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Ameliorated Shewhart type Mean Control Chart using Stratified Balanced Group Quartile Ranked Set Sampling Scheme

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SUMMARY

Sampling techniques play an important role in determining the efficiency of control charts. The study was designed to develop a new Shewhart-type $\bar{x}$ control chart to monitor processes using a newly developed cost-effective method of ranked set sampling namely Stratified Balanced Quartile Ranked Set Sampling (SBGQRSS). SBGQRSS is a recent sampling design proposed based on the traditional sampling of ranking set samples. ARL is utilized as a performance measure to evaluate the efficiency of SBGQRSS $\bar{x}$ control chart and other considered SRS, ranked set sampling (RSS) and extreme ranked set sampling (ERSS) charts by using Monte Carlo simulations. In most simulation scenarios, the SBGQRSS control chart is the best in comparison to RSS and its extensions.An application to real forest data illustrates the proposed method, with an evident increase in the sensitivity of the SBGQRSS based Shewhart-type $\bar{x}$ chart compared to other control charts.

Keywords: Shewhart control chart; Average run length; Simple Random Sampling; Minimax Ranked set sampling; Stratified Balanced Quartile Ranked Set Sampling; Monte Carlo simulation.

1. INTRODUCTION

Today, there are many technological resources to monitor many industrial processes in real-time. Control charts are the often used graphical techniques for continuous quality surveillance and have a long successful history and many practical applications are applied to scientific and industrial research.Their main objectives are to distinguish between the causes of randomness and the factors attributable to the variation in the process.The chance causes are generally very small and part of a stable system, but the assignable causes are caused by non-process factors.When the process occurs due to an assigned cause, it is unstable and may promote an inability to control it.Control charts help to quickly identify the causes of process disruptions so that investigations and several correction measures be taken before the pre-production of many non-conforming units. Indeed, one of its applications can be a "pre-warning" index for potential "out-of-control" processes.In general, control charts are an effective tool to eliminate process variations and estimate production process parameters (Montgomery, 2009). Many control charts are proposed, the most common being the Shewhart bar, which Shewhart introduced in 1924. The control chart comprises three horizontal lines: upper control limit (UCL), central line (CL), and lower control limit (LCL), and it is expected that the natural variations will be within these limits.Processes are considered stable when plot points are within the control limits mentioned. Points outside control limits indicate signals outside control and require corrections to restore processes to control and improve process quality. In practice, there are three types of control charts widely used, including the Shewhart control chart, the cumulative sum chart (CUSUM), and the exponential weighted moving average chart (EWMA) (Shewhart, 1924). These control tables are often compared to average run lengths (ARLs). The ARL

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represents the average number of samples displayed in the control chart until a sample outside the control is observed.Nevertheless, sampling must be recognized as playing an important role in assessing statistical quality. Factors like high cost or inspection time and destructive testing can limit the assessment of many elements. In this respect, efficient sampling designs and the smallest sample sizes are very useful for providing accurate results. In industrial applications, the ranked sets based sampling designs have proven to be an efficient alternative to more conventional methods (e.g., simple random sampling), particularly in developing statistical quality control charts. Most literature reports are based on simple random sampling (SRS), and estimates of the population mean are to some extent less effective, compared with new sampling techniques based on ranked sets, such as the ranked set sampling (RSS) and its variants.

In recent decades, sampling using ranked sets (RSS) has attracted a great deal of research interest. McIntyre proposed the concept of RSS in 1952 for estimating the average forage and pasture yields. McIntyre noted that RSS is much more effective than SRS when ranking observations is easier.Thus, if measuring the parameter of interest is not easy, and is expensive or time-consuming, but can easily be ranked using some auxiliary information, then RSS is used as an alternative to SRS.Later, Takashi andWakimoto developed the theory and characteristics of RSS in 1968. As alternatives to the RSS some variants, for example, the minimax ranked set sampling (MMRSS) proposed by Al-Nasser and Al-Omari, 2018 consist of drawing m simple random samples each of size $1, 2, 3, \ldots, m$ sampling units respectively. If the sample size is odd, the lowest ranked unit is measured otherwise the largest unit is considered. This sampling design provides a more reliable and efficient estimate of the average distribution of symmetrical data over RSS(Salazar and Sinha, 1997). In addition, the sampling of ranked extreme sets (ERSS) is based on selecting the units that are judged minimum in half of the sets and maximum in the other half.This sampling design is a practical (but not effective) alternative to RSS and MMRSS, provided it is easier to identify extreme units than intermediate classes.

Shewhart X-*bar* is considered a control chart that monitors the average quality characteristics of certain processes.Control charts are discussed extensively in many textbooks and papers, and are extended.Initially, Brown, 1991; and later Claro *et al.*, 2008; Haridy *et al.*, 2016; Al-Nasser and Gogah, 2017; Huang *et al.*, 2017;Gogah and Al-Nasser, 2018;Bouza and Al-Omari, 2018; Al-Nasser and Aslam, 2019; Montgomery, 2020 and Al-Nasser et al, 2020; have done extensive work on control charts

Today, many changes and improvements to RSS are being proposed: Samawi *et al.* (1996) proposed the extreme ranked set sampling (ERSS), Muttlak (1997) proposed the median ranked set sampling (MRSS), Jemain and Al-Omari (2006) suggested double quartile ranked set sampling scheme, Al-Nasser (2007) suggested L ranked set sampling (LRSS) which is a generalized robust sampling technique. In addition, Al-Nasser and Mustafa (2009) proposedL ranked set sampling (LRSS) and used robust extreme ranked set sampling (RERSS) as an alternative sampling strategy. Meanwhile, Mahdizadeh and Zamanzade (2020) and Al-Omari and Haq (2019)did an estimation of the parameters of some distributions using RSS, Al-Omari, and Buza (2015) present an in-depth review of RSS design, extension, theory, and application and refer to its references.Salazar and Sinha first proposed using RSS to create quality control charts to monitor process averages (1997). They found that RSS-based control charts performed better than classical SRS-based control charts.Proceeding in the same context, Muttlak and Al-Sabah (2003) and Al-Nasser and Al-Rawwash (2007), contemplated Shewhart-type mean control charts to improve performance by detecting major changes in process averages through RSS technologies. Under equal allocation, the RSS is found to be more precise than simple random sampling (SRS). Further accuracy gains can be achieved by the appropriate use of unequal allocations. In skewed distributions, the best accuracy is achieved by unequal allocation based on the Neyman approach, with the sample size corresponding to each ranking order proportional to the standard deviation. However, the absence of standard deviations in rank order makes Neyman's approach impractical. Chandra and Tiwari (2011) propose a simple and systematic approach for unequal allocation for RSS with skew distributions. Kaur *et al.* (1994, 1997) proposed the near optimum allocation models for skewed distributions to overcome the certain difficulties found in Neyman's optimum allocation procedure in which the knowledge of standard deviations of the order statistics was unknown. Their allocation procedure does not provide the integer allocation values. Bhoj and Chandra, 2019 proposed a practical unbalanced Ranked Set Sampling (RSS) model to estimate the population mean of positively skewed distributions with a relative precision very close to or equal to the optimal Neyman allocation model.

The paper is based on the concept of developing efficient Shewhart-type $\bar{x}$ control chart based on stratified balanced group quartile ranked set sampling (SBGQRSS) and the average run-length performance (ARL) of the chart is investigated and compared to the average run length of the control charts based on SRS, RSS, ERSS, and MMRSS with equal sample sizes. The new control chart proposed is considered to be more effective than the conventional SRS-based control chart. The rest of this paper is summarized as follows: Section 2 describes the sampling methodology under SBGQRSS. The proposed Shewhart-type $\bar{x}$ control chart using SBGQRSS is explained in Section 3. The evaluation of the run length and the comparison of performances are provided in Section 4.Section 5 provides an example of an application for supporting the proposed control chart by analyzing the real dataset on tree height and diameter. Finally, the 7th section ends the article with some concluding remarks.

2. STRATIFIED BALANCED GROUP QUARTILE RANKED SET SAMPLING

The Stratified Balanced Group Quartile Ranked Set Sampling abbreviated as SBGQRSS was proposed by Shah *et al.* (2020). The selection of the ranked set sample using this scheme involves the following steps:

- Divide the population into L strata using auxiliary variables as a basis for stratification. To obtain the full benefits of stratification, it is necessary to know the size of the h^{th} subgroup, defined by N_h at h=1,2,..., L. Also, information on the auxiliary variable is required, which is essential for the ranking of the units within each stratum.
- Select $m_h=3k$, randomly where $k = 1,2,...$and $h=1,2,..., L$ sets each of size m_h from each stratum, perform ranking.Finally allocate the $3k$ selected sets randomly into three groups, and each group should be of size equal to$k$ i.e. each group should consist of k sets.
- Select for measurement from:
  - 1st group the $q_1(m_h+1)^{th}$ smallest ranked unit,
  - 2nd group the $q_2(m_h+1)^{th}$ median ranked unit
  - 3rd group select the $q_3(m_h+1)^{th}$ largest ranked unit.

Note that we always take the nearest integer of $q_1(m_h+1)^{th}$, $q_2(m_h+1)^{th,}$ and $q_3(m_h+1)^{th}$where q_1=0.25, q_2=0.50, and q_3=0.75.In this way,a sample of size $m_h = 3k$ units is measured in one cycle from each stratum. The Steps (2-3) can be repeated times v to increase the sample size to $3kv$. The final sample size ni.e. the no. of units drawn from each stratum using SBGQRSS is equal to $\sum_{m=1}^{L} m_h$.

SBGQRSS is different from regular RSS, QRSS, and BGRSS methods (Immad *et al.* 2021).This method divides the sample into three balanced groups. Furthermore, the Q_1 $(m_h+1)^{th}$ unit is selected from the 1st Group, the Q_2 $(m_h+1)^{th}$ from the 2nd Group, and Q_3 $(m_h+1)^{th}$ from the 3rd Group respectively.

The SBGQRSS estimator of the population mean whenm_h is odd is defined as:

CASE 1: When m_h is odd i.e. for $k=1, 3, 5, 7,...$

$$\bar{X}_{SBGQRSSO} = \sum_{h=1}^{L} \frac{W_h}{m_h} \left[\sum_{i=1}^{k} X_{ih(q_1(m_h+1):m_h)} + \sum_{i=k+1}^{2k} X_{ih(q_2(m_h+1):m_h)} + \sum_{i=2k+1}^{3k} X_{ih(q_3(m_h+1):m_h)} \right]$$

where, $W_h = \frac{N_h}{N}$, N_h is the h^{th}stratum size and N is the total population size. The variance of SBGQRSSO is given by:

$$Var(\bar{X}_{SBGQRSSO}) = Var\left[\sum_{h=1}^{L} \frac{W_h}{m_h} \left[\sum_{i=1}^{k} X_{ih(q_1(m_h+1):m_h)} + \sum_{i=k+1}^{2k} X_{ih(q_2(m_h+1):m_h)} + \sum_{i=2k+1}^{3k} X_{ih(q_3(m_h+1):m_h)} \right] \right]$$

CASE 2: When m_h is even i.e. for $k=2, 4, 6, 8, ...$

$$\bar{X}_{SBGQRSSE} = \sum_{h=1}^{L} \frac{W_h}{m_h} \left[\sum_{i=1}^{k} X_{ih(q_1(m_h+1):m_h)} + \sum_{i=k+1}^{2k} \left(\frac{1}{2} \left[X_{ih(q_2(m_h):m_h)} + X_{ih(q_2(m_h+2):m_h)} \right] \right) + \sum_{i=2k+1}^{3k} X_{ih(q_3(m_h+1):m_h)} \right]$$

where, $W_h = \frac{N_h}{N}$, N_h is the stratum size and N is the total population size. The variance of SBGQRSSE is given by:

$$Var(\bar{X}_{SBGQRSSE}) = Var\left[\sum_{h=1}^{L}\frac{W_h}{m_h}\left[\sum_{i=1}^{k}X_{ih\left(q_1(m_h+1):m_h\right)} + \sum_{i=k+1}^{2k}\left(\frac{1}{2}\left[X_{ih\left(q_2(m_h):m_h\right)} + X_{ih\left(q_2(m_h+2):m_h\right)}\right]\right) + \sum_{i=2k+1}^{3k}X_{ih\left(q_3(m_h+1):m_h\right)}\right]\right]$$

However, the most common difficulty in the above procedure is executing the ranking protocol. As a result of ranking errors, units are assigned to ranks that differ from their true ranks according to the variables of interest. This leads to the measurement difference between units that have been quantified and units that should have been quantified. Such a ranking error is usually caused by the similarity between sampling units to be ranked. The closer these sampling units are to each other, the higher the probability of ranking errors. In particular, if the population distribution is symmetric, the ranked set sample mean has the same expected value even under the ranking error function. In other words, ranking errors do not affect the unbiased property of the ranked set sample mean as long as the parent distribution is symmetric.

3. ESTIMATING $\bar{x}$ CONTROL CHART USING SBGQRSS

Shewhart-type $\bar{x}$ control charts are determined using LCL and UCL, as well as CL. These three parts of estimates are required when the mean and variance of the population are not known.The mean and variance are estimated using the SBGQRSS methodology to construct the control chartwith the limits and central line given as:

$$Lower\ Control\ Limit\left(LCL\right) = \mu - k\sigma_{\bar{X}SBGQRSS}$$

$$Central\ Line\left(CL\right) = \mu$$

$$Upper\ Control\ Limit\left(UCL\right) = \mu + k\sigma_{\bar{X}SBGQRSS}$$

whereμ is the mean of the process under control state and $\sigma_{\bar{X}SBGQRSS}$ is the standard deviation obtained based on SBGQRSS technique. It is worth noting that in practice, since both population parameters may not be known, it is possible to estimate control limits based on the sample mean and sample standard deviation from SBGQRSS.and are given as:

$$Lower\ Control\ Limit\left(LCL\right) = \bar{X}SBGQRSS - k\sigma_{\bar{X}SBGQRSS}$$

$$Central\ Line\left(CL\right) = \bar{X}SBGQRSS$$

$$Upper\ Control\ Limit\left(UCL\right) = \bar{X}SBGQRSS + k\sigma_{\bar{X}SBGQRSS}$$

4. COMPARISONS BETWEEN SBGQRSS AND OTHERRSS VARIANTS

This section consists of a comprehensive simulation study that compares the performance of the SBGQRSS control chart with SRS, RSS, ERSS, and MMRSS control charts based on the average run length (ARL) for various values of shift denoted by (δ). The design MMRSS is a mixture of MRSS and ERSS. In this scheme, median ranked units are selected in those sets where they can be identified. In the remaining sets, ERSS is applied to select the units for actual quantification. To mention, in the extreme ranked set sampling (ERSS) procedure, we select *n* random samples of size *n* units from the population and rank the units within each sample concerning a variable of interest by visual inspection. If the sample size *n* is even, select from *n/2* samples the smallest unit and from the other *n/2* samples the largest unit for the actual measurement. If the sample size is odd, select from *(n-1)/2* samples the smallest unit, from the other *(n-1)/2* the largest unit, and from one sample the median of the sample for the actual measurement. The cycle may be repeated r times to get *nr* units. These *nr* units form the ERSS data.

W is defined as the number of observations shown in the diagram until the first observation exceeds the control limit. ARL when the process is under control is defined by:

$$ARL_0 = \frac{1}{\alpha}$$

where α is the probability of type I error, more appropriately the producers risk.Meanwhile, if the process is not controlled, the ARL is written in the type II error (β) or more appropriately the consumer's risk as follows:

$$ARL_1 = \frac{1}{1-\beta}$$

According to ARL standards, the process is controlled by mean and standard deviations, sometimes causing it to drift from control when there is an average shift in quantity $\delta\frac{\sigma_0}{m}$, where σ_0 is not negative and is chosen to dominate the average change μ. Simulation studies are performed based on normal assumptions with a zero mean and unit variance, and also on the basis that the ranking is perfect to evaluate quality control mechanisms using SRS, RSS, ERSS, MMRSS, and SBGQRSS schemes. The ***{qcc}*** package of the R studio software (version 4.1.2) was used to generate the control limits and the corresponding charts. Note that in SRS, the X-chart's ARL is 370. This is the probability of a single point being crossed beyond the control line when the process is actually under control. In other words, although the process is already controlled, all 370 samples monitored will flash an out-of-control signal once. In 2003, Muttlak and Al-Sabah used the same methodology to simulate using one million iterations for each value of δ for all sampling schemes. Each iteration simulates samples of size $m = 3, 4, 5, 6$ and sets the average shift from 0 to 4 to cover under and out-of-control processes. To carry out the simulation study, the steps are given as under:

Step I: Mean and variance of the samples:

Generate 1000000 SBGQRSS samples of size 6, 9, 12, and 15 for an in-control process i.e. from $N(0,1)$, and calculate the mean and variance of the samples.

Table 1. The exact value of the variance under normal distribution with SBGQRSS

Sample Size (m)	Variance
6	0.214
9	0.103
12	0.022
15	0.004

Step II: Setting up the control limits

Select the initial value k for the fixed value ARL_0 (here $k=3$) and evaluate the control chart limits (LCL, CL, and UCL).

Step III: Evaluate the out-of-control ARL

1. This is done by verifying the average of the process outside the control. If the process is declared in control, the third step is repeated. If a process is declared uncontrollable, register the number of samples up to the length of the run that is under control.
2. Repeat Steps 1 and 2 thousand times to calculate the control ARL.
3. If you assume that the length of the control run is R. Then the ARL= R/1000000
4. Compute ARL for δ = 0, 0.25, 0.50, . . . , 4.0

Table 2. Average Run Length using several ranked set variants when m=6

δ	SRS	RSS	ERSS	MMRSS	SBGQRSS
0.00	370.39	349.23	345.11	341.19	323.10
0.25	281.15	238.91	234.68	222.68	210.23
0.50	155.22	122.19	117.96	114.96	101.98
0.75	81.21	52.50	48.27	44.27	40.12
1.00	43.89	21.32	17.09	15.09	8.23
1.50	14.96	3.52	3.10	2.97	1.92
2.00	6.30	1.55	1.23	1.19	1.11
2.50	3.24	1.20	1.11	1.05	0.98
3.00	2.00	1.318	1.51	1.42	0.87
3.50	1.94	1.275	1.271	1.11	1.07
4.00	1.71	1.051	1.049	1.031	1.01

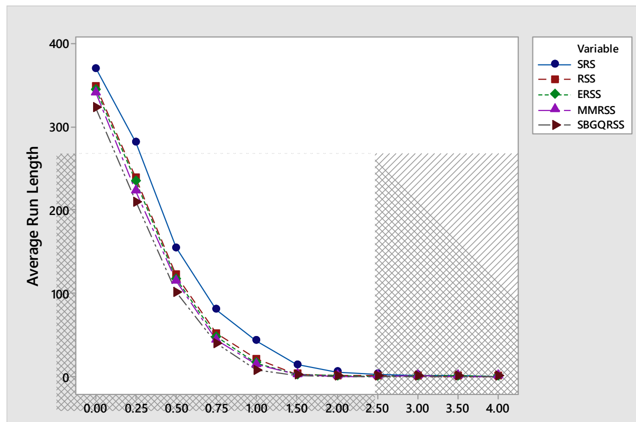

Fig. 1. ARL for different variants of SRS and RSS for m=6

Table 3. ARL using several ranked set variants when m=9

δ	SRS	RSS	ERSS	MMRSS	SBGQRSS
0.00	377.80	356.21	352.01	348.01	329.56
0.25	286.77	243.69	239.37	227.13	214.43
0.50	158.32	124.63	120.32	117.26	104.02
0.75	82.83	53.55	49.24	45.16	40.92
1.00	44.77	21.75	17.43	15.39	8.39
1.50	15.26	3.59	3.16	3.03	1.96
2.00	6.43	1.58	1.25	1.21	1.13
2.50	3.30	1.22	1.13	1.07	1.06
3.00	2.04	1.05	1.54	1.45	1.25
3.50	1.47	1.03	1.24	1.14	1.11
4.00	1.20	1.01	1.01	1.09	1.03

Thus from the above tables and figures, we conclude that the efficient quality control chart is explored by using the SBGQRSS technique to improve process monitoring. The ARL is employed to compare the proposed SBGQRSS mean control chart with the

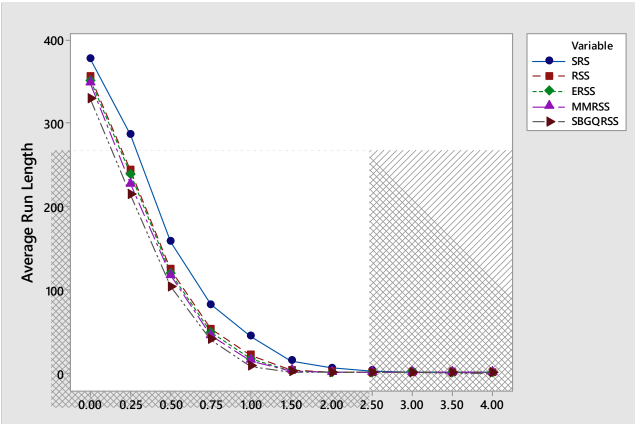

Fig. 2. ARL for different variants of SRS and RSS for m=9

Table 4. ARL using several ranked set variants when *m=12*

δ	SRS	RSS	ERSS	MMRSS	SBGQRSS
0.00	383.46	361.56	357.29	353.23	334.51
0.25	291.07	247.34	242.96	230.54	217.65
0.50	160.70	126.50	122.12	119.02	105.58
0.75	84.08	54.35	49.97	45.83	41.54
1.00	45.44	22.07	17.69	15.62	8.52
1.50	15.49	3.64	3.21	3.07	1.99
2.00	6.52	1.60	1.27	1.23	1.15
2.50	3.35	1.24	1.15	1.09	1.01
3.00	2.07	1.07	1.56	1.47	1.24
3.50	1.49	1.05	1.26	1.16	1.12
4.00	1.22	1.02	1.05	1.03	1.02

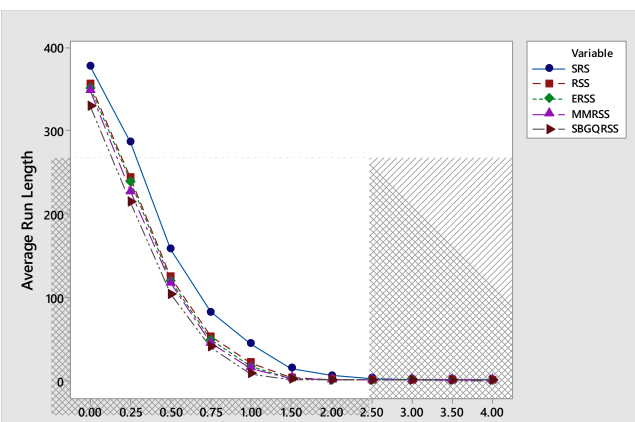

Fig. 3. ARL for different variants of SRS and RSS for m=12

mean control charts under ERSS, RSS, and SRS for the same sample sizes, shifts, and the number of iterations.

5. NUMERICAL ILLUSTRATION

SBGQRSS is an inexpensive sampling method for natural resource research in agriculture, ecology, forestry, and the environment.The total height of a single tree is one of the most common variables used in forest research and is an important prerequisite for the development of a forest management plan. Also, the tree volume strongly depends on the height–d.b.h. relationship, which varies with varying site quality. Accurate information about tree volume is vital in quality timber cruising, calculating sustainable

Table 5. ARL using several ranked set variants when *m=15*

δ	SRS	RSS	ERSS	MMRSS	SBGQRSS
0.00	391.13	368.79	364.44	360.30	341.20
0.25	296.90	252.29	247.82	235.15	222.00
0.50	163.91	129.03	124.57	121.40	107.69
0.75	85.76	55.44	50.97	46.75	42.37
1.00	46.35	22.51	18.05	15.94	8.69
1.50	15.80	3.72	3.27	3.14	2.03
2.00	6.65	1.64	1.30	1.26	1.17
2.50	3.42	1.27	1.17	1.11	1.03
3.00	2.11	1.09	1.59	1.50	0.92
3.50	1.52	1.07	1.29	1.18	0.79
4.00	1.25	1.05	1.07	1.94	1.03

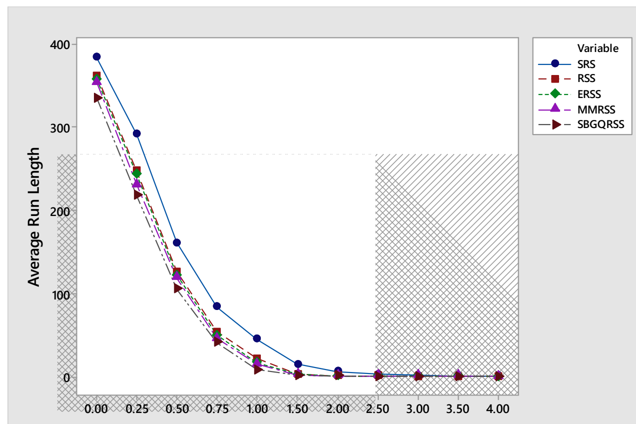

Fig. 4. ARL for different variants of SRS and RSS for m=15

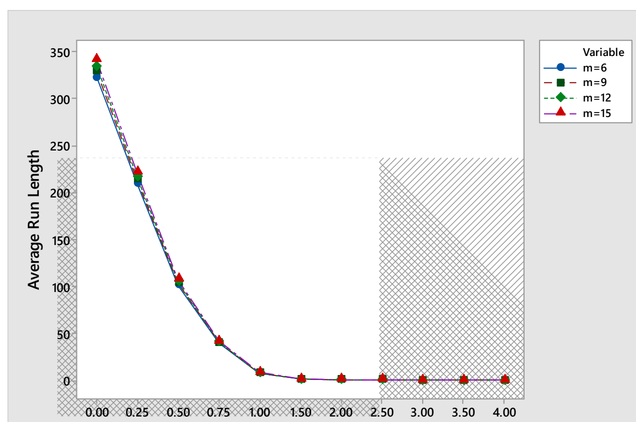

Fig. 5. Comparison between ARL using SBGQRSS technique when m=6, 9, 12, and 15

and quality wood supply, and developing forest management plans This section considers a secondary real data set from a research site maintained by the Faculty of Forestry, SKUAST-Kashmir to investigate the performance of the suggested Shewhart-type $\bar{x}$ control charts. The dataset consists of two variables: the height of the Cupressus tree measured in meters (m) say Y, and the diameter of the breast measured in cm. We only consider the height of the trees in the numerical illustration. Our goal is to estimate the average height of 469 randomly selected Cupressus trees.A previous study based on the data revealed that the distribution was not normal. We used this data after removing 37potential outlying observations to satisfy the assumption of normality. The histogram and the Normal probability plot of the height of the cupress generated using the ***{rcompanion}*** and ***{ggqqplot}*** in R are shown in Figures 6 and 7, respectively. Based on these figures, it is possible to assume that the tree height distribution is approximated by the normal distribution. The summary statistics of this dataset are given in Table 6.

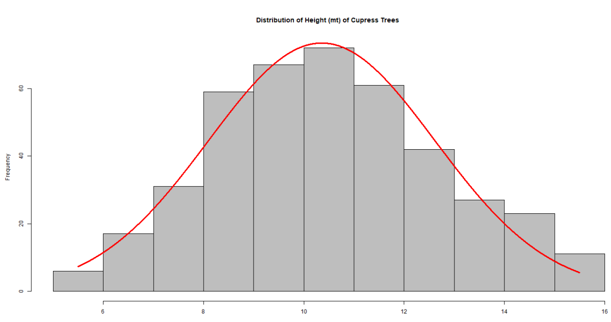

Fig. 6. Histogram of 420cupress trees heights (m)

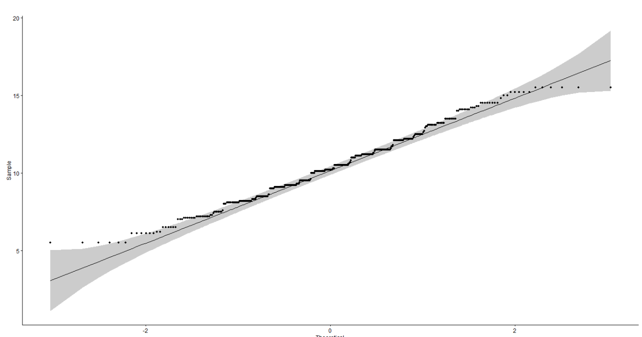

Fig. 7. Normal q-q plot of 432cupress trees heights (m)

Draw a random sample of 72 trees with$9k^2$; $k=2$ units in each of the two strata, to draw a sample of $m_h=3k$; $k=2$ units in one cycle from each stratum using SBGQRSS scheme using collar diameter as an auxiliary variable for ranking the units within each balanced group. Similarly, draw samples of size 12 using the traditional SRS and variants of RSS to compare the estimates and the control charts for each of the considered schemes.

Table 7. A summary of the selected samples using different techniques (m=12)

$\bar{Y}_{SRS}$	10.416	σ^2_{SRS}	4.616
$\bar{Y}_{RSS}$	10.975	σ^2_{RSS}	3.191
$\bar{Y}_{ERSS}$	9.933	σ^2_{ERSS}	6.532
$\bar{Y}_{MMRSS}$	10.749	σ^2_{MMRSS}	5.749
$\bar{Y}_{SBGQRSS}$	10.279	$\sigma^2_{SBGQRSS}$	2.358

Fig. 8. Control charts under various variants of the RSS

Table 6. Summary statistics of the dataset

Variable	N	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Height	432	10.35	0.108	2.25	5.50	8.52	10.20	11.77	15.50
DBH	432	92.87	1.670	34.73	8.56	69.09	88.28	112.49	230.20

6. CONCLUSION

The SBGQRSS technology is used to study efficient quality control diagrams and improve process monitoring. The average run length is employed to compare the proposed SBGQRSS $\bar{x}$ control chart with the existing $\bar{x}$ control chart under MMRSS, ERSS, RSS, and SRS for the same sample sizes, shifts, and the number of iterations.The significant role of ranked set sampling and its variants viz. RSS, ERSS, MMRSS, and SBGQRSS become apparent when the process gradually loses control. (i.e., when gets larger than zero). The distinguishing differences between RSS and the proposed new sampling technique are emphasized in the comparison of results when m increases.

REFERENCES

D.C. Montgomery (2009). Introduction to Statistical Quality Control. 6th ed. New York: John Wiley and Sons.

Shewhart, W.A. (1924). Some applications of statistical methods to the analysis of physical and engineering data. *Bell System Technical Journal,* **3**, 43-87.

O. Kuoh How, C.M. Harvey, R.L. Shehab, J.D. Dechert, A. Darisipudi, (2004). "The effects of three statistical control charts on task performance," Production Planning and Control, **15**(3), pp. 313-23.

Mcintyre, G. (1952). A method for unbiased selective sampling, using ranked sets. *Australian journal of agricultural research*, **3**, 385-390.

Takahasi, K. and Wakimoto, K. (1968). On unbiased estimates of the population mean based on the sample stratified by means of ordering. *Annals of the institute of statistical mathematics*, **20**, 1-31.

Gogah, F. and Al-Nasser, A.D. (2018). Median ranked acceptance sampling plans for the exponential distribution. *AfrikaMatematika*, **29**, 477-497.

H.A. Muttlak (1997). Median ranked set sampling, *Journal of Applied Statistical Sciences,* **6.4**, pp. 245-255.

Brown, D.W. (1991). *Statistical process control: theory and practice*. Chapman and Hall, London, UK.

Claro, F.A., Costa, A.F., and Machado, M.A. (2008). Double sampling control chart for a first order autoregressive process. *PesquisaOperacional*, **28**, 545–562.

Haridy, S., Ou, Y., Wu, Z., and Khoo, M.B. (2016). A single x chart outperforming the joint x & r and x & s charts for monitoring mean and variance. *Quality Technology & Quantitative Management*, **13**, 289-308.

Huang, W.-H., Yeh, A.B., and Wang, H. (2018). A control chart for the lognormal standard deviation. *Quality Technology & Quantitative Management*, **15**, 1-36.

Al-Nasser, A.D., and Gogah, F.S. (2017). On using the median ranked set sampling for developing reliability test plans under generalized exponential distribution. *Pakistan Journal of Statistics and Operation Research*, **13**, 757-774.

Bouza, C.N., and Al-Omari, A.I. (2018). *Ranked set sampling: 65 years improving the accuracy in data gathering*. Academic Press, San Diego, CA, USA.

Al-Nasser, A.D., and Aslam, M. (2019). Development of a new control chart based on ranked repetitive sampling. *Ranked Set Sampling*. Academic Press, 9-24.

Al-Nasser, A.D., Ciavolino, E., and Al-Omari, A.I. (2020). Extreme ranked repetitive sampling control charts. *PesquisaOperacional*, 40.

Montgomery, D.C. (2020). *Introduction to statistical quality control***.** John Wiley & Sons, Hoboken.

Samawi, H.M., Ahmed, M.S., and Abu-Dayyeh, W. (1996). Estimating the population mean using extreme ranked set sampling. *Biometrical Journal*, **38**, 577-586.

Jemain, A.A., and Al-Omari, A.I. (2006). Double quartile ranked set samples. *Pakistan Journal of Statistics*, **22**, 217.

Al-Nasser, A.D.,and Al-Rawwash, M. (2007). A control chart based on ranked data. *Journal of Applied Sciences***,** **7**, 1936-1941.

Al-Nasser, A.D. and Mustafa, A.B. (2009). Robust extreme ranked set sampling. *Journal of Statistical Computation and Simulation*, **79**, 859–867.

Mahdizadeh, M., And Zamanzade, E. (2020). Estimation of a symmetric distribution function in multistage ranked set sampling. *Statistical Papers*, **61**, 851-867.

Al-Omari, A.I., And Haq, A. (2019). A new sampling method for estimating the population mean. *Journal of Statistical Computation and Simulation*, **89**, 1973-1985.

Al-Omari, A.I., And Bouza, C.N. (2015). Ratio estimators of the population mean with missing values using ranked set sampling. *Environmetrics*, **26**, 67–76.

Salazar, R. And Sinha, A. (1997). Control chart x based on ranked set sampling. *ComunicacionTecica*, **1**, 1-97.

Muttlak, H. andAl-Sabah, D.W. (2003). Statistical quality control based on ranked set sampling. *Journal of Applied Statistics*, **3**, 1055-1078.

Al-Nasser, A.D., and Al-Rawwash, M. (2007). A control chart based on ranked data. *Journal of Applied Sciences***,** **7**, 1936-1941.

Shah, I.A., Mir, S.A., Khan, I. (2020). New quartile based variant of the ranked set sampling scheme**.** *Journal of Indian Society of Agricultural Statistics*. **74**(3), 255-264

Kaur, A., Patil, G.P. and Taillie, C. (1994). Unequal allocation models for ranked set sampling with skew distributions. *Technical Report 94-0930, Center for Statistical Ecology and Environmental Statistics*, Pennsylvania State University, University Park

Kaur, A., Patil, G.P. and Taillie, C. (1997). Unequal allocation models for ranked set sampling with skew distributions. *Biometrics*, **53**, 123-130

Tiwari, N. and Chandra, G. (2011). A systematic procedure for unequal allocation forskewed Distributions in Ranked Set Sampling. *Journal of the Indian Society of Agricultural Statistics*, **65**(3), 331-338

Bhoj, D., & Chandra, G. (2019). Simple unequal allocation procedure for ranked set sampling with skew distributions. *Journal of Modern Applied Statistical Methods*, **18**(2), eP2811. doi: 10.22237/jmasm/1604189700