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A Computational Approach for Estimation of a Finite Population Mean under Two-Phase Sampling in Presence of Two Auxiliary Variables

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SUMMARY

In this paper, a transformed class of ratio-cum-product estimators has been developed for estimating the mean of a finite population using two auxiliary variables in two-phase sampling. The mathematical expressions for bias and mean square error (MSE) of the proposed class, as well as for the other pre-existing estimators, have been obtained to the first order of approximation. Some of the existing estimators are shown to be the members of the proposed class. The proposed class of estimators has been compared with the other existing estimators using the MSE criterion. The theoretical results have been empirically validated by using real population datasets, and also by conducting a simulation study.

Keywords: Auxiliary variable; Bias; Mean square error; Percent relative efficiency; Study variable; Two-phase sampling.

1. INTRODUCTION

In sample surveys, information on auxiliary variable(s) is generally used for obtaining efficient estimators of population parameters (such as the population mean) under various sampling designs, for instance, simple random sampling (SRS), two-phase sampling, and stratified random sampling. The information on auxiliary variable(s) is quite effectively utilized for developing ratio, product and regression estimators of the population parameters. Some remarkable developments towards the theory of estimation of population mean under SRS design, on utilizing prior information on auxiliary variable(s), have been made by Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh (2003), Kadilar and Cingi (2004), Gupta and Shabbir (2007), Vishwakarma and Kumar (2015), Zeeshan *et al.* (2021). Moreover, in the absence of prior information on auxiliary variable(s), the most applicable design is the two-phase sampling design, which was initially developed by Neyman (1938). Several authors have contributed towards the development of estimators under two-phase sampling design, for instance, Sukhatme (1962), Srivastava (1970), Chand (1975), Sisodia and Dwivedi (1982), Singh and Upadhyaya (1995), Singh *et al.* (2007), Singh and Ruiz Espejo (2007), Choudhury and Singh (2012) , Vishwakarma and Kumar (2016), Kumar and Vishwakarma (2017), Dubey *et al.* (2020), Kumar and Tiwari (2022), Kumar *et al.* (2022), and Tiwari *et al.* (2023).

Considering the importance and utility of two-phase sampling design, an attempt is made in this paper to develop a transformed class of ratio-cum-product estimators for estimating the population mean $\overline{Y}$ of the study variable Y in two-phase sampling using two auxiliary variables X and Z. The bias and mean square error (MSE) of the proposed class is derived to the first order of approximation, and efficiency comparisons of the proposed class are made with some pre-existing estimators. Also, the theoretical results have been validated using an empirical analysis as well as by conducting a simulation study.

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2. MATERIALS AND METHODS

2.1 Some Pre-Existing Estimators of the Population Mean

Sukhatme (1962) defined the following ratio estimator for the population mean $\bar{Y}$ of the study variate *Y* under two-phase sampling:

$$\bar{y}_R^d = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right) \tag{1}$$

where $\bar{x}' = \sum_{i=1}^{n'} X_i / n'$ denotes the first-phase sample mean of the auxiliary variable *X*. Also, $\bar{y} = \sum_{i=1}^{n} Y_i / n$ and $\bar{x} = \sum_{i=1}^{n} X_i / n$ denote the second-phase sample means of the variables *Y* and *X,* respectively.

The usual product estimator for the population mean $\bar{Y}$ under two-phase sampling is given by:

$$\bar{y}_P^d = \bar{y}\left(\frac{\bar{x}}{\bar{x}'}\right) \tag{2}$$

Srivastava (1970) developed the following ratio estimator for $\bar{Y}$ under two-phase sampling:

$$\bar{y}_{ds} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha} \tag{3}$$

where α is a scalar, which is obtained on minimizing the mean square error (MSE) of $\bar{y}_{ds}$.

Chand (1975) defined the following chain ratio-type estimator for $\bar{Y}$ using two auxiliary variables *X* and *Z*, such that *X* is closely related to *Y* as compared to that of *Z* (i.e., $\rho_{YX} > \rho_{YZ} > 0$):

$$\bar{y}_R^{dc} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}'}\right) \tag{4}$$

where $\bar{Z} = \sum_{i=1}^{N} Z_i / N$ denotes the population mean of the variable *Z,* and $\bar{z}' = \sum_{i=1}^{n'} Z_i / n'$ denotes the first phase sample mean of the variable Z.

The usual chain product-type estimator of $\bar{Y}$ in two phase sampling using two auxiliary variables *X* and *Z,* is given by:

$$\bar{y}_P^{dc} = \bar{y}\left(\frac{\bar{x}}{\bar{x}'}\right)\left(\frac{\bar{z}'}{\bar{Z}}\right) \tag{5}$$

Singh and Upadhyaya (1995) suggested the following class of modified chain-type estimators for $\bar{Y}$ by using information on known population coefficient of variation of variable Z:

$$\bar{y}_{SU}^{dc} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}+C_Z}{\bar{z}'+C_Z}\right)^{\alpha} \tag{6}$$

where α is an unknown constant.

Singh *et al.* (2007) suggested the following chain ratio-type estimator for $\bar{Y}$ by utilizing the information on correlation coefficient between the variable *X* and *Z*:

$$\bar{y}_{SEA} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}+\rho_{XZ}}{\bar{z}'+\rho_{XZ}}\right) \tag{7}$$

Singh and Ruiz Espejo (2007) suggested the following ratio-product type estimator for $\bar{Y}$ under two-phase sampling:

$$\bar{y}_{RP}^d = \bar{y}\left[k\frac{\bar{x}'}{\bar{x}} + (1-k)\frac{\bar{x}}{\bar{x}'}\right] \tag{8}$$

Choudhury and Singh (2012) suggested the following class of chain ratio–product type estimators for $\bar{Y}$ under two-phase sampling:

$$\bar{y}_{RP}^{dc} = \bar{y}\left[k\frac{\bar{x}'}{\bar{x}}\frac{\bar{Z}}{\bar{z}'} + (1-k)\frac{\bar{x}}{\bar{x}'}\frac{\bar{z}'}{\bar{Z}}\right] \tag{9}$$

Singh and Choudhury (2012) suggested the following exponential chain ratio and product type estimators for $\bar{Y}$ under two-phase sampling:

$$\bar{y}_{\mathrm{Re}}^{dc} = \bar{y}\exp\left\{\frac{(\bar{x}'/\bar{z}')\bar{Z} - \bar{x}}{(\bar{x}'/\bar{z}')\bar{Z} + \bar{x}}\right\} \tag{10}$$

$$\bar{y}_{Pe}^{dc} = \bar{y}\exp\left\{\frac{\bar{x} - (\bar{x}'/\bar{z}')\bar{Z}}{\bar{x} + (\bar{x}'/\bar{z}')\bar{Z}}\right\} \tag{11}$$

The biases of various pre-existing estimators, under first order of approximation, have been computed and are presented below:

$$B(\bar{y}_R^d) = \bar{Y}\left[f_3(C_X^2 - \rho_{YX}C_YC_X)\right] \tag{12}$$

$$B(\bar{y}_P^d) = \bar{Y}\left[f_3\rho_{YX}C_YC_X\right] \tag{13}$$

$$B(\bar{y}_{ds}) = \bar{Y}\left[f_3\,\alpha\left\{\frac{(\alpha+1)}{2}C_X^2 - \rho_{YX}C_YC_X\right\}\right] \tag{14}$$

$$B(\bar{y}_R^{dc}) = \bar{Y}\left[f_3(C_X^2 - \rho_{YX}C_YC_X) + f_2(C_Z^2 - \rho_{YZ}C_YC_Z)\right] \tag{15}$$

$$B(\bar{y}_P^{dc}) = \bar{Y}\left[f_3\rho_{YX}C_YC_X + f_2\rho_{YZ}C_YC_Z\right] \tag{16}$$

$$B(\bar{y}_{SU}^{dc}) = \bar{Y}\left[f_3(C_X^2 - \rho_{YX}C_YC_X) + f_2\left\{\frac{\alpha(\alpha+1)}{2}\zeta^2C_Z^2 - \alpha\zeta\,\rho_{YZ}C_YC_Z\right\}\right] \tag{17}$$

$$B(\bar{y}_{SEA}) = \bar{Y}\left[f_3(C_X^2 - \rho_{YX}C_YC_X) + f_2(\xi^2C_Z^2 - \xi\,\rho_{YZ}C_YC_Z)\right] \tag{18}$$

$$B(\bar{y}_{RP}^d) = \bar{Y}\left[f_3\left\{k\,C_X^2 - (2k-1)\rho_{YX}\,C_YC_X\right\}\right] \tag{19}$$

$$B\left(\bar{y}_{RP}^{dc}\right)=\bar{Y}\left[k\left(f_3C_X^2+f_2C_Z^2-2f_3\,\rho_{YX}\,C_YC_X-2f_2\,\rho_{YZ}\,C_YC_Z\right)+f_3\,\rho_{YX}\,C_YC_X+f_2\,\rho_{YZ}\,C_YC_Z\right] \quad (20)$$

$$B\left(\bar{y}_{\text{Re}}^{dc}\right)=\bar{Y}\left[\frac{3}{8}(f_3C_X^2+f_2C_Z^2)-\frac{1}{2}(f_3\rho_{YX}C_YC_X+f_2\rho_{YZ}C_YC_Z)\right] \quad (21)$$

$$B\left(\bar{y}_{Pe}^{dc}\right)=\bar{Y}\left[\frac{1}{2}(f_3\rho_{YX}C_YC_X+f_2\rho_{YZ}C_YC_Z)-\frac{1}{8}(f_3C_X^2+f_2C_Z^2)\right] \quad (22)$$

Furthermore, the mean square errors (MSEs) of various estimators, under first order of approximation, are described below:

$$MSE(\bar{y}_R^d)=\bar{Y}^2\left(f_1C_Y^2+f_3C_X^2-2f_3\rho_{YX}C_YC_X\right) \quad (23)$$

$$MSE(\bar{y}_P^d)=\bar{Y}^2\left(f_1C_Y^2+f_3C_X^2+2f_3\rho_{YX}C_YC_X\right) \quad (24)$$

$$MSE(\bar{y}_{ds})=\bar{Y}^2\left\{f_1C_Y^2+f_3\left(\alpha^2C_X^2-2\alpha\rho_{YX}C_YC_X\right)\right\} \quad (25)$$

$$MSE(\bar{y}_R^{dc})=\bar{Y}^2\left\{f_1C_Y^2+f_3C_X^2+f_2C_Z^2-2f_3\rho_{YX}C_YC_X-2f_2\rho_{YZ}C_YC_Z\right\} \quad (26)$$

$$MSE(\bar{y}_P^{dc})=\bar{Y}^2\left\{f_1C_Y^2+f_3C_X^2+f_2C_Z^2+2f_3\rho_{YX}C_YC_X+2f_2\rho_{YZ}C_YC_Z\right\} \quad (27)$$

$$MSE(\bar{y}_{SU}^{dc})=\bar{Y}^2\left\{f_1C_Y^2+f_2\left(\alpha^2\zeta^2C_Z^2-2\alpha\zeta\rho_{YZ}C_YC_Z\right)+f_3\left(C_X^2-2\rho_{YX}C_YC_X\right)\right\} \quad (28)$$

$$MSE(\bar{y}_{SEA})=\bar{Y}^2\left\{f_1C_Y^2+f_3\,C_X^2+\xi^2f_2C_Z^2-2f_3\rho_{YX}C_YC_X-2\xi\,f_2\rho_{YZ}C_YC_Z\right\} \quad (29)$$

$$MSE(\bar{y}_{RP}^d)=\bar{Y}^2\left\{f_1C_Y^2+4k^2f_3C_X^2-4kf_3\left(C_X^2+\rho_{YX}C_YC_X\right)+f_3\left(C_X^2+2\rho_{YX}C_YC_X\right)\right\} \quad (30)$$

$$MSE(\bar{y}_{RP}^{dc})=\bar{Y}^2\left\{f_1C_Y^2+\left(2k-1\right)^2\left(f_3C_X^2+f_2C_Z^2\right)-2\left(2k-1\right)\left(f_3\rho_{YX}C_YC_X+f_2\rho_{YZ}C_YC_Z\right)\right\} \quad (31)$$

$$MSE(\bar{y}_{\text{Re}}^{dc})=\bar{Y}^2\left\{f_1C_Y^2+\frac{1}{4}\left(f_3C_X^2+f_2C_Z^2\right)-\left(f_3\rho_{YX}C_YC_X+f_2\rho_{YZ}C_YC_Z\right)\right\} \quad (32)$$

$$MSE(\bar{y}_{Pe}^{dc})=\bar{Y}^2\left\{f_1C_Y^2+\frac{1}{4}\left(f_3C_X^2+f_2C_Z^2\right)+\left(f_3\rho_{YX}C_YC_X+f_2\rho_{YZ}C_YC_Z\right)\right\} \quad (33)$$

Moreover, the minimum attainable MSEs of the estimators $\bar{y}_{ds}$, $\bar{y}_{SU}^{dc}$, $\bar{y}_{RP}^{d}$ and $\bar{y}_{RP}^{dc}$ are given, respectively, by:

$$MSE(\bar{y}_{ds})_{\min}=\bar{Y}^2C_Y^2\left(f_1-f_3\rho_{YX}^2\right) \quad (34)$$

$$MSE(\bar{y}_{SU}^{dc})_{\min}=\bar{Y}^2\left\{f_1C_Y^2+f_3\left(C_X^2-2\rho_{YX}C_YC_X\right)-f_2\rho_{YZ}^2C_Y^2\right\} \quad (35)$$

$$MSE(\bar{y}_{RP}^{d})_{\min}=\bar{Y}^2C_Y^2\left(f_1-f_3\rho_{YX}^2\right) \quad (36)$$

$$MSE(\bar{y}_{RP}^{dc})_{\min}=\bar{Y}^2\left\{f_1C_Y^2-\frac{\left(f_3\rho_{YX}C_YC_X+f_2\rho_{YZ}C_YC_Z\right)^2}{f_3C_X^2+f_2C_Z^2}\right\} \quad (37)$$

The notations used are described below:

$$f_1=\left(\frac{1}{n}-\frac{1}{N}\right),\; f_2=\left(\frac{1}{n'}-\frac{1}{N}\right),\; f_3=f_1-f_2=\left(\frac{1}{n}-\frac{1}{n'}\right),$$

$$\xi=\frac{\bar{Z}}{(\bar{Z}+\rho_{XZ})},\; \zeta=\frac{\bar{Z}}{(\bar{Z}+C_Z)},\; C_Y^2=\frac{S_Y^2}{\bar{Y}^2},\; C_X^2=\frac{S_X^2}{\bar{X}^2},$$

$$C_Z^2=\frac{S_Z^2}{\bar{Z}^2},\; \rho_{YX}=\frac{S_{YX}}{S_YS_X},\; \rho_{YZ}=\frac{S_{YZ}}{S_YS_Z},\; \rho_{XZ}=\frac{S_{XZ}}{S_XS_Z},$$

$$S_Y^2=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)^2,\; S_X^2=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(X_i-\bar{X}\right)^2,$$

$$S_Z^2=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(Z_i-\bar{Z}\right)^2,\; S_{YX}=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)\left(X_i-\bar{X}\right),$$

$$S_{YZ}=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)\left(Z_i-\bar{Z}\right),$$

$$S_{XZ}=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(X_i-\bar{X}\right)\left(Z_i-\bar{Z}\right).$$

2.2 Proposed Class of Estimators

Motivated by Singh *et al.* (2007), and Singh and Ruiz Espejo (2007), we propose the following transformed class of ratio-cum-product estimators for population mean $\bar{Y}$ in two-phase sampling:

$$T=\bar{y}\left[k\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\alpha\bar{Z}+\gamma}{\alpha\bar{z}'+\gamma}\right)+\left(1-k\right)\left(\frac{\bar{x}}{\bar{x}'}\right)\left(\frac{\alpha\bar{z}'+\gamma}{\alpha\bar{Z}+\gamma}\right)\right] \quad (38)$$

where the terms *α*, *γ* and *k* denote the scalars. The optimum values of these scalars are obtained on minimizing the MSE of proposed class *T*.

Some of the pre-existing estimators, as mentioned in Sub-section 2.1, are shown to be the members of the proposed class *T* by assigning specific values to the scalars *k*, *α* and *γ* in (38), as depicted in Table 1.

Table 1. Members of the proposed class T

Sl. No.	Authors	Estimators	Values assigned to the scalars in the proposed class T		
			k	α	γ
1	Sukhatme (1962)	$\bar{y}_R^d = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)$	1	0	1
		$\bar{y}_P^d = \bar{y}\left(\frac{\bar{x}}{\bar{x}'}\right)$	0	0	1
2	Chand (1975)	$\bar{y}_R^{dc} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}'}\right)$	1	1	0
		$\bar{y}_P^{dc} = \bar{y}\left(\frac{\bar{x}}{\bar{x}'}\right)\left(\frac{\bar{z}'}{\bar{Z}}\right)$	0	1	0
3	Singh *et al.* (2007)	$\bar{y}_{SEA} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}+\rho_{XZ}}{\bar{z}'+\rho_{XZ}}\right)$	1	1	ρ_{XZ}
4	Singh and Ruiz Espejo (2007)	$\bar{y}_{RP}^d = \bar{y}\left[k\frac{\bar{x}'}{\bar{x}}+(1-k)\frac{\bar{x}}{\bar{x}'}\right]$	k	0	1
5	Choudhury and Singh (2012)	$\bar{y}_{RP}^{dc} = \bar{y}\left[k\frac{\bar{x}'}{\bar{x}}\frac{\bar{Z}}{\bar{z}'}+(1-k)\frac{\bar{x}}{\bar{x}'}\frac{\bar{z}'}{\bar{Z}}\right]$	k	1	0

2.3 Bias and MSE of the Proposed Class

The mathematical expressions for bias and MSE of the proposed class T is obtained on considering the following assumptions:

$$\bar{y} = \bar{Y}(1+e_0), \qquad \bar{x} = \bar{X}(1+e_1), \qquad \bar{x}' = \bar{X}(1+e_1'), \qquad \bar{z}' = \bar{Z}(1+e_2').$$

Hence, we have

$$\left.\begin{aligned} &E(e_0)=E(e_1)=E(e_1')=E(e_2')=0,\\ &E(e_0^2)=f_1C_Y^2, E(e_1^2)=f_1C_X^2,\ E(e_1'^2)=f_2C_X^2,\\ &E(e_2'^2)=f_2C_Z^2, E(e_0e_1)=f_1\rho_{YX}C_YC_X,\\ &E(e_0e_1')=f_2\rho_{YX}C_YC_X, E(e_0e_2')=f_2\rho_{YZ}C_YC_Z,\\ &E(e_1e_1')=f_2C_X^2, E(e_1e_2')=f_2\rho_{XZ}C_XC_Z,\\ &E(e_1'e_2')=f_2\rho_{XZ}C_XC_Z. \end{aligned}\right\} \tag{39}$$

Now, expressing T in terms of e_0, e_1, e_1' and e_2', we have

$$T = \bar{Y}(1+e_0)\left[kU_1+(1-k)U_2\right], \tag{40}$$

where

$$U_1 = (1+e_1')(1+e_1)^{-1}\left(1+\psi e_2'\right)^{-1},$$

$$U_2 = (1+e_1)(1+e_1')^{-1}(1+\psi e_2') \text{ and } \psi = \alpha\bar{Z}/(\alpha\bar{Z}+\gamma).$$

Hence, on simplifying (40), taking the expectation, and using results of (39), the bias of proposed class T, to the first order of approximation, is obtained as follows:

$$\begin{aligned} B(T) &= E(T)-\bar{Y}\\ &= \bar{Y}\Big[k\left(\psi^2 f_2C_Z^2 + f_3C_X^2 - 2\psi f_2\rho_{YZ}C_YC_Z - \right.\\ &\qquad \left. 2f_3\rho_{YX}C_YC_X\right) + f_3\rho_{YX}C_YC_X + \psi f_2\rho_{YZ}C_YC_Z\Big] \end{aligned} \tag{41}$$

Again, on simplifying (40), and retaining the first order error terms, we have

$$T-\bar{Y} = \bar{Y}\left[e_0 + (2k-1)(e_1'-e_1-\psi e_2')\right] \tag{42}$$

Squaring both sides of (42), taking the expectation, and using results of (39), we obtain the *MSE* of proposed class T, to the first order of approximation, as follows:

$$\begin{aligned} MSE(T) = \bar{Y}^2\Big[&f_1C_Y^2 + (2k-1)^2\left(f_3C_X^2+\psi^2 f_2C_Z^2\right) - \\ &2(2k-1)\left(f_3\rho_{YX}C_YC_X + \psi f_2\rho_{YZ}C_YC_Z\right)\Big] \end{aligned} \tag{43}$$

The optimum value of ψ in (43) is obtained on minimizing the $MSE(T)$ with respect to ψ, and hence is obtained as follows:

$$\psi_{opt} = \frac{\rho_{YZ}C_Y}{C_Z(2k-1)} \tag{44}$$

Again, on substituting the optimum value of ψ from (44) in (43), the *MSE(T)* in terms of k is obtained as follows:

$$\begin{aligned} MSE(T) = \bar{Y}^2\Big[&f_1C_Y^2 - f_2\rho_{YZ}^2C_Y^2 - 2(2k-1)f_3\rho_{YX}C_YC_X + \\ &(2k-1)^2 f_3C_X^2\Big] \end{aligned} \tag{45}$$

Further, on minimizing (45) with respect to k, the optimum value of k is obtained as follows:

$$k_{opt} = \frac{1}{2}\left(1+\frac{\rho_{YX}C_Y}{C_X}\right) \tag{46}$$

Hence, on substituting the value of k from (46) in (45), the minimum attainable MSE of the proposed class T is obtained as follows:

$$MSE(T)_{\min} = \bar{Y}^2C_Y^2(f_1 - f_2\rho_{YZ}^2 - f_3\rho_{YX}^2) \tag{47}$$

Thus, we establish the resulting theorem.

Theorem 2.3.1 *To the first order of approximation, we have*

$$MSE(T) \geq \bar{Y}^2C_Y^2(f_1 - f_2\rho_{YZ}^2 - f_3\rho_{YX}^2) \tag{48}$$

with equality holding if $k=\frac{1}{2}\left(1+\frac{\rho_{YX}C_Y}{C_X}\right)$, and $\psi=\frac{\rho_{YZ}C_Y}{C_Z\left(2k-1\right)}$.

2.4 Efficiency Comparisons

The variance of sample mean $\bar{y}$ under simple random sampling without replacement (SRSWOR) scheme is given by

$$Var(\bar{y})=f_1\bar{Y}^2C_Y^2 \tag{49}$$

The proposed class T is compared with the pre-existing estimators on the basis of MSE criterion, by utilizing the equations (23) to (33), (43), and (49) as follows:

(i) $MSE\left(T\right)<Var\left(\bar{y}\right)$ if

$$C_Y>\frac{1}{2}\left[\frac{(2k-1)(f_3C_X^2+\psi^2f_2C_Z^2)}{f_3\rho_{YX}C_X+\psi f_2\rho_{YZ}C_Z}\right] \tag{50}$$

(ii) $MSE\left(T\right)<MSE\left(\bar{y}_R^d\right)$ if

$$C_Y<\frac{1}{2}\left[\frac{4k(1-k)f_3C_X^2-(2k-1)^2\psi^2f_2C_Z^2}{2kf_3\rho_{YX}C_X+(2k-1)\psi f_2\rho_{YZ}C_Z}\right] \tag{51}$$

(iii) $MSE\left(T\right)<MSE\left(\bar{y}_P^d\right)$ if

$$C_Y<\frac{1}{2}\left[\frac{4k(1-k)f_3C_X^2-(2k-1)^2\psi^2f_2C_Z^2}{2(k-1)f_3\rho_{YX}C_X+(2k-1)\psi f_2\rho_{YZ}C_Z}\right] \tag{52}$$

(iv) $MSE\left(T\right)<MSE\left(\bar{y}_{ds}\right)$ if

$$C_Y>\frac{1}{2}\left[\frac{\left\{(2k-1)^2-\alpha^2\right\}f_3C_X^2+(2k-1)^2\psi^2f_2C_Z^2}{\left\{(2k-1)-\alpha\right\}f_3\rho_{YX}C_X+(2k-1)\psi f_2\rho_{YZ}C_Z}\right] \tag{53}$$

(v) $MSE\left(T\right)<MSE\left(\bar{y}_R^{dc}\right)$ if

$$C_Y>\frac{1}{2}\left[\frac{4k(k-1)f_3C_X^2+\left\{\psi^2(2k-1)^2-1\right\}f_2C_Z^2}{2(k-1)f_3\rho_{YX}C_X+\left\{\psi(2k-1)-1\right\}f_2\rho_{YZ}C_Z}\right] \tag{54}$$

(vi) $MSE\left(T\right)<MSE\left(\bar{y}_P^{dc}\right)$ if

$$C_Y>\frac{1}{2}\left[\frac{4k(k-1)f_3C_X^2+\left\{\psi^2(2k-1)^2-1\right\}f_2C_Z^2}{2kf_3\rho_{YX}C_X+\left\{\psi(2k-1)+1\right\}f_2\rho_{YZ}C_Z}\right] \tag{55}$$

(vii) $MSE\left(T\right)<MSE\left(\bar{y}_{SU}^{dc}\right)$ if

$$C_Y>\frac{1}{2}\left[\frac{4k(k-1)f_3C_X^2+\left\{\psi^2(2k-1)^2-\alpha^2\zeta^2\right\}f_2C_Z^2}{2f_3\rho_{YX}C_X(k-1)+\left\{\psi(2k-1)-\alpha\zeta\right\}f_2\rho_{YZ}C_Z}\right] \tag{56}$$

(viii) $MSE\left(T\right)<MSE\left(\bar{y}_{SEA}\right)$ if

$$C_Y>\frac{4k(k-1)f_3C_X^2+\left\{(2k-1)^2\psi^2-\xi^2\right\}f_2C_Z^2}{4(k-1)f_3\rho_{YX}C_X+(\psi-2\xi)f_2\rho_{YZ}C_Z} \tag{57}$$

(ix) $MSE\left(T\right)<MSE\left(\bar{y}_{RP}^d\right)$ if

$$C_Y>\frac{\left(2k-1\right)^2\psi C_Z}{\rho_{YZ}} \tag{58}$$

(x) $MSE\left(T\right)<MSE\left(\bar{y}_{RP}^{dc}\right)$ if

$$C_Y>\frac{(2k-1)(\psi+1)C_Z}{2\rho_{YZ}} \tag{59}$$

(xi) $MSE\left(T\right)<MSE\left(\bar{y}_{\mathrm{Re}}^{dc}\right)$ if

$$C_Y>\frac{1}{4}\left[\frac{\left\{4(2k-1)^2-1\right\}f_3C_X^2+\left\{4(2k-1)^2\psi^2-1\right\}f_2C_Z^2}{\left\{2(2k-1)-1\right\}f_3\rho_{YX}C_X+\left\{2(2k-1)\psi-1\right\}f_2\rho_{YZ}C_Z}\right] \tag{60}$$

(xii) $MSE\left(T\right)<MSE\left(\bar{y}_{Pe}^{dc}\right)$ if

$$C_Y>\frac{1}{4}\left[\frac{\left\{4(2k-1)^2-1\right\}f_3C_X^2+\left\{4(2k-1)^2\psi^2-1\right\}f_2C_Z^2}{\left\{2(2k-1)+1\right\}f_3\rho_{YX}C_X+\left\{2(2k-1)\psi+1\right\}f_2\rho_{YZ}C_Z}\right] \tag{61}$$

3. RESULTS AND DISCUSSION

3.1 Empirical Analysis

In this section, the empirical results are obtained on considering five real population datasets. The descriptions of the populations, along with the values of various parameters, are elaborated below:

Population I- [Source: Handique (2012)]

Y: Forest timber volume in cubic meter (Cum) in 0.1 ha sample plot,

X: Average tree height in the sample plot in meter (m),

Z: Average crown diameter in the sample plot in meter (m),

N=2500, $n'=200, n=25$, $\bar{Y}=4.63$, $\bar{X}=21.09$, $\bar{Z}=13.55$, $\rho_{YX}=0.79$, $\rho_{YZ}=0.72$, $\rho_{XZ}=0.66, C_Y=0.95$, $C_X=0.98$, $C_Z=0.64$.

Population II- [Source: Murthy (1967)]

Y: Area under wheat in 1964,

X: Area under wheat in 1963,

Z: Cultivated area in 1961,

N=34, $n' = 10$, $n = 7$, $\bar{Y} = 199.44$, $\bar{X} = 208.89$, $\bar{Z} = 747.59$, $\rho_{YX} = 0.9801$, $\rho_{YZ} = 0.9043$, $\rho_{XZ} = 0.9097$, $C_Y^2 = 0.5673$, $C_X^2 = 0.5191$, $C_Z^2 = 0.3527$.

Population III- [Source- Sahoo and Swain (1980)]

Y: Yield of rice per plant,

X: Number of tillers,

Z.: Percentage of sterility,

N = 50, n' = 30, n = 15, $\bar{Y}$ = 12.842, $\bar{X}$ = 9.04, $\bar{Z}$ = 18.77, $\rho_{YX} = 0.7133$, ρ_{YZ} = -0.2509, $\rho_{XZ} = 0.0224$, C_Y = 0.3957, C_X = 0.2627, C_Z = 0.0970.

Population IV- [Source: Srivnstava *et al.* (1989)]

Y: The measurement of weight of children,

X: Mid arm circumference of children,

Z: Skull circumference of children,

N = 55, n' = 30, n = 18, $\bar{Y}$ = 17.08, $\bar{X}$ = 16.92, $\bar{Z}$ = 50.44, ρ_{YX} = 0.54, ρ_{YZ} = 0.51, ρ_{XZ} = –0.08, C_Y^2 = 0.0161, C_X^2 = 0.0049, C_Z^2 = 0.0007.

Population V- [Source: Sukhatme and Chand (1977)]

Y: Apple trees of bearing age in 1964,

X: Bushels of apples harvested in 1964,

Z: Bushels of apples harvested in 1959,

N=200, $n' = 30$, $n = 20$, $\bar{Y} = 1031.82$, $\bar{X} = 2934.58$, $\bar{Z} = 3651.49$, $\rho_{YX} = 0.93$, $\rho_{YZ} = 0.77$, $\rho_{XZ} = 0.84$, $C_Y^2 = 2.55280$, $C_X^2 = 4.02504$, $C_Z^2 = 2.09379$.

The optimum values of the scalars k and ψ are computed for the above mentioned populations, and the findings are depicted in Table 2.

Table 2. Optimum values of the scalars k and ψ for various populations

Scalars	Populations				
	I	II	III	IV	V
$k_{opt} = \frac{1}{2}\left(1+\frac{\rho_{YX}C_Y}{C_X}\right)$	0.8829	1.0123	1.0372	0.9894	0.8703
$\psi_{opt} = \frac{\rho_{YZ}C_Y}{C_Z(2k-1)}$	1.3956	1.1194	-0.9526	2.5042	1.1480

The percent absolute relative biases (PARBs) of various estimators of $\bar{Y}$ are computed and elaborated in Table 3. The PARBs are obtained on using the following formula:

$$PARB(\phi) = \left|\frac{Bias(\phi)}{\bar{Y}}\right| \times 100 = \left|\frac{E(\phi)-\bar{Y}}{\bar{Y}}\right| \times 100,$$

where

$\phi = \bar{y}, \bar{y}_R^d, \bar{y}_P^d, \bar{y}_{ds}, \bar{y}_R^{dc}, \bar{y}_P^{dc}, \bar{y}_{SU}^{dc}, \bar{y}_{SEA}, \bar{y}_{RP}^d, \bar{y}_{RP}^{dc}, \bar{y}_{\mathrm{Re}}^{dc}, \bar{y}_{Pe}^{dc}, T$.

Table 3. PARBs of various estimators of $\bar{Y}$

Estimator	Population				
	I	II	III	IV	V
$\bar{y}$	0.0000	0.0000	0.0000	0.0000	0.0000
$\bar{y}_R^d$	0.7872	0.0547	0.0171	0.0002	1.7399
$\bar{y}_P^d$	*	*	*	*	*
$\bar{y}_{ds}$	0.3014	0.0280	0.0092	0.0001	0.6443
$\bar{y}_R^{dc}$	0.7742	0.4204	0.0083	0.0013	2.6284
$\bar{y}_P^{dc}$	*	*	*	*	*
$\bar{y}_{SU}^{dc}$	0.7757	0.2655	0.0301	0.0017	2.1166
$\bar{y}_{SEA}$	0.7665	0.4230	0.0082	0.0013	2.6269
$\bar{y}_{RP}^d$	0.9964	0.0834	0.0270	0.0003	2.1586
$\bar{y}_{RP}^{dc}$	0.9925	0.6681	0.0121	0.0017	3.4801
$\bar{y}_{\mathrm{Re}}^{dc}$	0.0566	0.7995	0.0262	0.0021	0.2659
$\bar{y}_{Pe}^{dc}$	*	*	*	*	*
T	1.1052	0.2004	0.0283	0.0005	4.6741

* Data is not applicable for the concerned estimators.

Furthermore, the percent relative efficiencies (PREs) of various estimators of $\bar{Y}$ are computed and depicted in Table 4. The PREs are obtained with respect to the sample mean $\bar{y}$ on using the following formula:

$$PRE(\varphi, \bar{y}) = \frac{Var(\bar{y})}{MSE(\varphi)} \times 100,$$

where

$\phi = \bar{y}, \bar{y}_R^d, \bar{y}_P^d, \bar{y}_{ds}, \bar{y}_R^{dc}, \bar{y}_P^{dc}, \bar{y}_{SU}^{dc}, \bar{y}_{SEA}, \bar{y}_{RP}^d, \bar{y}_{RP}^{dc}, \bar{y}_{\mathrm{Re}}^{dc}, \bar{y}_{Pe}^{dc}, T$.

Table 4. PREs of various estimators of $\bar{Y}$

Estimator	Population				
	I	II	III	IV	V
$\bar{y}$	100.00	100.00	100.00	100.00	100.00
$\bar{y}_R^d$	200.01	156.91	156.66	120.96	139.09
$\bar{y}_P^d$	*	*	*	*	*
$\bar{y}_{ds}$	223.02	156.96	157.09	120.98	147.13
$\bar{y}_R^{dc}$	227.27	730.81	144.80	131.91	279.93
$\bar{y}_P^{dc}$	*	*	*	*	*
$\bar{y}_{SU}^{dc}$	227.40	778.27	161.20	138.65	289.32
$\bar{y}_{SEA}$	227.04	730.07	144.81	131.92	279.96
$\bar{y}_{RP}^d$	223.02	156.96	157.09	120.98	147.13
$\bar{y}_{RP}^{dc}$	254.61	763.30	144.88	132.32	322.95
$\bar{y}_{\mathrm{Re}}^{dc}$	212.00	259.55	131.18	120.57	247.82
$\bar{y}_{Pe}^{dc}$	*	*	*	*	*
T	**257.61**	**779.54**	**161.66**	**138.66**	**326.41**

* Data is not applicable for the concerned estimators.
Bold values indicate the maximum PREs.

3.2 Outcomes of the Analysis

The following results are obtained from Table 3:

(i) In population I, we have

$$PARB\left(\bar{y}_{\mathrm{Re}}^{dc}\right) < PARB\left(\bar{y}_{ds}\right) < PARB\left(\bar{y}_{SEA}\right) < PARB\left(\bar{y}_R^{dc}\right) < PARB\left(\bar{y}_{SU}^{dc}\right) < PARB\left(\bar{y}_R^{d}\right) < PARB\left(\bar{y}_{RP}^{dc}\right) < PARB\left(\bar{y}_{RP}^{d}\right) < PARB\left(T\right)$$

(ii) In population II, we have

$$PARB\left(\bar{y}_{ds}\right) < PARB\left(\bar{y}_R^{d}\right) < PARB\left(\bar{y}_{RP}^{d}\right) < PARB\left(T\right) < PARB\left(\bar{y}_{SU}^{dc}\right) < PARB\left(\bar{y}_R^{dc}\right) < PARB\left(\bar{y}_{SEA}\right) < PARB\left(\bar{y}_{RP}^{dc}\right) < PARB\left(\bar{y}_{\mathrm{Re}}^{dc}\right)$$

(iii) In population III, we have

$$PARB\left(\bar{y}_{SEA}\right) < PARB\left(\bar{y}_R^{dc}\right) < PARB\left(\bar{y}_{ds}\right) < PARB\left(\bar{y}_{RP}^{dc}\right) < PARB\left(\bar{y}_R^{d}\right) < PARB\left(\bar{y}_{\mathrm{Re}}^{dc}\right) < PARB\left(\bar{y}_{RP}^{d}\right) < PARB\left(T\right) < PARB\left(\bar{y}_{SU}^{dc}\right)$$

(iv) In population IV, we have

$$PARB\left(\bar{y}_{ds}\right) < PARB\left(\bar{y}_R^{d}\right) < PARB\left(\bar{y}_{RP}^{d}\right) < PARB\left(T\right) < PARB\left(\bar{y}_R^{dc}\right) < PARB\left(\bar{y}_{SEA}\right) < PARB\left(\bar{y}_{SU}^{dc}\right) < PARB\left(\bar{y}_{RP}^{dc}\right) < PARB\left(\bar{y}_{\mathrm{Re}}^{dc}\right)$$

(v) In population V, we have

$$PARB\left(\bar{y}_{\mathrm{Re}}^{dc}\right) < PARB\left(\bar{y}_{ds}\right) < PARB\left(\bar{y}_R^{d}\right) < PARB\left(\bar{y}_{SU}^{dc}\right) < PARB\left(\bar{y}_{RP}^{d}\right) < PARB\left(\bar{y}_{SEA}\right) < PARB\left(\bar{y}_R^{dc}\right) < PARB\left(\bar{y}_{RP}^{dc}\right) < PARB\left(T\right)$$

Moreover, the following results are obtained from Table 4:

1) In all the five populations, the PREs of the proposed class T are maximum as compared to the sample mean ($\bar{y}$) and the other pre-existing estimators.
2) In all the five populations, the PREs of Srivastava (1970) estimator $\bar{y}_{ds}$ are same as that of Singh and Ruiz Espejo (2007) estimator $\bar{y}_{RP}^d$.
3) In all the five populations, the PREs of Chand (1975) estimator $\bar{y}_R^{dc}$ are nearly the same as that of Singh *et al.* (2007) estimator $\bar{y}_{SEA}$.
4) Among the members of proposed class T , the PREs of Choudhury and Singh (2012) estimator $\bar{y}_{RP}^{dc}$ are more as compared to other members in populations I, II, IV and V.
5) In all the five populations, the PREs of the estimators $\bar{y}_P^d$, $\bar{y}_P^{dc}$, and $\bar{y}_{Pe}^{dc}$ are not applicable as the theoretical condition $(\rho_{YX}\, C_Y/C_X) < -1/2$ is not satisfied.

3.3 Simulation Study

A simulation study is conducted to examine the influence of pre-defined parametric values viz. ρ_{YX} , ρ_{YZ} , C_Y , C_X and C_Z on the PREs of the proposed class *T*, as well as on the PREs of the other pre-existing estimators. The graphical analysis is used for assessing and demonstrating the relative performances of various estimators by using Mathematica software.

The pre-defined parametric values used in simulation analysis are: C_Y = 0.75, C_X = 0.70 and C_Z = 0.65. Also, the values of ρ_{YX} and ρ_{YZ} are considered in the entire interval [-1,1]. The graphical

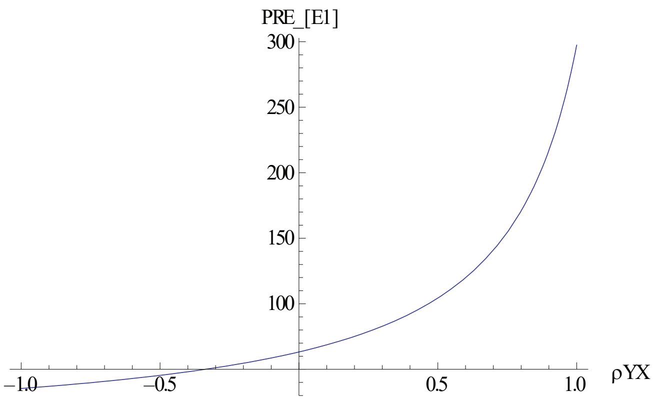

Fig. 1. PRE of E1 (i.e., $\bar{y}_R^d$)

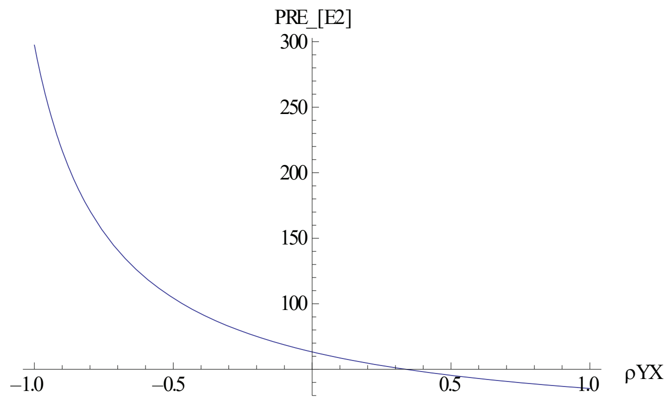

Fig. 2. PRE of E2 (i.e., $\bar{y}_P^d$)

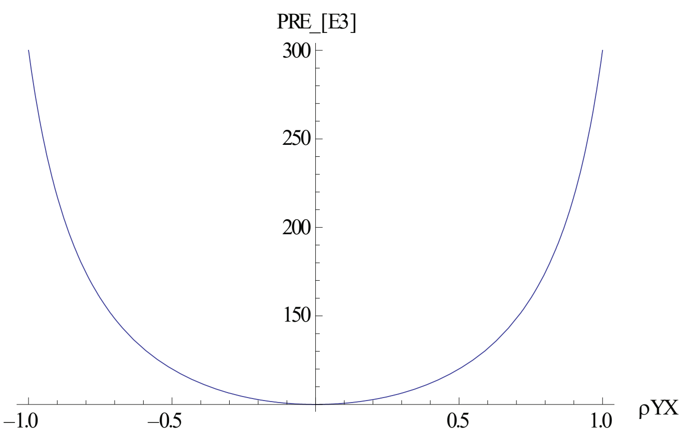

Fig. 3. PRE of E3 (i.e., $\bar{y}_{ds}$)

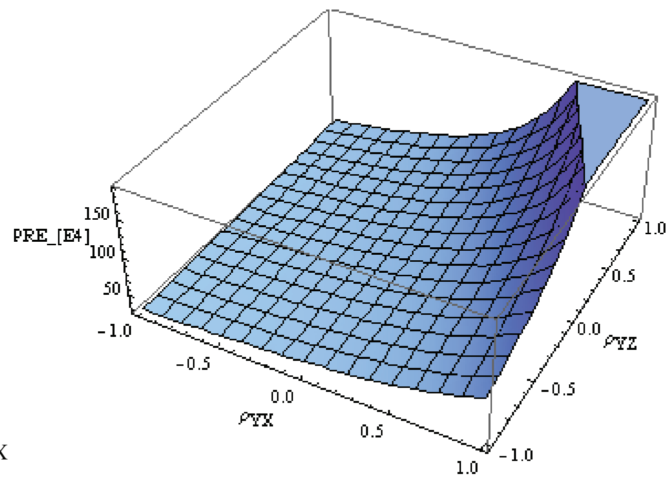

Fig. 4. PRE of E4 (i.e., $\bar{y}_R^{dc}$)

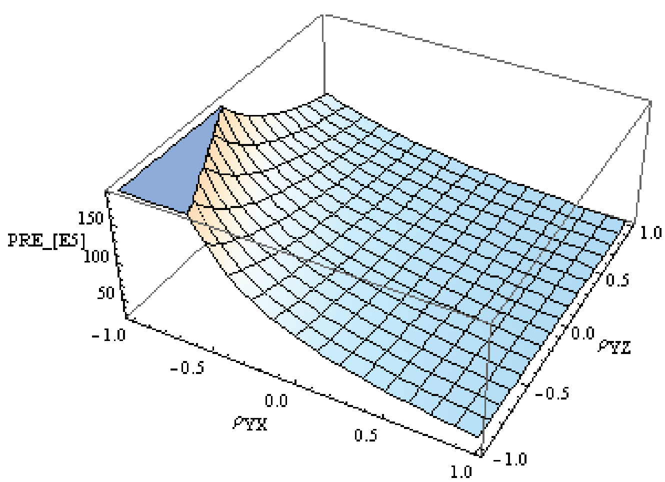

Fig. 5. PRE of E5 (i.e., $\bar{y}_P^{dc}$)

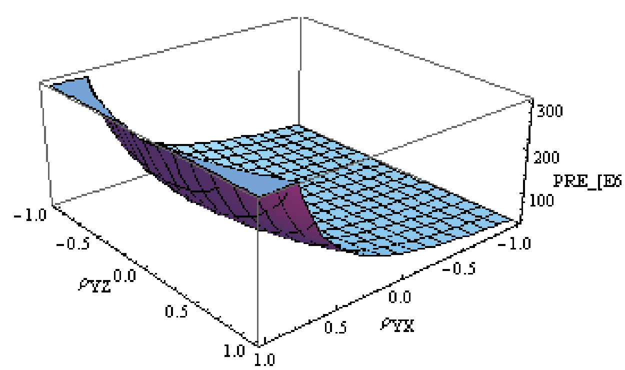

Fig. 6. PRE of E6 (i.e., $\bar{y}_{SU}^{dc}$)

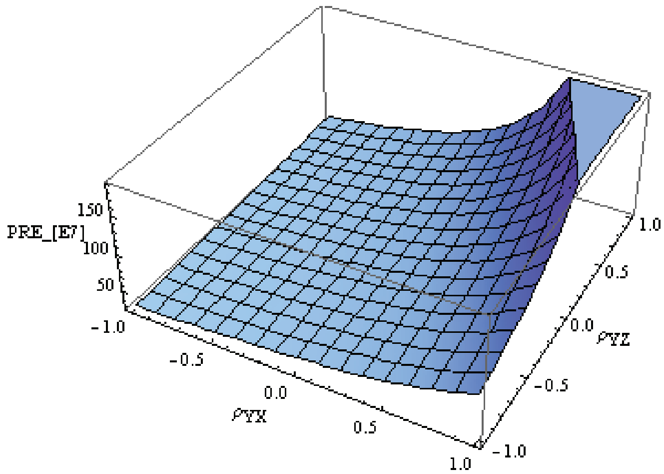

Fig. 7. PRE of E7 (i.e., $\bar{y}_{SEA}$)

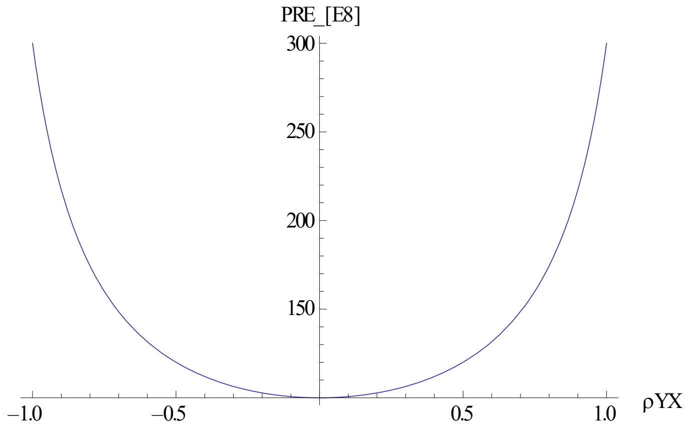

Fig. 8. PRE of E8 (i.e., $\bar{y}^{d}_{RP}$)

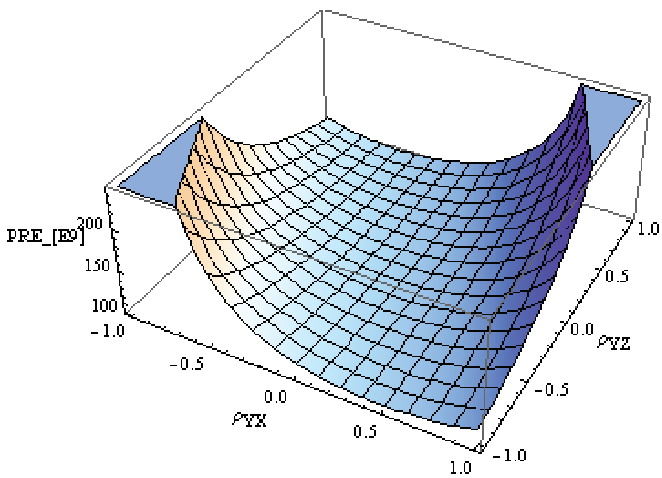

Fig. 9. PRE of E9 (i.e., $\bar{y}^{dc}_{RP}$)

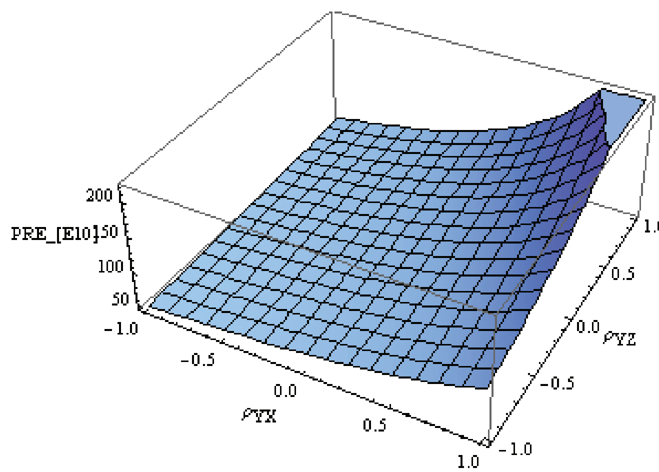

Fig. 10. PRE of E10 (i.e., $\bar{y}^{dc}_{\mathrm{Re}}$)

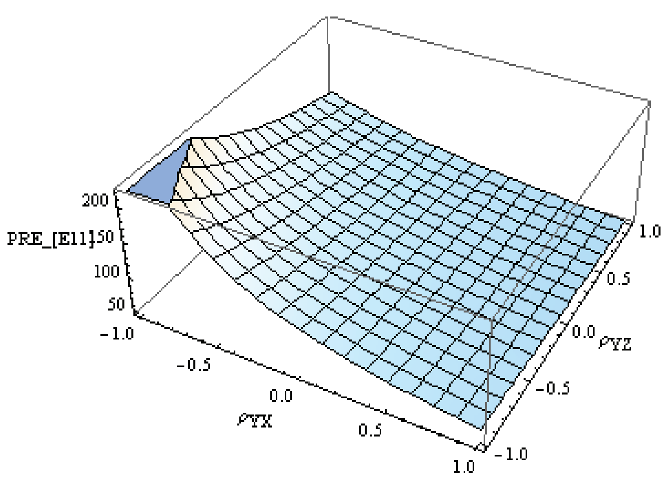

Fig. 11. PRE of E11 (i.e., $\bar{y}^{dc}_{Pe}$)

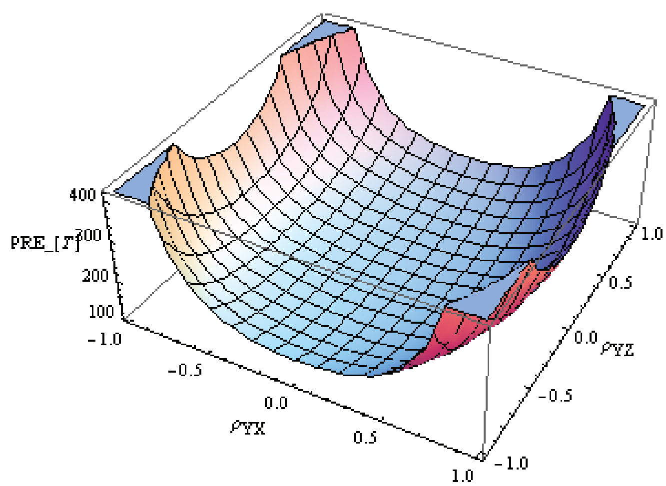

Fig. 12. PRE of proposed class *T*

representations of the PREs of various estimators of $\bar{Y}$, for the pre-defined parametric values, are obtained and demonstrated from Fig. 1 to Fig. 12. Also, for the sake of convenience, the notations used for pre-existing estimators in graphs are as follows:

$$\bar{y}_R^d = E1,\ \ \bar{y}_P^d = E2,\ \ \bar{y}_{ds} = E3,\ \ \bar{y}_R^{dc} = E4,\ \ \bar{y}_P^{dc} = E5,$$
$$\bar{y}_{SU}^{dc} = E6,\ \bar{y}_{SEA} = E7,\ \ \bar{y}_{RP}^d = E8,\ \ \bar{y}_{RP}^{dc} = E9,\ \ \bar{y}_{\text{Re}}^{dc} = E10,$$
$$\bar{y}_{Pe}^{dc} = E11.$$

From Fig. 1 to Fig. 12, we observe that the values of PREs of pre-existing estimators and proposed class *T* are changing for the change in values of ρ_{YX} and ρ_{YZ} in the entire interval [-1,1]. Moreover, from Fig. 3 and Fig. 8, it is seen that the performances of estimators E3 (i.e., $\bar{y}_{ds}$) and E8 (i.e., $\bar{y}_{RP}^d$) are the same, as observed earlier in the results of empirical analysis. Furthermore, Fig. 4 and Fig. 7 exhibit that the performances of estimators E4 (i.e., $\bar{y}_R^{dc}$) and E7 (i.e., $\bar{y}_{SEA}$) are the same, as seen earlier in the results of empirical analysis. Also, Fig. 12 reveal that the proposed class *T* achieves the highest PRE as compared to the other pre-existing estimators, and hence it is superior as compared to other estimators for the estimation of population mean $\bar{Y}$ of the study variable *Y*.

4. CONCLUSION

In the present paper, a transformed class of ratio-cum-product estimators has been developed for estimating the population mean $\bar{Y}$ of the study variable *Y* in two-phase sampling. It has been established in Sub-section 2.2 that the proposed class *T* constitutes a wide range of members for specific choices of the scalars k, α and γ. Also, in Sub-section 2.3, the mathematical expressions for bias and MSE of the proposed class *T* are derived to the first order of approximation. Moreover, in Sub-section 3.1, the PARBs and PREs are computed for various estimators of $\bar{Y}$ on considering five real population datasets. Finally, in Sub-section 3.3, a simulation study is conducted on considering pre-defined parametric values of the parameters of study variable *Y* and auxiliary variables (X, Z). Furthermore, on the basis of results of empirical analysis and simulation study, it is revealed that the proposed class *T* achieves the highest PREs as compared to the PREs of usual unbiased estimator (i.e., the sample mean $\bar{y}$) and the pre-existing estimators in the concerned populations.

Hence, in view of the theoretical findings, and the results of empirical and simulation analysis, we conclude that the proposed class *T* is superior, as compared to the sample mean $\bar{y}$ and the pre-existing estimators, for the estimation of population mean $\bar{Y}$ of the study variable *Y*.

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REFERENCES

Chand, L. (1975). Some ratio type estimators based on two or more auxiliary variables. Ph.D. dissertation. Iowa State University, Ames, Iowa (USA).

Choudhury, S., and Singh, B.K., (2012). A class of chain ratio–product type estimators with two auxiliary variables under double sampling scheme. *Journal of the Korean Statistical Society*, 41, 247-256.

Dubey, S. K., Sisodia, B.V.S. and Sharma, M.K. (2020). Some transformed and composite chain ratio-type estimators using two auxiliary variables. *Journal of the Indian Society of Agricultural Statistics*, 74(1), 17-22.

Gupta, S. and Shabbir, J. (2007). On the use of transformed auxiliary variables in estimating population mean by using two auxiliary variables. *Journal of Statistical Planning and Inference*, 137, 1606-1611.

Handique, B.K. (2012). A class of regression-cum-ratio estimators in two-phase sampling for utilizing information from high resolution satellite data. *ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, Volume I-4, 71-76.

Kadilar, C. and Cingi, H. (2004). Ratio estimators in simple random sampling. *Applied Mathematics and Computation*, 151, 893-902.

Kumar, M., Dubey, S.K., Rai, V.N. and Sisodia, B.V.S. (2022). A transformed class of estimators of a finite population mean using two auxiliary variables in two-phase sampling. *Journal of the Indian Society of Agricultural Statistics*, 76(3), 105-114.

Kumar, M. and Tiwari, A. (2022). A composite class of ratio estimators for a finite population mean in two-phase sampling. *International Journal of Statistics and Reliability Engineering*, 9(3), 364-370.

Kumar, M. and Vishwakarma, G. K. (2017). Estimation of mean in double sampling using exponential technique on multi-auxiliary variates. *Communications in Mathematics and Statistics*, 5(4), 429-445.

Murthy, M. N. (1967). Sampling theory and methods. Calcutta, India: Statistical Publishing Soc.

Neyman, J. (1938). Contribution to the theory of sampling human populations. *Journal of the American Statistical Association*, 33, 101-116.

Sahoo, L. N. and Swain, A. K. P. C. (1980) Unbiased ratio-cum-product estimator. *Sankhya*, 42, 56-62.

Singh, B.K. and Choudhury, S. (2012). Exponential chain ratio and product type estimators for finite population mean under double sampling scheme. *Global Journal of Science Frontier Research: Mathematics and Decision Sciences*, 12(6), 13-24.

Singh, G.N. (2003). On the improvement of product method of estimation in sample surveys. *Journal of the Indian Society of Agricultural Statistics*, 56(3), 267-275.

Singh, G.N. and Upadhyaya, L.N. (1995): A class of modified chain type estimators using two auxiliary variables in two phase sampling. *Metron*, LIII, 117-125.

Singh, H.P. and Ruiz Espejo, M. (2007). Double sampling ratio-product estimator of a finite population mean in sample surveys. *Journal of Applied Statistics*, 34(1), 71-85.

Singh, R., Chauhan, P. and Sawan, N. (2007). A family of estimators for estimating population mean using known correlation coefficient in two phase sampling. *Statistics in Transition*, 8(1), 89-96.

Sisodia, B.V.S. and Dwivedi, V. K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable. *Journal of the Indian Society of Agricultural Statistics*, 33(2), 13-18.

Sisodia, B.V.S. and Dwivedi, V.K. (1982). A class of ratio cum product-type estimator in double sampling. *Biometrical Journal*, 24(4), 419-424.

Srivnstava, R.S., Srivastava S.P., and Khare B.B. (1989). Chain ratio-type estimator for ratio of two population means using auxiliary characters. *Communications in Statistics: Theory and Methods*, 18(10), 3917–3926.

Srivastava, S.K. (1970). A two-phase sampling estimator in sample surveys. *Australian Journal of Statistics*, 12(1), 23-27.

Sukhatme, B.V. (1962). Some ratio-type estimators in two-phase sampling. *Journal of the American Statistical Association*, 57, 628-632.

Sukhatme, B.V. and Chand, L. (1977). Multivariate ratio-type estimators. *Proceedings of the American Statistical Association, Social Statistics Section*, 927-931.

Tiwari, A., Kumar, M. and Dubey, S.K. (2023). A generalized approach for estimation of a finite population mean in two-phase sampling. *Asian Journal of Probability and Statistics*, 21(3), 45-58.

Upadhyaya, L.N. and Singh, H.P. (1999). Use of transformed auxiliary variable in estimating the finite population mean. *Biometrical Journal*, 41(5), 627-636.

Vishwakarma, G.K. and Kumar, M. (2015). A general family of dual to ratio-cum-product estimators of population mean in simple random sampling. *Chilean Journal of Statistics*, 6(2), 69-79.

Vishwakarma, G.K. and Kumar, M. (2016). A new approach to mean estimation using two auxiliary variates in two-phase sampling. *Proceedings of the National Academy of Sciences, India, Section A: Physical Sciences*, 86(1), 33-39.

Zeeshan, S.M., Vishwakarma, G.K. and Kumar, M. (2021). An efficient variant of dual to product and ratio estimators in sample surveys. *Communications Faculty of Sciences University of Ankara Series A1: Mathematics and Statistics*, 70(2), 997-1010.