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A General Class of Modified Ratio type Estimators of Population Mean

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SUMMARY

In the present manuscript, an enhanced ratio-type estimator of population mean of the study variable has been proposed using the known parameters of the auxiliary variable. The sample was selected from the population using simple random sampling without replacement (SRSWOR). The expressions for large sample properties are obtained, up to the first order of approximation. The optimum value of the characterizing scalar (α) has been obtained and the minimum value of the MSE of the proposed estimator for this optimum value has also been obtained. Comparisons with existing estimators are made numerically using data sets considered earlier by Murthy (1967) and Mukhopadhyay (2009). To check the tolerance power of proposed estimator, it has been compared with the existing estimators on and around the optimum value of α. Graphical illustrations show that the proposed ratio-type estimator performs better than the existing estimators of population mean under certain conditions.

Keywords: Auxiliary variable, Bias, Characterizing scalar, Mean square error, Ratio-type estimator, Study variable.

1. INTRODUCTION

The sustainable growth of any organization is only possible with the collection and analysis of up-to-date data. This task of data collection can be done either through the complete quantify survey or by using sample survey technique. To estimate any parameter usually the best estimator is the corresponding statistic. Thus for estimating population mean, sample mean is the most suitable estimator but it has a reasonably large sampling variance. Our aim is to search for the estimator with higher efficiency that is having minimum variance or mean squared error. In sampling theory, the information on auxiliary variables related to the study variable is utilised to increase the efficiency of the estimators of the finite population mean of the study of variable.

When the information on auxiliary variable(x), which is highly correlated with study variable (y), is available, then such information may be utilized to obtain a more efficient estimator of the population mean of the study variable. Thus, ratio method of estimation envisaged by Cochran (1940) is an attempt in this direction. The extensive utilization of the ratio method of estimation is because of its intuitional plead and computational clarity. Population mean is one of the very important measures of central tendency in various fields. In the present manuscript a general class of modified ratio type estimators of population mean of study variable has been proposed and its large sample properties like bias and mean square error have been studied up to first degree of approximation.

2. MATERIALS AND METHODS

2.1 Sampling Techniques and Terminology

Let the finite population under consideration consists of N distinct and identifiable units and let (x_i, y_i) where $i=1, 2, 3 \ldots n$ be a bivariate sample of size n taken from (X, Y) using a SRSWOR scheme. Let $\bar{X}$ and $\bar{Y}$ respectively be the population means of the auxiliary and the study variables, and let $\bar{x}$ and $\bar{y}$ be their corresponding sample means. It is well known and has been seen in practice that in simple random sampling scheme, sample means $\bar{x}$ and $\bar{y}$ are

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unbiased estimators of population means of $\bar{X}$ and $\bar{Y}$ respectively.

Brief description of Notations and Terminology used:

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2$$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2$$

$$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)\left(X_i - \bar{X}\right)$$

$$C_x = \frac{S_x}{\bar{X}},\ C_y = \frac{S_y}{\bar{Y}},\ C_{yx} = \frac{S_{yx}}{\bar{X}\bar{Y}}$$

$$\gamma = \left(\frac{1}{n} - \frac{1}{N}\right)$$

$$C_{yx} = \frac{S_{yx}}{\bar{X}\bar{Y}} \quad \rho = \frac{S_{xy}}{S_x S_y}$$

β_1 = Coefficient of skewness of x,

M_d = Population median of x

$\hat{\bar{Y}}_{mgm}$ – Proposed modified ratio estimator of $\bar{Y}$,

Bias ($\hat{\bar{Y}}_{mgm}$) – Bias of the estimator

MSE ($\hat{\bar{Y}}_{mgm}$) – Mean square error of the estimator,

2.2 Brief Review of Some Existing Estimators

Cochran (1940) used the positively correlated auxiliary variable with the study variable and proposed the following usual ratio estimator of population mean as,

$$\hat{\bar{Y}}_r = \bar{y}\frac{\bar{X}}{\bar{x}}$$

The above estimator is a biased estimator of population mean and its bias and mean squared error, up to the first order of approximation respectively are,

$$B(\hat{\bar{Y}}_r) = \gamma\bar{Y}[C_x^2 - \rho C_y C_x]$$

$$MSE(\hat{\bar{Y}}_r) = \gamma\bar{Y}^2[C_y^2 + C_x^2 - 2\rho C_y C_x]$$

In literature various modified estimators of population mean of study variable using auxiliary variables have been given by various authors like Sisodia and Dwivedi (1981), Singh and Tailor (2003), Yan and Tian (2010), Subramani and Kumarpandiyan (2013) and Jerajuddin and Kishun (2016).

Table 1. Some existing ratio-type estimators from literature with their constants and mean squared errors

Sr. No.	Estimators	MSE
1	$\hat{\bar{Y}}_{sd} = \bar{y}\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$ Sisodia and Dwivedi [1981]	$\gamma\bar{Y}^2(C_y^2 + k_{sd}^2C_x^2 - 2k_{sd}\rho C_x C_y)$ Where $k_{sd} = \left(\frac{\bar{X}}{\bar{X}+C_x}\right)$
2	$\hat{\bar{Y}}_{st} = \bar{y}\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$ Singh and Tailor [2003]	$\gamma\bar{Y}^2(C_y^2 + k_{st}^2C_x^2 - 2k_{st}\rho C_x C_y)$ Where $k_{st} = \left(\frac{\bar{X}}{\bar{X}+\rho}\right)$
3	$\hat{\bar{Y}}_{yt} = \bar{y}\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$ Yan and Tian [2010]	$\gamma\bar{Y}^2(C_y^2 + k_{yt}^2C_x^2 - 2k_{yt}\rho C_x C_y)$ where $k_{yt} = \left(\frac{\bar{X}}{\bar{X}+\beta_1}\right)$
4	$\hat{\bar{Y}}_{sk} = \bar{y}\left[\frac{\bar{X}+M_d}{\bar{x}+M_d}\right]$ Subramani and Kumarpandiyan [2013]	$\gamma\bar{Y}^2(C_y^2 + k_{sk}^2C_x^2 - 2k_{sk}\rho C_x C_y)$ where $k_{sk} = \left(\frac{\bar{X}}{\bar{X}+M_d}\right)$
5	$\hat{\bar{Y}}_{jk} = \bar{y}\left[\frac{\bar{X}+n}{\bar{x}+n}\right]$ Jerajuddin and Kishun [2016]	$\gamma\bar{Y}^2(C_y^2 + k_{jk}^2C_x^2 - 2k_{jk}\rho C_x C_y)$ where $k_{jk} = \left(\frac{\bar{X}}{\bar{X}+n}\right)$ and n is sample size

2.3 Proposed Ratio-Type Estimator

Considering the fact that the auxiliary variable X is closely related to (having positive correlation with) the study variable Y and it is assumed that the population total and/or mean of X is known, we have made effort to introduce a new kind of ratio estimator, selected from the population under SRSWOR which is more efficient as compared to the other existing modified ratio estimators.

The proposed modified ratio-type estimator for population mean $\bar{Y}$ is;

$$\hat{\bar{Y}}_{mgm} = \bar{y}\left[\frac{\bar{X}+\theta}{\bar{x}+\theta}\right]^{\alpha} \tag{1}$$

where, α is a suitably chosen constant to be defined such that the mean squared error of the proposed estimator is minimum and θ is the known constant based on information available for the auxiliary variable.

To study the bias and MSE of the proposed estimator, we define;

$\bar{y} = \bar{Y}(1+\varepsilon_0)$ And $\bar{x} = \bar{X}(1+\varepsilon_1)$

So we have, $E(\varepsilon_0) = 0$, $E(\varepsilon_1) = 0$

$$E(\varepsilon_0^{\,2}) = \left(\frac{1}{n} - \frac{1}{N}\right)C_y^{\,2} = \gamma C_y^{\,2},$$

$$E(\varepsilon_1^{\,2}) = \left(\frac{1}{n} - \frac{1}{N}\right)C_x^{\,2} = \gamma C_x^{\,2},$$

$$E(\varepsilon_0\varepsilon_1) = \left(\frac{1}{n} - \frac{1}{N}\right)C_{yx} = \gamma C_{yx}$$

Expressing [1] in terms of $\varepsilon_i's$, we have

$$\hat{\bar{Y}}_{mgm} = \bar{Y}(1+\varepsilon_0)\left[\frac{\bar{X}+\theta}{\bar{X}(1+\varepsilon_1)+\theta}\right]^{\alpha}$$

$$\hat{\bar{Y}}_{mgm} = \bar{Y}(1+\varepsilon_0)\left[1+k\varepsilon_1\right]^{-\alpha}$$

where, $k = \dfrac{\bar{X}}{\bar{X}+\theta}$ (2)

Now, here we assume that $|\varepsilon_1| < 1$ so that $[1+k\varepsilon_1]^{-\alpha}$ may be expanded. Now expanding the right-hand side of (2), we have,

$$\hat{\bar{Y}}_{mgm} = \bar{Y}\left[1+\varepsilon_0 - \alpha k\varepsilon_1 - \alpha k\varepsilon_0\varepsilon_1 + \left(\frac{\alpha(\alpha-1)k^2\varepsilon_1^{\,2}}{2!}\right) + \left(\frac{\alpha(\alpha-1)k^2\varepsilon_1^{\,2}}{2!}\right)\varepsilon_0 - \ldots\right] \quad (3)$$

Taking expectations on both sides of (3) and retaining the terms up to the second degree of approximation, we get the bias of $\hat{\bar{Y}}_{mgm}$ as;

$$E\left(\hat{\bar{Y}}_{mgm}\right) = \bar{Y} + \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}\left[\frac{\alpha(\alpha-1)k^2C_x^{\,2}}{2!} - \alpha k\rho C_x C_y\right]$$

$$Bias\left(\hat{\bar{Y}}_{mgm}\right) = \gamma\bar{Y}\left[\frac{\alpha(\alpha-1)k^2C_x^{\,2}}{2!} - \alpha k\rho C_x C_y\right] \quad (4)$$

Up to the first order of approximation, the MSE of proposed estimator $\hat{\bar{Y}}_{mgm}$ is

$$MSE(\hat{\bar{Y}}_{mgm}) = \gamma\bar{Y}^2\left[C_y^{\,2} + \alpha^2k^2C_x^{\,2} - 2\alpha kC_{yx}\right] \quad (5)$$

which is minimum for the value of $\alpha = \dfrac{C_{yx}}{kC_x^{\,2}}$ (6)

Thus, $\left[\dfrac{C_{yx}}{kC_x^{\,2}}\right]$ is the optimum value of (α)

Therefore, the minimum value of MSE $\left(\hat{\bar{Y}}_{mgm}\right)$ for optimum value of (α) is given by,

Hence, $MSE_{\min}(\hat{\bar{Y}}_{mgm}) = \gamma\bar{Y}^2\left[C_y^{\,2} - \dfrac{C_{yx}^{\,2}}{C_x^{\,2}}\right]$ (7)

3. RESULTS AND DISCUSSION

3.1 Empirical Study

To judge the performances of the proposed estimator and the existing estimators of population mean using auxiliary variable, we have considered some natural population from different sources. We used four data sets earlier considered by Murthy (1967) and Mukhopadhyay (2009). The values of the population parameters are given in Table 2. The optimum value of α for different values of K for each population is given in Table 3. To know the behaviour of proposed estimator around optimum value of α, the values of the MSE of the proposed estimator for different values of α (in both positive and negative direction of the optimum value of α) with a fixed value of $\theta = \beta_1$ for each population are given in Table 4, Table 5, Table 6 and Table 7.

Table 3. Optimum value of α for different values of K for each population

Population	θ	K	Optimum value of Alpha
I	$M_d = 7.5$	0.59808	0.741733
	$\beta_1 = 1.0$	0.914735	0.484966
II	$M_d = 1.4$		
	$\beta_1 = 1.3$	0.686747	0.50017
III	$M_d = 1.2$	0.657143	0.474465
	$\beta_1 = 1.9$	0.547619	0.569358
IV	$M_d = 7.0$	0.574468	0.71527
	$\beta_1 = 0.8$	0.921951	0.445685

Table 2. Values of population parameters

Murthy (1967)	N	n	$\bar{Y}$	$\bar{X}$	ρ	c_y	c_x	c_{xy}	M_d	β_1
Population I	80	20	51.8	11.3	0.94	0.35	0.75	0.25	7.57	1.05
Population II	80	20	51.8	2.85	0.91	0.35	0.95	0.31	1.48	1.30
Mukhopadhyay (2009)	N	n	$\bar{Y}$	$\bar{X}$	ρ	c_y	c_x	c_{xy}	M_d	β_1
Population III	40	8	50.8	2.30	0.80	0.33	0.84	0.22	1.25	1.97
Population IV	40	8	50.8	9.45	0.83	0.33	0.68	0.19	7.07	0.88

Table 4. MSE of proposed and Existing estimators for different values of for Population I

Alpha	Estimators					
	Proposed	SD	ST	YT	SK	JK
0.1	8.702	17.332	16.670	16.035	4.108	2.636
0.2	5.717	17.332	16.670	16.035	4.108	2.636
0.3	3.682	17.332	16.670	16.035	4.108	2.636
0.4	2.597	17.332	16.670	16.035	4.108	2.636
0.5	2.462	17.332	16.670	16.035	4.108	2.636
0.6	3.277	17.332	16.670	16.035	4.108	2.636
0.7	5.041	17.332	16.670	16.035	4.108	2.636
0.8	7.756	17.332	16.670	16.035	4.108	2.636
0.9	11.420	17.332	16.670	16.035	4.108	2.636
1	16.035	17.332	16.670	16.035	4.108	2.636

Table 5. MSE of proposed and Existing estimators for different values of α for Population II

Alpha	Estimators					
	Proposed	SD	ST	YT	SK	JK
0.1	8.876	18.015	18.777	13.612	12.604	6.442
0.2	5.972	18.015	18.777	13.612	12.604	6.442
0.3	3.926	18.015	18.777	13.612	12.604	6.442
0.4	2.738	18.015	18.777	13.612	12.604	6.442
0.5	2.407	18.015	18.777	13.612	12.604	6.442
0.6	2.933	18.015	18.777	13.612	12.604	6.442
0.7	4.317	18.015	18.777	13.612	12.604	6.442
0.8	6.558	18.015	18.777	13.612	12.604	6.442
0.9	9.656	18.015	18.777	13.612	12.604	6.442
1	13.612	18.015	18.777	13.612	12.604	6.442

Table 6. MSE of proposed and Existing estimators for different values of α for Population III

Alpha	Estimators					
	Proposed	SD	ST	YT	SK	JK
0.1	22.381	41.963	43.419	20.031	31.522	11.625
0.2	17.752	41.963	43.419	20.031	31.522	11.625
0.3	14.214	41.963	43.419	20.031	31.522	11.625
0.4	11.769	41.963	43.419	20.031	31.522	11.625
0.5	10.415	41.963	43.419	20.031	31.522	11.625
0.6	10.154	41.963	43.419	20.031	31.522	11.625
0.7	10.985	41.963	43.419	20.031	31.522	11.625
0.8	12.908	41.963	43.419	20.031	31.522	11.625
0.9	15.924	41.963	43.419	20.031	31.522	11.625
1	20.031	41.963	43.419	20.031	31.522	11.625

Table 7. MSE of proposed and Existing estimators for different values of α for Population IV

Alpha	Estimators					
	Proposed	SD	ST	YT	SK	JK
0.1	20.575	45.514	44.108	44.108	14.256	12.922
0.2	15.076	45.514	44.108	44.108	14.256	12.922
0.3	11.605	45.514	44.108	44.108	14.256	12.922
0.4	10.162	45.514	44.108	44.108	14.256	12.922
0.5	10.749	45.514	44.108	44.108	14.256	12.922
0.6	13.363	45.514	44.108	44.108	14.256	12.922
0.7	18.007	45.514	44.108	44.108	14.256	12.922
0.8	24.679	45.514	44.108	44.108	14.256	12.922
0.9	33.379	45.514	44.108	44.108	14.256	12.922
1	44.108	45.514	44.108	44.108	14.256	12.922

3.2 Graphical Illustration

The behaviour of the proposed estimator for some deviation in the optimum value of α (both in positive and negative side) with a fixed value of $\theta = \beta_1$ for each population is presented graphically.

These graphical illustrations clearly depicts the idea that the proposed estimator shows minimum value of MSE among all the other estimators considered for comparison study for the data sets obtained from Murthy (1967) and Mukhopadhyay (2009). Fig 1 shows that the MSE of the proposed estimator is minimum than the estimators defined by Sisodia and Dwivedi (1981), Singh and Tailor (2003), Yan and Tian (2010) for a great deviation from the optimum value of α. Whereas the tolerance power of the proposed estimator to stay best for a deviation in the optimum value of α is less in comparison to estimators defined by Subramani and Kumarpandiyan (2013) and Jerajuddin and Kishun (2016)

Therefore, from above numerical and graphical study, we can conclude that the proposed estimator has minimum mean squared errors for all the data sets. Thus, the proposed estimator is most efficient among all other existing ratio-type estimators. Hence, the proposed estimator should be used for improved estimation of population mean.

4. CONCLUSION

The major findings of the empirical study highlights that the proposed ratio type estimator of population mean has minimum value of MSE among

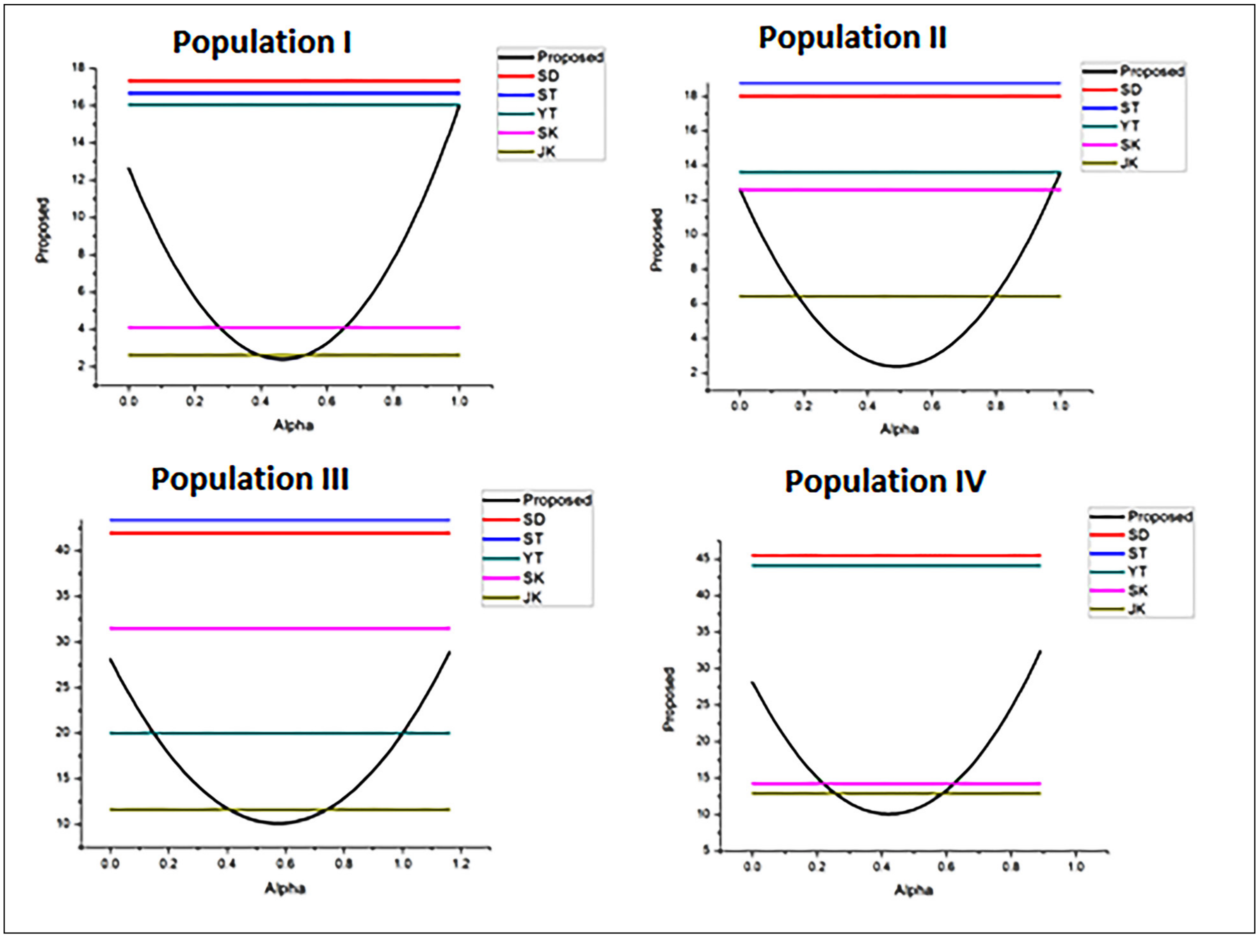

Fig. 1. MSE of Proposed and Existing Estimators for the different values of α

all the other existing ratio-type estimators considered in the analysis. Also, to show the merits of the proposed ratio type estimator the values of MSE are calculated for some deviation in the optimum value of the alpha, both in positive and negative side. The results shows that the MSE of the proposed estimator is much smaller in comparison to all existing ratio-type estimators for the given population data sets. This clearly depicts that the proposed estimator provides more relevant and admissible results in estimating the population mean.

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