

# Calibration Estimator of Finite Population Mean using Auxiliary Information under Adaptive Cluster Sampling

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### SUMMARY

Adaptive cluster sampling (ACS) technique is usually used for estimation of the abundance of an exclusive, clustered biological population. Commonly, neighbouring units are added to the sample if it satisfies a pre-determined criterion. Use of auxiliary information to increase the precision of estimators is a very general practice. This paper deals with the use of auxiliary information for the development of efficient estimator of finite population mean under ACS design using the well-known Calibration Approach given by Deville and Särndal (1992). The statistical performance of the calibration estimators of population mean under ACS are evaluated through a simulation study with respect to conventional Horvitz Thomson (HT) estimator of population mean which do not utilize the auxiliary information. The results of the simulation study conducted on a rare and clustered population often cited in Smith *et al.* (1995) show that proposed calibration estimators are more efficient than conventional HT estimator of the population mean under ACS with respect to percentage Relative Bias (%RB) and percentage Relative Root Mean Squared Error (%RRMSE).

Keywords: Calibration, Auxiliary information, Rare attribute, Clustered.

### 1. INTRODUCTION

Survey statisticians frequently deal with situations in which the sampling units bearing characteristics of interest are sparsely scattered, but in a clustered manner for a geographically distributed population. For instance, estimation of rare birds, number of trees of a rare species, production of non-hybrid crops, infected with a rare disease, drug use etc. Estimating rare characteristics present difficult sampling and estimation problems. Conventional sampling methods like simple random sampling, stratified sampling, clustered sampling are unable to deal with abovementioned situations. The basic reason behind this is that in case of rare and clustered species estimation, we may not get enough units in the sample meeting with specified criteria. In such cases, the investigators are motivated to search for sampling and estimation methods that go beyond the conventional set of techniques. Thompson (1990, 1991a, 1991b) was the first to introduce Adaptive Cluster Sampling (ACS) into the literature on survey sampling. Thompson

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and Seber (1996) followed it up. With an adaptive sampling scheme, the procedure for selecting units to include in the sample may depend on values of the variable of interest observed during the survey, i.e. the sampling is "adapted" to the data (Thompson and Seber, 1996). ACS technique is usually used for estimation of the abundance of exclusive, clustered biological population which are geographically rare and hidden. ACS design allows observed values to trigger increased sampling effort during the survey. Commonly, neighbouring units are added to the sample if it satisfies a pre-determined criterion. ACS designs assign high probabilities to samples that include areas with high density of characteristics under study. This intuitively appealing design can have lower variance than conventional designs. The increase in precision may depend on the spatial distribution of the population, the condition determining when to adapt sampling effort etc. Compared to conventional sampling designs, adaptive sampling can result in higher efficiency (i.e. higher precision for fixed cost) and higher rates of

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encountering occupied habitat and detecting rare species (Brown, 2003; Smith *et al.*, 2004). Adaptive sampling designs have been applied for survey of a wide range of rare species including Pacific hake larvae (Lo *et al.*, 1997), rare deforestation events (Magnussen *et al.*, 2005), terrestrial herpetofauna (Noon *et al.*, 2006), subtidal macroalgae (Goldberg *et al.*, 2007), plant disease organisms (Ojiambo and Scherm, 2008), red sea urchin (Skibo *et al.*, 2008), and freshwater mussels (Smith *et al.*, 2003; Outeiro *et al.*, 2008; Hornbach *et al.*, 2010). Turk and Borkowski (2005) provide a review of works carried out in the ACS design during 1990–2003.

Use of auxiliary information to improve the precision of the estimates is a very common practice in survey sampling. Several authors attempted to deal with ratio estimator of the population mean using auxiliary information of the variable under ACS design (Dryver and Chao, 2007; Chao et al., 2008; Chutiman et al., 2013; Chutiman and Chiangpradit, 2014). The calibration approach, proposed by Deville and Särndal (1992), is frequently used for developing estimators of important population parameters incorporating auxiliary information. The calibration approach focuses on the design weights given to the sampling units for estimation. These calibration weights satisfy a set of calibration constraints that make use of the specified auxiliary information. In fact, the generalized regression estimator (GREG) (Cassel et al., 1976) is a special case of the calibration estimator choosing the Chi-square distance function (Deville and Särndal, 1992). In the past few decades, calibration estimation has gained significant attention not only in the field of survey methodology but also in survey practice. Following Deville and Särndal (1992), lot of work has been carried out in calibration estimation i.e. Singh et al. (1998, 1999), Wu and Sitter (2001), Kott (2006), Aditya et al. (2016, 2017), Salinas et al. (2018), Biswas et al. (2020) etc. Särndal (2007) and Kim and Park (2010) provided a comprehensive review of the calibration approach. In this article, an attempt has been made to develop calibration estimator for estimation of population mean using known auxiliary information under ACS design. In order to study the statistical performance of the proposed calibration estimators under ACS design framework, a simulation study was carried out using a real dataset on the blue-winged teal bird population taken from often cited Smith et al. (1995).

# 2. STANDARD ESTIMATION PROCEDURE UNDER ADAPTIVE CLUSTER SAMPLING

Consider a population  $U = \{1, ..., j, ..., N\}$  with the assumption that N is known. Y is a variable defined on the population U and taking real values as  $y_1, y_2, ..., y_N$ . The objective is to estimate the population mean i.e.  $\overline{Y} = \frac{1}{N} \sum_{j=1}^{N} y_j$ . In order to take a sample by ACS design, first of all, an initial sample of  $n_j$  units are selected from a finite population by simple

random sampling without replacement (SRSWOR). Then, for each selected units a predefined condition of the character of interest Y say  $y_i > C$  is to be verified. If the condition is satisfied by the sampled unit, then the rest of the unit's neighbourhood is added to the sample. Now, those neighbourhood units which satisfy the condition C are also added in the sample. The process is continued until we get the edge units i.e. which do not satisfy condition C. Finally, we get the sample in which we get n clusters (not necessarily distinct), one for each initially selected unit. If the selected unit in the initial sample does not satisfy the predefined condition C, then there is no adaptive selection and it is considered as a cluster of size one. Neighbourhoods can be defined in a variety of ways. The first-order neighbourhood is the most prevalent technique where neighbourhood consists of the unit itself and the four adjacent units sharing a common boundary.

For illustration purpose, we have taken the often cited blue-winged teal bird population for ACS design (Fig. 1) given by Smith *et al.* (1995). This figure represents 200 quadrants of 25 km<sup>2</sup> area each with counts given for quadrats having non-zero bird counts. Pictorial representation of the selection of an adaptive cluster sample from this population is also given in Fig. 1. An initial simple random sample of 10 quadrats is selected indicated with  $\otimes$ . For the predefined condition C i.e.  $y_j \ge 1$ , all neighbourhood quadrats would be sampled with ACS design denoted with  $\bigcirc$ . Quadrats with  $\bigcirc$  having the teal counts that do not satisfy C are the edge units.

Let, the population can be partitioned into K distinct networks, where the *i*<sup>th</sup> network is denoted as  $A_i$ . Let,  $m_i$  denote the number of units in the network  $A_i$ . Define  $y_i^*$  as the total of y-values in the *i*<sup>th</sup> network as  $y_i^* = \sum_{j \in A_i} y_j$ . Let, the initial sample mean

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Fig. 1. An adaptive cluster sample (C:  $y_i \ge 1$ ) from the Blue-winged teal population as quoted in Smith *et al.* (1995)

is  $\overline{y}_1 = \frac{1}{n_1} \sum_{j=1}^{n_1} y_j$ . Thompson (1990) defined the partial

inclusion probability as given by

$$\pi_i' = 1 - \left[ \binom{N - m_i}{n_1} \right] / \binom{N}{n_1}.$$

The probability  $\pi'_i$  can be interpreted as the probability that the initial sample intersects network  $A_i$ , the network containing unit *i*. It is same for each unit in the network. It is also referred as the marginal initial intersection probability i.e. the probability that at least one unit in the initial sample intersects the *i*<sup>th</sup> network. The probability that network *i* and *j* are both intersected i.e. the joint initial intersection probability is given by

$$\pi_{ij}' = 1 - \left[ \binom{N - m_i}{n_1} + \binom{N - m_j}{n_1} - \binom{N - m_i - m_j}{n_1} \right] / \binom{N}{n_1}$$

Thompson (1990) proposed the modified Horvitz-Thompson estimator of the population mean using the initial intersection probability under ACS design as given by

$$\hat{\bar{Y}}_{HT(AC)} = \frac{1}{N} \sum_{i=1}^{K} \frac{y_i^* J_i}{\pi_i'} = \frac{1}{N} \sum_{i=1}^{k} \frac{y_i^*}{\pi_i'} = \frac{1}{N} \sum_{i=1}^{k} d_i y_i^*$$
(2.1)

where,  $J_i$  takes value 1 if the initial sample intersects the i<sup>th</sup> network and 0 otherwise, k be the number of distinct networks intersected by the initial sample and  $d_i = 1/\pi_i'$ . Thompson (1990) showed that  $\hat{Y}_{HT(AC)}$  is unbiased and the sampling variance of the  $\hat{Y}_{HT(AC)}$  is given by

$$V\left(\hat{\bar{Y}}_{HT(AC)}\right) = \frac{1}{N^2} \sum_{i=1}^{K} \sum_{j=1}^{K} \Delta_{ij} \left(d_i y_i^*\right) \left(d_j y_j^*\right)$$
(2.2)  
where,  $\Delta_{ij} = \pi'_{ij} - \pi'_i \pi'_j$ .

## 3. PROPOSED CALIBRATION ESTIMATORS UNDER ADAPTIVE CLUSTER SAMPLING DESIGN

Let,  $U=\{1, 2, ..., N\}$  be the finite population under consideration and Y is the study variable as defined earlier. Let, X be a linearly related auxiliary variable with real values  $x_1, x_2, ..., x_N$ . Let us assume,  $X = \sum_{j=1}^{N} x_j = \sum_{i=1}^{K} x_i^*$  is known, where  $x_i^* = \sum_{j \in A_i} x_j$  and K is

the total number of distinct networks in the population. In this study, we have proposed calibration estimator for estimation of finite population mean under ACS design using the available auxiliary information when the members bearing a characteristic of interest are sparsely scattered in a geographically distributed population in unknown manners. Let, the study variable  $y_j$  was observed for all  $j \in s$ , where *s* is the set all sampling units obtained by ACS design. Using the well-known Calibration Approach (Deville and Särndal, 1992), an attempt was made to improve the Horvitz-Thompson estimator ( $\hat{T}_{HT(AC)}$ ) of the population mean,  $\overline{Y}$ , under ACS design. The proposed calibration estimator of the population mean is given by

$$\hat{\bar{Y}}_{CAL(AC)} = \frac{1}{N} \sum_{i=1}^{k} w_i y_i^*$$
(3.1)

where,  $w_i$  is the desired calibrated weight.

The new set of weights  $w_i$  are chosen as close as possible to the design weight  $d_i$  under calibration equation. For this purpose, following Deville and Särndal (1992), we minimize the chi-square type distance function given by

$$\sum_{i=1}^{k} \left[ (w_i - d_i)^2 / d_i q_i \right]$$
(3.2)

subject to the constraint  $\overline{X} = \frac{1}{N} \sum_{i=1}^{k} w_i x_i^*$ , where  $q_i$  are

suitably chosen constants. The loss function is

$$\varphi(w_i,\lambda) = \sum_{i=1}^k \frac{(w_i - d_i)^2}{d_i q_i} - \lambda \left( N\overline{X} - \sum_{i=1}^k w_i x_i^* \right)$$
(3.3)

Now, by minimizing above loss function, new set of calibration weights  $W_i$  are obtained as following

$$w_{i} = d_{i} + \left( d_{i}q_{i}x_{i}^{*} / \sum_{i=1}^{k} d_{i}q_{i}x_{i}^{*^{2}} \right) \left( X - \sum_{i=1}^{k} d_{i}x_{i}^{*} \right)$$
(3.4)

where, population total  $X = \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} x_i^*$  is assumed to

be known and  $x_i^* = \sum_{j \in A_i} x_j$ .

The proposed calibration estimator based on the revised weights is given by

$$\hat{\bar{Y}}_{CAL(AC)} = \frac{1}{N} \sum_{i=1}^{k} d_{i} y_{i}^{*} + \frac{1}{N} \frac{\sum_{i=1}^{k} d_{i} q_{i} x_{i}^{*} y_{i}^{*}}{\sum_{i=1}^{k} d_{i} q_{i} x_{i}^{*^{2}}} \left( X - \sum_{i=1}^{k} d_{i} x_{i}^{*} \right)$$
$$= \hat{\bar{Y}}_{HT(AC)} + \hat{B}_{CAL(AC)} \left( \overline{X} - \hat{\overline{X}}_{HT(AC)} \right), \qquad (3.5)$$

where,  $\hat{\overline{X}}_{HT(AC)} = \frac{1}{N} \sum_{i=1}^{k} d_i x_i^*$  and

$$\hat{B}_{CAL(AC)} = \sum_{i=1}^{k} d_i q_i x_i^* y_i^* / \sum_{i=1}^{k} d_i q_i x_i^{*2} .$$

Calibration estimators are asymptotically design unbiased and equivalent to Generalized Regression (GREG) estimator (Deville and Särndal, 1992). Thus, the proposed calibration estimator in the Equation (3.5) is also asymptotically design unbiased and equivalent to GREG estimator. Accordingly, for the asymptotic variance and variance estimator of the proposed calibration estimator, the expressions given by Särndal *et al.* (1992) in case of GREG estimator can be used with modifications in the population regression errors and the sample regression residuals. The approximate variance of the proposed calibration estimator is given by

$$AV\left(\hat{\bar{Y}}_{CAL(AC)}\right) = \frac{1}{N^2} \sum_{i=1}^{K} \sum_{j=1}^{K} \Delta_{ij} \left(d_i E_i\right) \left(d_j E_j\right)$$
(3.6)

where,

$$E_{i} = y_{i}^{*} - Bx_{i}^{*}, \quad B = \left(\sum_{i=1}^{K} q_{i}x_{i}^{*}y_{i}^{*}\right) / \left(\sum_{i=1}^{K} q_{i}x_{i}^{*^{2}}\right) \text{ and }$$
  
$$\Delta_{ij} = \pi_{ij}^{'} - \pi_{i}^{'}\pi_{j}^{'}.$$

The expression of the estimator of variance is given as

$$\hat{V}\left(\hat{\bar{Y}}_{CAL(AC)}\right) = \frac{1}{N^2} \sum_{i=1}^{k} \sum_{j=1}^{k} \frac{\Delta_{ij}}{\pi_{ij}} (w_i e_i) (w_j e_j)$$
(3.7)

where,

$$e_i = y_i^* - \hat{B}x_i^*, \quad \hat{B} = \left(\sum_{i=1}^k w_i q_i x_i^* y_i^*\right) / \left(\sum_{i=1}^k w_i q_i x_i^{*^2}\right).$$

When we choose  $q_i = (x_i^*)^{-1}$  the revised weights given Equation (3.4) simplify to

$$w_{i} = d_{i} + \left( \frac{d_{i}}{\sum_{i=1}^{n} d_{i} x_{i}^{*}} \right) \left( X - \sum_{i=1}^{k} d_{i} x_{i}^{*} \right)$$
(3.8)

The proposed calibration estimator based on these revised weights is given by

$$\hat{\bar{Y}}_{CAL(AC)} = \frac{1}{N} \sum_{i=1}^{k} d_{i} y_{i}^{*} + \frac{1}{N} \frac{\sum_{i=1}^{k} d_{i} y_{i}^{*}}{\sum_{i=1}^{k} d_{i} x_{i}^{*}} \left( X - \sum_{i=1}^{k} d_{i} x_{i}^{*} \right) = \frac{\sum_{i=1}^{k} d_{i} y_{i}^{*}}{\sum_{i=1}^{k} d_{i} x_{i}^{*}} \overline{X}.$$
(3.9)

Now, when we choose  $q_i = 1$  the revised weights as given in Equation (3.4) simplify to

$$w_{i} = d_{i} + \left( d_{i} x_{i}^{*} / \sum_{i=1}^{k} d_{i} x_{i}^{*2} \right) \left( X - \sum_{i=1}^{k} d_{i} x_{i}^{*} \right). (3.10)$$

The proposed calibration estimator based on this revised weights is given by

$$\hat{Y}_{CAL(AC)} = \frac{1}{N} \sum_{i=1}^{k} d_i y_i^* + \frac{1}{N} \frac{\sum_{i=1}^{k} d_i x_i^* y_i^*}{\sum_{i=1}^{k} d_i x_i^{*^2}} \left( X - \sum_{i=1}^{k} d_i x_i^* \right).$$
(3.11)

The formula of approximate variance and estimator of the variance of these proposed calibration estimators can be simplified accordingly using Equations (3.6) and (3.7).

## 4. SIMULATION STUDY

In order to study the statistical performance of the proposed calibration estimators under the ACS design framework, a simulation study was carried out using a real dataset. The blue-winged teal bird population (Fig. 1) given in the often cited Smith et al. (1995) has been utilized. Blue-winged teal birds are counted from aircraft to monitor density and test ecological hypotheses. Because the distribution of blue-winged teal bird populations is spatially clustered, precise estimation of density is difficult. ACS is effective in this situation since it allows observed values to trigger increased sampling effort during the survey. Under the simulation study, the number of blue-winged teal bird observed in 25 km<sup>2</sup> quadrats in 200 quadrats has been considered as the character of interest Y. Thus, the population size is considered to be N=200. Empty cell represents zero counts of the bird. It can be observed that the population mean of this blue-winged teal population is  $\overline{Y}$  =70.60 i.e. approximately 71 birds per quadrats.

In the simulation study, in order to study the statistical performance of proposed calibration estimators, an auxiliary variable X which is highly correlated with the study variable Y has been generated. Here, it has been assumed that X follows Normal distribution considering:

Mean of	Standard deviation	Correlation coefficient between <i>X</i>		
$X(\overline{X})$	of $X(\sigma_x)$	and $Y(\rho_{xy})$		
30	10	0.9		

We used Monte Carlo simulation to draw samples from the enumerated populations. From the study population, a total of 5000 independent samples were selected from the population of N=200 quadrats using ACS design for different choices of initial sample by Simple Random Sampling Without Replacement (SRSWOR). If the observed *y* value of a sampled unit satisfies the condition C i.e.  $y_i > 0$ , then the rest of the unit's neighbourhood was added to the sample. If any other units in that neighbourhood satisfy the condition C, then their neighbourhoods were also added to the sample. The process was continued until a cluster of units is obtained for each unit selected in the initial sample. Different choices of initial sample sizes have been taken as  $n_I = 10$ , 15, 20, and 25.

From each of the 5000 independent samples, estimates of the proposed calibration estimators  $(\bar{Y}_{CAL(AC)})$  of the population mean (as given in Equation 3.8 considering  $q_i = (x_i^*)^{-1}$ ) as well as modified Horvitz-Thompson estimator  $(\bar{Y}_{HT(AC)})$  (as given in Equation 2.1) were calculated. The sample mean estimator of the initial sample  $(\bar{Y}_{SRS})$  was also computed. Values of these estimates were averaged over 5000 replicate samples under Monte Carlo simulation. The estimators of population mean under ACS were evaluated on the basis of three measures viz. percentage Relative Bias (%RB) and percentage Relative Root Mean Squared Error (%RRMSE) as given by

$$RB(\hat{\theta}) = \frac{1}{S} \sum_{i=1}^{S} \left( \frac{\hat{\theta}_i - \theta}{\theta} \right) \times 100 \quad and \quad RRMSE(\hat{\theta}) = \sqrt{\frac{1}{S} \sum_{i=1}^{S} \left( \frac{\hat{\theta}_i - \theta}{\theta} \right)^2} \times 100.$$

where,  $\hat{\theta}_i$  are the value of the estimator of population parameter  $\theta$  for the character under study obtained at  $i^{\text{th}}$ sample in the simulation study and  $\theta$  is considered to be the population parameter i.e. population mean. The Percentage Relative Efficiency (PRE) of the proposed calibration estimator ( $\hat{Y}_{CAL(AC)}$ ) of the population mean ( $\bar{Y}$ ) in comparison to the Horvitz-Thompson estimator ( $\hat{Y}_{HT(AC)}$ ) as well as the sample mean estimator of initial sample ( $\hat{Y}_{SRS}$ ) based on the %RRMSE can be obtained as

$$PRE\left(\hat{\overline{Y}}_{CAL(AC)}\right) = \left[RRMSE\left(\hat{\overline{Y}}_{i}\right) / RRMSE\left(\hat{\overline{Y}}_{CAL(AC)}\right)\right] \times 100$$

where, *i*=HT(AC) or SRS.

## 5. RESULTS AND DISCUSSION

The results of the simulation study are presented here. Table 1 presents the absolute of percentage Relative Bias (%RB) of all the estimators of the population mean ( $\overline{Y}$ ) under ACS for different initial sample sizes ( $n_1$ ). Table 2 presents the percentage Relative Root Mean Squared Error (%RRMSE) and Percentage Relative Efficiency (PRE) of the proposed calibration estimator of the population mean with respect to SRS and HT estimators under ACS design.

**Table 1.** Absolute %RB of all the estimators of the population mean ( $\overline{Y}$ ) for different initial sample sizes ( $n_l$ ) under adaptive cluster sampling

<i>n</i> <sub>1</sub>	$\hat{\overline{Y}}_{SRS}$	$\hat{\vec{Y}}_{HT(AC)}$	$\hat{\overline{Y}}_{CAL(AC)}$
10	4.79	1.67	4.60
15	2.12	0.47	3.23
20	1.78	0.90	3.43
25	1.82	1.35	0.55

**Table 2.** %RRMSE of all the estimators and PRE of the proposed calibration estimator ( $\hat{\vec{Y}}_{CAL(AC)}$ ) of the population mean ( $\vec{Y}$ ) for different initial sample sizes  $(n_l)$  under adaptive cluster sampling

		%RRMSE	$PRE(\hat{\overline{Y}}_{CAL(AC)})$		
	$\hat{\overline{Y}}_{SRS}$	$\hat{\overline{Y}}_{HT(AC)}$	$\hat{\overline{Y}}_{CAL(AC)}$	$\hat{\overline{Y}}_{SRS}$	$\hat{\overline{Y}}_{HT(AC)}$
10	296.90	147.49	138.42	214.49	106.55
15	235.71	113.14	109.02	216.21	103.78
20	202.98	92.19	89.94	225.68	102.50
25	176.91	76.98	75.58	234.07	101.85

From Table 1, it is evident that with respect to absolute %RB, the Horvitz-Thompson estimator i.e.  $\overline{Y}_{HT(AC)}$  gives the least amount of bias in the estimation of the population mean  $(\overline{Y})$  under ACS as compared to the sample mean estimator of initial sample ( $\hat{\overline{Y}}_{_{RRS}}$ ). Absolute %RB of the proposed calibration estimator of the population mean is higher. Absolute %RB decreases with the increase in initial sample sizes  $(n_1)$ . Table 2 reveals that the proposed calibration estimator  $(\hat{\overline{Y}}_{CAL(AC)})$  of the population mean  $(\overline{Y})$  gives lesser %RRMSE than that of the Horvitz-Thompson estimator  $(\hat{ar{Y}}_{_{HT(AC)}})$  as well as the sample mean estimator of initial sample ( $\hat{\vec{Y}}_{srs}$ ). PRE of the proposed calibration estimator of the population mean under ACS design with respect to SRS and HT estimators clearly suggest superiority for all sample size combinations. Thus, the proposed calibration estimators of the population mean developed under ACS design were always more efficient based on %RRMSE and PRE than the usual Horvitz-Thompson estimator and sample mean estimator of the initial sample of the population mean. %RRMSE

decreases with the increase in initial sample sizes  $(n_l)$ . The sample mean estimator of the initial sample  $(\overline{\hat{Y}}_{SRS})$  always performs poorly under ACS design.

#### 6. CONCLUSIONS

In this study, two forms of calibration estimators of population mean has been proposed under ACS design using known auxiliary information following Calibration Approach by Deville and Särndal (1992). In order to study the statistical performance of the proposed calibration estimators, a simulation study was carried out using a real dataset on the blue-winged teal bird population. The results of the simulation study showed that the proposed calibration estimators of the population mean ( $\overline{Y}$ ) were performing better than the usual sample mean estimator of initial sample  $(\hat{Y}_{SRS})$ as well as Horvitz-Thompson estimator ( $\hat{\vec{Y}}_{HT(AC)}$ ) under ACS design. The %RB of the proposed calibration estimator of the population mean  $(\overline{Y})$  is little higher than that of the Horvitz-Thompson estimator ( $\hat{\vec{Y}}_{HT(AC)}$ ) and, in general, it is decreasing with the increase of the initial sample sizes  $(n_1)$ . Simulation results also reveal that the proposed calibration estimators of the population mean under ACS design was always more efficient with respect to %RRMSE and PRE than the usual Horvitz-Thompson estimators  $(\hat{\bar{Y}}_{HT(AC)})$  of the population mean ( $\overline{Y}$ ) given by Thompson (1990).

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