}\phi_1 \\ \vdots \\ \phi_k\end{bmatrix}\right)$ and $\left(F = \begin{bmatrix} f_{1h}\left(\underline{x}_h\right) \\ \vdots \\ f_{nh}\left(\underline{x}_h\right)\end{bmatrix}\right)$ are the column vectors.

4.1 Determination of Constants ϕ_l; $\left(l = 1, 2, ..., k\right)$

Let us put the value of δ_h^* from equation (17) into the equations $\left(I_1\right)$, $\left(I_2\right)$, …, $\left(I_k\right)$ *i.e.*

$$\sum_{h=1}^{L}\delta_h^* f_{lh}\left(x_1, x_2, ..., x_{n_h}\right) = \sum_{h=1}^{L} w_h g_{lh}\left(X_1, X_2, ..., X_{N_h}\right); \quad l = 1, 2, ..., k$$

$$\Rightarrow \sum_{h=1}^{L}\delta_h^* f_{lh}\left(\underline{x}_h\right) = \sum_{h=1}^{L} w_h g_{lh}\left(\underline{X}_h\right) \quad ; \; l = 1, 2, ..., k$$

$$\Rightarrow \sum_{h=1}^{L}\left\{w_h + w_h q_h \sum_{l=1}^{k}\phi_l f_{lh}\left(\underline{x}_h\right)\right\} f_{lh}\left(\underline{x}_h\right) = \sum_{h=1}^{L} w_h g_{lh}\left(\underline{X}_h\right); \quad l = 1, 2, ..., k$$

$$\Rightarrow \sum_{h=1}^{L} w_h q_h\left(\sum_{l=1}^{k}\phi_l f_{lh}\left(\underline{x}_h\right)\right) f_{lh}\left(\underline{x}_h\right) = \sum_{h=1}^{L} w_h g_{lh}\left(\underline{X}_h\right) - \sum_{h=1}^{L} w_h f_{lh}\left(\underline{x}_h\right) \quad l = 1, 2, ..., k$$

From the above, we get a system of equations with k constraints. The system of equations can be written as

$$\phi_1\sum_{h=1}^{L} w_h q_h f_{1h}^2\left(\underline{x}_h\right) + \cdots + \phi_k\sum_{h=1}^{L} w_h q_h f_{kh}\left(\underline{x}_h\right) f_{1h}\left(\underline{x}_h\right) = \sum_{h=1}^{L} w_h g_{1h}\left(\underline{X}_h\right) - \sum_{h=1}^{L} w_h f_{1h}\left(\underline{x}_h\right) \quad \left(E_1\right)$$

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$$\phi_1\sum_{h=1}^{L} w_h q_h f_{1h}\left(\underline{x}_h\right) f_{kh}\left(\underline{x}_h\right) + \cdots + \phi_k\sum_{h=1}^{L} w_h q_h f_{kh}^2\left(\underline{x}_h\right) = \sum_{h=1}^{L} w_h g_{kh}\left(\underline{X}_h\right) - \sum_{h=1}^{L} w_h f_{kh}\left(\underline{x}_h\right) \quad \left(E_k\right)$$

Let us now write the system of equations in the matrix form

$$\begin{bmatrix} \sum_{h=1}^{L} w_h q_h f_{1h}^2\left(\underline{x}_h\right) & \cdots & \sum_{h=1}^{L} w_h q_h f_{kh}\left(\underline{x}_h\right) f_{1h}\left(\underline{x}_h\right) \\ \vdots & \ddots & \vdots \\ \sum_{h=1}^{L} w_h q_h f_{1h}\left(\underline{x}_h\right) f_{kh}\left(\underline{x}_h\right) & \cdots & \sum_{h=1}^{L} w_h q_h f_{kh}^2\left(\underline{x}_h\right)\end{bmatrix}\begin{bmatrix}\phi_1 \\ \vdots \\ \phi_k\end{bmatrix} = \begin{bmatrix}\sum_{h=1}^{L} w_h g_{1h}\left(\underline{X}_h\right) - \sum_{h=1}^{L} w_h f_{1h}\left(\underline{x}_h\right) \\ \vdots \\ \sum_{h=1}^{L} w_h g_{kh}\left(\underline{X}_h\right) - \sum_{h=1}^{L} w_h f_{kh}\left(\underline{x}_h\right)\end{bmatrix}$$

$$\Rightarrow \quad A\phi = B \tag{18}$$

where

$$A = \begin{bmatrix} \sum_{h=1}^{L} w_h q_h f_{1h}^2\left(\underline{x}_h\right) & \cdots & \sum_{h=1}^{L} w_h q_h f_{kh}\left(\underline{x}_h\right) f_{1h}\left(\underline{x}_h\right) \\ \vdots & \ddots & \vdots \\ \sum_{h=1}^{L} w_h q_h f_{1h}\left(\underline{x}_h\right) f_{kh}\left(\underline{x}_h\right) & \cdots & \sum_{h=1}^{L} w_h q_h f_{kh}^2\left(\underline{x}_h\right) \end{bmatrix}$$

and

$$B = \begin{bmatrix} \sum_{h=1}^{L} w_h g_{1h}\left(\underline{X}_h\right) - \sum_{h=1}^{L} w_h f_{1h}\left(\underline{x}_h\right) \\ \vdots \\ \sum_{h=1}^{L} w_h g_{kh}\left(\underline{X}_h\right) - \sum_{h=1}^{L} w_h f_{kh}\left(\underline{x}_h\right) \end{bmatrix}.$$

Let us assume that the matrix A is non-singular matrix. Since the matrix A is non-singular, the inverse of matrix A exists and hence

$$A^{-1} = \frac{1}{|A|} adj(A)$$

where $|A|$ is the determinant of matrix A.

The solution of the system of equations (18) is given as

$$\phi = A^{-1}B \tag{19}$$

Thus, the suggested generalized-type calibration estimator becomes

$$\bar{y}_{st,c}(g) = \sum_{h=1}^{L} w_h \bar{y}_h + \sum_{h=1}^{L} w_h q_h \left(A^{-1}B\right)^T F\bar{y}_h \tag{20}$$

4.2 Some Special Cases

Case 1: If chi-square type distance is minimized subject to only one constraint *i.e.* $\sum_{h=1}^{L} \delta_h^* \bar{x}_h = \bar{X}$, then we have

$$f_{1h}\left(\underline{x}_h\right) = \bar{x}_h \text{ and } g_{1h}\left(\underline{X}_h\right) = \bar{X}_h .$$

In such a case, the matrices A, B and F are given as

$$A = \left[\sum_{h=1}^{L} w_h q_h \bar{x}_h^2\right]_{1\times 1}, \qquad B = \left[\bar{X} - \sum_{h=1}^{L} w_h \bar{x}_h\right]_{1\times 1} \quad \text{and}$$

$$F = \left[\bar{x}_h\right]_{1\times 1}.$$

Therefore, the deduced generalized-type calibration estimator is given by

$$\bar{y}_{st,c}(g_1) = \sum_{h=1}^{L} w_h \bar{y}_h + \sum_{h=1}^{L} w_h q_h \left\{\left(\sum_{h=1}^{L} w_h q_h \bar{x}_h^2\right)^{-1} \left(\bar{X} - \sum_{h=1}^{L} w_h \bar{x}_h\right)\right\}^T \bar{x}_h \bar{y}_h$$

$$\Rightarrow \bar{y}_{st,c}(g_1) = \sum_{h=1}^{L} w_h \bar{y}_h + \left(\frac{\sum_{h=1}^{L} w_h q_h \bar{x}_h \bar{y}_h}{\sum_{h=1}^{L} w_h q_h \bar{x}_h^2}\right)\left(\bar{X} - \sum_{h=1}^{L} w_h \bar{x}_h\right)$$

which is same as the calibration estimator proposed by Singh *et al.* (1998).

Case 2: If chi-square type distance is minimized subject to two calibration constraints *i.e.* $\sum_{h=1}^{L} \delta_h^* \bar{x}_h = \bar{X} = \sum_{h=1}^{L} w_h \bar{X}_h$ and $\sum_{h=1}^{L} \delta_h^* = 1$, then we have

$$f_{1h}\left(\underline{x}_h\right) = \bar{x}_h \text{ and } g_{1h}\left(\underline{X}_h\right) = \bar{X}_h$$

$$f_{2h}\left(\underline{x}_h\right) = 1 \text{ and } g_{2h}\left(\underline{X}_h\right) = 1$$

The matrices A, B and F are given as

$$A = \begin{bmatrix} \sum_{h=1}^{L} w_h q_h \bar{x}_h^2 & \sum_{h=1}^{L} w_h q_h \bar{x}_h \\ \sum_{h=1}^{L} w_h q_h \bar{x}_h & \sum_{h=1}^{L} w_h q_h \end{bmatrix}, \qquad B = \begin{bmatrix} \bar{X} - \sum_{h=1}^{L} w_h \bar{x}_h \\ 0 \end{bmatrix}$$

and $F = \begin{bmatrix} \bar{x}_h \\ 1 \end{bmatrix}$

Thus, the deduced generalized-type estimator becomes

$$\bar{y}_{st,c}(g_2) = \sum_{h=1}^{L} w_h \bar{y}_h + \frac{\sum_{h=1}^{L} w_h q_h \sum_{h=1}^{L} w_h q_h \bar{x}_h \bar{y}_h - \sum_{h=1}^{L} w_h q_h \bar{x}_h \sum_{h=1}^{L} w_h q_h \bar{y}_h}{|A|}\left(\bar{X} - \sum_{h=1}^{L} w_h \bar{x}_h\right)$$

$$\text{where } |A| = \left(\sum_{h=1}^{L} w_h q_h \bar{x}_h^2\right)\left(\sum_{h=1}^{L} w_h q_h\right) - \left(\sum_{h=1}^{L} w_h q_h \bar{x}_h\right)^2$$

which is the expression for the calibration estimator proposed by Nidhi *et al.* (2017).

Note: In the similar manner, one can get some other well known existing calibration estimators and some new calibration estimators of the population mean $\bar{Y}$ by minimizing the chi-square type distance Δ^* subject to the three constraints, four constraints, …, k constraints.

5. AN EXTENSION UNDER CONSIDERATION

In the previous subsections, we have pioneered out a generalized-type calibration estimator of the population mean $\overline{Y}$ under the calibration constraints based on a single auxiliary characteristic. An extension can also be carried out where $f_{lh}(\bullet)$ and $g_{lh}(\bullet)$ are the functions of some other momentous characteristics. For instance, one can consider both the cost of enumeration C and information on a single auxiliary variable X at the same time. Let $f_{ih}(\underline{x}_h, \underline{c}_h)$ be the functions of the auxiliary variable and the cost of enumeration for the sample drawn from the h^{th} stratum. Let $g_{ih}(\underline{X}_h, \underline{C}_h)$ be the functions of the auxiliary variable and the cost of enumeration for the population in the h^{th} stratum. Thus, the resulting generalized-type calibration estimator of the population mean $\overline{Y}$ is given by

$$\overline{y}_{st,c}(G) = \sum_{h=1}^{L} w_h \overline{y}_h + \sum_{h=1}^{L} w_h q_h \left(A^{*-1} B^*\right)^T F^* \overline{y}_h \qquad (21)$$

where

$$A^* = \begin{bmatrix} \sum_{h=1}^{L} w_h q_h f_{1h}^2(\underline{x}_h, \underline{c}_h) & \cdots & \sum_{h=1}^{L} w_h q_h f_{kh}(\underline{x}_h, \underline{c}_h) f_{1h}(\underline{x}_h, \underline{c}_h) \\ \vdots & \ddots & \vdots \\ \sum_{h=1}^{L} w_h q_h f_{1h}(\underline{x}_h, \underline{c}_h) f_{nh}(\underline{x}_h, \underline{c}_h) & \cdots & \sum_{h=1}^{L} w_h q_h f_{kh}^2(\underline{x}_h, \underline{c}_h) \end{bmatrix},$$

$$B^* = \begin{bmatrix} \sum_{h=1}^{L} w_h g_{1h}(\underline{X}_h, \underline{C}_h) - \sum_{h=1}^{L} w_h f_{1h}(\underline{x}_h, \underline{c}_h) \\ \vdots \\ \sum_{h=1}^{L} w_h g_{kh}(\underline{X}_h, \underline{C}_h) - \sum_{h=1}^{L} w_h f_{kh}(\underline{x}_h, \underline{c}_h) \end{bmatrix} \text{ and}$$

$$F^* = \begin{bmatrix} f_{1h}(\underline{x}_h, \underline{c}_h) \\ \vdots \\ f_{nh}(\underline{x}_h, \underline{c}_h) \end{bmatrix}.$$

6. SIMULATION STUDY

The calibration estimators proposed by Singh *et al.* (1998) and Nidhi *et al.* (2017) are the special cases of generalized calibration estimator $\overline{y}_{st,c}(g)$. Thus, we have conducted a simulation study to assess the efficiency of calibration approach based estimators derived by Singh *et al.* (1998) and Nidhi *et al.* (2017) with that of the usual estimator of the population mean. To study the efficiency of the calibration estimators, we have taken a finite population of 3100 distinct and identifiable units. This population is composed of 4 strata with strata sizes of 800, 300, 1200 and 800 units. Now, we decide to select a sample of 620 units from the population. The sample from each stratum is taken using proportional allocation scheme with $\frac{n_h}{N_h} = \frac{1}{5}$; $h = 1, 2, 3, 4.$ the procedure suggested by Reddy *et al.* (2010) has been used to generate the data for the variables Y and X. Table 1 shows the description of the distribution of the population.

Table 1. Particulars of Population

Stratum No. (h)	Stratum size (N_h)	Sample Size (n_h)	Distribution of Study Variable, *i. e.*, $Y \sim N(\mu_Y, \sigma_Y^2)$	Distribution of Auxiliary Variable, *i.e.*, $X \sim N(\mu_X, \sigma_X^2)$
I	800	160	$N(50, 25)$	$N(110, 25)$
II	300	60	$N(45, 49)$	$N(150, 49)$
III	1200	240	$N(70, 81)$	$N(125, 81)$
IV	800	160	$N(100, 36)$	$N(180, 36)$

Table 2 depicts the approximate MSE (*AMSE)* of the usual estimator $\overline{y}_{st}$ and calibration approach based estimators derived by Singh *et al.* (1998) and Nidhi *et al.* (2017). The percentage relative efficiency (PRE) of the calibration approach based estimators derived by Singh *et al.* (1998) and Nidhi *et al.* (2017) with respect to the usual estimator $\overline{y}_{st}$ has also been shown. The formulae for computing the *AMSE* are given below:

$$AMSE\left(\overline{y}_{st}\right) = \frac{1}{5000} \sum_{l=1}^{5000} \left[\overline{y}_{stl} - \overline{Y}\right]^2$$

$$AMSE\left[\overline{y}_{st,c}(g_1)\right] = \frac{1}{5000} \sum_{l=1}^{5000} \left[\overline{y}_{st,c}(g_1)_l - \overline{Y}\right]^2$$

$$AMSE\left[\overline{y}_{st,c}(g_2)\right] = \frac{1}{5000} \sum_{l=1}^{5000} \left[\overline{y}_{st,c}(g_2)_l - \overline{Y}\right]^2$$

Note: Repetition of the sample is considered as 5000.

Table 2. *AMSE* and *PRE* of Estimators $\bar{y}_{st}$, $\bar{y}_{st,c}(g_1)$ and $\bar{y}_{st,c}(g_2)$

Estimator	*AMSE*	*PRE*
$\bar{y}_{st}$	0.0705	100.00
$\bar{y}_{st,c}(g_1)$	0.0316	223.44
$\bar{y}_{st,c}(g_2)$	0.0288	244.99

7. CONCLUDING REMARKS

In the present research, we have suggested a generalized-type calibration estimator of the population mean in stratified random sampling. The expressions for the new stratum weights have been derived using the chi-square type distance measure. The new stratum weight is so chosen that it minimizes the chi-square type distance subject to the calibration constraints based on a single auxiliary variable. The generalized-type calibration estimator can produce a number of well-known existing calibration estimators and some new calibration estimators of the mean of a stratified population. The calibration estimators derived by Singh *et al.* (1998) and Nidhi *et al.* (2017) have been shown as some special cases of the suggested generalized-type calibration estimator. A simulation study has been performed to compare the efficiency of some calibration estimators considered as special cases of the suggested generalized-type calibration estimator with that of the usual estimator. Table 2 reveals that the calibration estimators $\bar{y}_{st,c}(g_1)$ and $\bar{y}_{st,c}(g_2)$ perform well as compared to the usual estimator $\bar{y}_{st}$. An extensive version of the suggested calibration-type estimator has also been pioneered out.

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