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Neutrosophic Analysis of the Experimental Data using Neutrosophic Latin Square Design

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SUMMARY

While dealing with the observed or measured data in surveys or experiments, it is not uncommon to deal with vague, incomplete, and imprecise information for whatever reasons. In this regard, researchers have proposed various emerging approaches such as fuzzy, intuitionistic fuzzy and neutrosophic logic and analysis, which provide better understanding, analysis and interpretations of the data. Neutrosophic logic is an extension of fuzzy logic where a variable x is described by triplet values, i.e., $x=(t,i,f)$, where t is the degree of "truth", f is the degree of "false" and i is the level of "indeterminacy" AlAita, Abdulrahmanand Aslam, Muhammad (2023). A neutrosophic data x can be expressed as $x = d + i$, where d is the determinate (sure) part of x, and i is the indeterminate (unsure) part of x. Experimental design and analysis is a systematic, rigorous approach to problem solving that applies principles and techniques at the data collection stage so as to ensure the generation of valid, defensible, and supportable conclusions. Latin square designs (LSDs) are used to compare treatment factor levels represented by the Latin letters and using two blocking factors in rows and columns to simultaneously control two sources of nuisance variability. In this paper, we will define a neutrosophic Latin square design (NLSD), neutrosophic LSD model and consider the neutrosophic analysis of the NLSD experimental data for testing the abrasion resistance of rubber-coated fabric in a Martindale wear tester.

Keywords: Imprecise data; Neutrosophic statistics; Neutrosophic Latin square design; Neutrosophic Analysis.

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1. INTRODUCTION TO THE PROBLEM

Experimental design and analysis is a systematic, rigorous approach to problem solving that applies principles and techniques at the data collection stage so as to ensure the generation of valid, defensible, and supportable conclusions. In addition, all of this is achieved with optimizing resources, in particular, time and money.

Recently, Smarandache Neutrosophic Set (2007); Smarandache, F. (2014); Smarandache, F. (2019) has proposed neutrosophic logic and neutrosophic math which is an extension of fuzzy logic where variable x is described by triplet values, i.e., $x=(t,i,f)$, where t is the degree of truth, f is the degree of false and i is the level of indeterminacy. A neutrosophic data x can be expressed as $x = d + i$, where d is the determinate (sure) part of x, and i is the indeterminate (unsure) part of x. For example, a data value $x = 8 + i$, where $i \in [0,0.6]$, is equivalent to $x \in [8,8.6]$. That means, for sure $x \geq 8$ (meaning that the determinate part of x is 8), while the indeterminate part $i \in [0,0.6]$ means the possibility for number x to be greater than or equal to 8 but less than or equal to 8.6.

In conducting experiments for data collection, experimenter deals withmore than one treatment factor or with blocking. These include paired comparison designs, randomized block designs, two-way layouts with fixed and random effects, Latin square designs, incomplete block designs and split-plot designs. Kumari, Srishti *et al.* have introduced the neutrosophic completely randomized design that is a generalization of the completely randomized design. They also studied

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the flexible way of handling imprecise elements in completely randomized design. AlAita *et al.* (2023) have proposed a novel method for ANCOVA using neutrosophic statistics. They applied the neutrosophic analysis of covariance (NANCOVA) in three different designs, neutrosophic completely randomized design, neutrosophic randomized complete block design, and neutrosophic split-plot design. Further, authors have implemented and explained proposed method using numerical examples. The proposed NANCOVA method is found to be flexible and effective in the presence of uncertainty when compared to the existing method. AlAita *et al.* (2023) have proposed neutrosophic statistics analysis for split-plot and split-block designs. In the method, neutrosophic hypothesis is formulated, a decision rule is suggested, and neutrosophic ANOVA Table is given. Further, a numerical example and a simulation study demonstrate the effectiveness of the proposed method.

In this paper, we consider the neutrosophic analysis of the experimental data, which was collected using the neutrosophic Latin square design for testing the abrasion resistance of rubber-coated fabric in a Martindale wear tester experiment Davies, O.L. ed. (1954).

2. NEUTROSOPHIC LATIN SQUARE DESIGN

A Neutrosophic Latin square design (NLSD) has two blocking factors represented by the rows and columns of the square and one experimental factor with all factors having k^N (a neutrosophic number) levels and represented by the Latin letters in the square. A NLSD of order $k^N \times k^N$ is a square of k^N Latin letters arranged such that each Latin letter appears only once in each row and only once in each column. The levels of two blocking factors are randomly assigned to the rows and columns; the treatments of the experiment factor are randomly assigned to the Latin letters $A, B, C \ldots$. Thus, a neutrosophic Latin square design is a classical Latin square design where the data collected from the design is neutrosophic data.

2.1 Neutrosophic Latin Square Design Model

The linear model for the Neutrosophic Latin square design is described as

$$Y_{ijl}^N = \mu^N + \alpha_i^N + \beta_j^N + \tau_l^N + \varepsilon_{ijl},$$

where $i, j = 1, 2, \ldots, k^N$, l is the Latin letter in the (i, j) cell of the Latin square, Y_{ijl}^N = The l^{th} neutrosophic treatment observation from the (i, j) cell, μ^N = The general neutrosophic location parameter (mean), τ_l^N = The l^{th} neutrosophic treatment effect, α_i^N = The i^{th} neutrosophic row effect, β_j^N = The j^{th} neutrosophic column effect, ε_{ijl} = The random error associated with the l^{th} neutrosophic treatment observation from the (i, j) cell and assumed to be independent following neutrosophic Normal distribution, i.e., $N_N(0, \sigma^2)$, k^N = Total number of Latin letters representing neutrosophic treatments. Total number of runs (observations) in the NLSD experiment is $N^N = k^N \times k^N$.

2.2 Hypothesis Tests for the Neutrosophic Treatment, Row and Column Effects

Neutrosophic Treatment Effects

Null Hypothesis: $\tau_1^N = \tau_2^N = \ldots = \tau_k^N = 0$.

Alternative Hypothesis: At least one of τ_l^N's is not equal to zero.

Test statistic follows the F-distribution, i.e.,

$$F\left(k^N - 1,\ (k^N - 1)(k^N - 2)\right) = (k^N - 1)(k^N - 2) NTrSS / \left(k^N - 1\right) NESS.$$

Neutrosophic Row Effects

Null Hypothesis: $\alpha_1^N = \alpha_2^N = \ldots = \alpha_k^N = 0$.

Alternative Hypothesis: At least one of α_i^N's is not equal to zero.

Test statistic follows the F-distribution, i.e.,

$$\left(k^N - 1,\ (k^N - 1)(k^N - 2)\right) = (k^N - 1)(k^N - 2) NRSS / \left(k^N - 1\right) NESS.$$

Neutrosophic Column Effects

Null Hypothesis: $\beta_1^N = \beta_2^N = \ldots = \beta_k^N = 0$.

Alternative Hypothesis: At least one of β_j^N's is not equal to zero.

Test statistic follows the F-distribution, i.e.,

$$\left(k^N - 1,\ (k^N - 1)(k^N - 2)\right) = (k^N - 1)(k^N - 2) NCSS / \left(k^N - 1\right) NESS.$$

2.3 Calculation of Sum of Squares

Neutrosophic treatment sum of squares (NTrSS) = $\sum_l k^N \left(Y_{..l}^N - \mu^N\right)^2$, d.f.= $k^N - 1$.

Neutrosophic row sum of squares (NRSS) = $\sum_{i} k^N \left(\mu_{i..}^N - \mu^N\right)^2$, d.f.= $k^N - 1$.

Neutrosophic column sum of squares (NCSS) = $\sum_{j} k^N \left(\mu_{.j.}^N - \mu^N\right)^2$, d.f.= $k^N - 1$.

Neutrosophic total sum of squares (NTSS) = $\sum_{i,j,l} \left(Y_{ijl}^N - \mu^N\right)^2$, d.f.= $N^N - 1$.

Neutrosophic error sum of squares (NESS) = $NTSS - \text{NTrSS} - \text{NRSS} - NCSS$, d.f.= $(k^N - 1)(k^N - 2)$.

3. DESCRIPTION OF THE EXPERIMENT

The experiment Davies, O.L. ed. (1954) for testing the abrasion resistance of rubber-coated fabric in a Martindale wear tester considered four types of material denoted by A-D. The response variable was the loss in weight in 0.1 milligrams (mg) over a standard period of time. There are four positions selected on the tester so that four sample can be tested at the same time where past experience has indicated that there were slight differences between the four positions. Each time the tester was used, setup could be a bit different, that means, there might be a systematic difference from application to application. Therefore, application was treated as a blocking variable. The experiment was designed to remove the variation due to position (column) and application (row). The layout of the design used in this experiment is given in Table 1.

Table 1. Layout of the Latin Square Design for Material types A-D, Applications A1-A4 and Positions P1-P4 in the Wear-Tester Experiment

	P1	P2	P3	P4
A1	C	D	B	A
A2	A	B	D	C
A3	D	C	A	B
A4	B	A	C	D

The data collected on loss in weight in 0.1 milligrams (mg) over a standard period of time is given in Table 2.

Table 2. Weight Loss (0.1 mg) in the Wear-Tester Experiment

	P1	P2	P3	P4
A1	235 C	236 D	218 B	268 A
A2	251 A	241 B	227 D	229 C
A3	234 D	273 C	274 A	226 B
A4	195 B	270 A	230 C	225 D

The corresponding neutrosophic weight loss data is presented in Table 3.

Table 3. Neutrosophic Weight Loss (0.1 mg) in the Wear-Tester Experiment

	P1	P2	P3	P4
A1	[232.65,237.35] C	[233.64,238.36] D	[215.82,220.18] B	[265.32,270.68] A
A2	[248.49,253.51] A	[238.59,243.41] B	[224.73,229.27] D	[226.71,231.29] C
A3	[231.66,236.34] D	[270.27,275.73] C	[271.26,276.74] A	[223.74,228.26] B
A4	[193.05,196.95] B	[267.30,272.70] A	[227.70,232.30] C	[222.75,227.25] D

4. RESULTS

The experimental data is analyzed using the neutrosophic Latin square design. Neutrosophic sums of squares of four treatments factors, four positions and four applications are calculated. Results are discussed below.

4.1 Descriptive Statistics

The neutrosophic average weight losses due to treatments, applications and positions are presented in Table 4.The maximum average weight loss is noted between 263.09 mg and 268.41 mg due to Treatment A and the minimum between 179.69 mg and 183.32 mg due to treatment D. For position 2, the maximum average loss is between 252.45 mg and 257.55 mg, while it is minimum for position 1 between 226.46 mg and 231.04 mg. For position 3 and position 4, the average weight losses are nearly of the same order. The maximum average weight loss for the application 4 is between 244.28 mg and 249.22 mg and the minimum between 236.86 mg and 241.64 mg for the application 1.

Table 4. Neutrosophic Average Weight Loss (0.1 mg) in the Wear-Tester Experiment

Treatment	A	B	C	D
Mean	[263.09, 268.41]	[217.80,2 22.20]	[239.33, 244.17]	[228.20, 232.81]
Position	P1	P2	P3	P4
Mean	[226.46, 231.04]	[252.45, 257.55]	[234.88, 239.62]	[234.63, 239.37]
Application	A1	A2	A3	A4
Mean	[236.86, 241.64]	[234.63, 239.37]	[249.23,2 54.27]	[227.70, 232.30]

The neutrosophic variance of the average weight losses due to treatments, applications and positions are presented in Table 5. The maximum variance of average weight loss is noted between 199.85 $(mg)^2$ and 508.34 $(mg)^2$ due to Treatment C and the minimum between 1.01 $(mg)^2$ and 83.99 $(mg)^2$ due to treatment D. For position 3, the maximum variance of average loss is between 341.75 $(mg)^2$ and 1007.94 $(mg)^2$, while it is minimum for position 2 between 84.93 $(mg)^2$ and 479.06 $(mg)^2$. The maximum variance of average weight loss for the application 4 is between 281.80 $(mg)^2$ and 719.48 $(mg)^2$ and the minimum between 30.92 $(mg)^2$ and 202.04 $(mg)^2$ for the application 2.

Table 5. Neutrosophic Variance of Average Weight Loss $(0.1\ mg)^2$ in the Wear-Tester Experiment

Treatment	A	B	C	D
Mean	[27.68, 183.21]	[186.61, 405.17]	[199.85, 508.34]	[1.01, 83.99]
Position	P1	P2	P3	P4
Mean	[396.90, 604.45]	[84.93, 479.06]	[341.75, 1007.94]	[223.39, 847.41]
Application	A1	A2	A3	A4
Mean	[210.32, 488.92]	[30.92, 202.04]	[410.74, 602.43]	[281.80, 719.48]

The neutrosophic effects of treatments, positions and applications are given in Table 6. It may be noted that the large neutrosophic effects of treatment A was between 21.1975 mg and 31.3025 mg and for treatment D was between -62.21 mg and -53.79 mg. Similarly, we can describe the neutrosophic effects of applications and positions.

Table 6. Neutrosophic treatment, Position and Application effects in the Wear-Tester Experiment. [Weight Loss (0.1 mg)]

Application	Position	Treatment
[-5.0375,4.5375] A1	[-15.4325,-6.0675] P1	[21.1975,31.3025] A
[-1.0775,8.5775] A2	[10.5550,20.4450] P2	[-7.2650,2.2650] B
[0.1600,9.8400] A3	[-7.0175,2.5175] P3	[-22.1150,-12.8850] C
[2.3875,12.1125] A4	[-7.2650,2.2650] P4	[-62.2100,-53.7900] D

Charts for the neutrosophic means of treatments, positions and applications are given in the following:

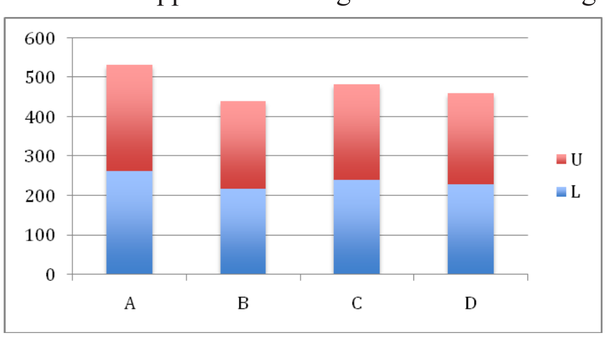

Chart 1. Average weight loss (in 0.1 mg) for treatments A-D

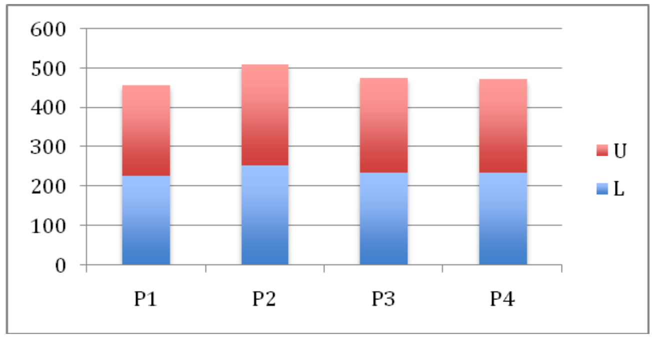

Chart 2. Average weight loss (in 0.1 mg) for positions P1-P4

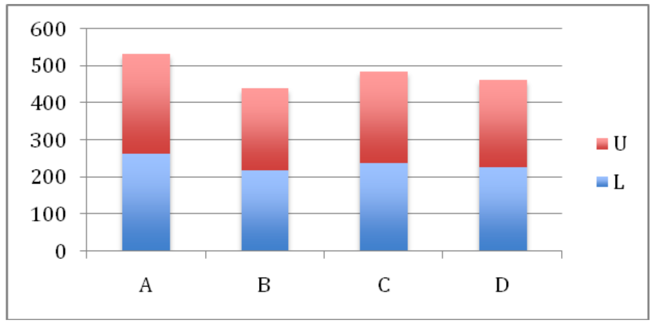

Chart 3. Average weight loss (in 0.1 mg) for applications A1-A4

4.2 Hypotheses Tests

The neutrosophic analysis of variance (NANOVA) table is prepared for the neutrosophic Latin square design and is shown in Table 7. It may be noted for the understanding of the calculations that every time, calculated values need to be checked and interchanged if necessary as the lower and upper bound. In sum of squares, if lower bound is negative, it is taken as zero because sum of squares cannot be negative. Also, F-statistic and p-value can not be determined if error sum of squares is zero as noted in Table 7. From Table 7, it is noted that the calculated F-statistic values and p-value are respectively [*,4.16] and [*,0.06] for the neutrosophic treatments, [*,1.75] and [*,0.26] for the neutrosophic positions and [*,1.33] and [*,0.35] for the neutrosophic applications.

Since the maximum p-value for comparing neutrosophic treatments is 0.06, neutrosophic treatment means (treatment effects) are significantly different at 6% significance level. However, since the p-values for the neutrosophic applications and neutrosophic positions are, respectively, 0.35 and 0.26 are very large, we fail to conclude that the application and position effects are significantly different.

Further, to give an example of NANOVA where lower and upper bounds for error sum of squares are

Table 7. Neutrosophic ANOVA table for comparing treatments

Source	DF	SS	F(3,6)	P – value
Treatments	3	[2786.20,7191.94]	[*,4.16]	[*, 0.06]
Applications	3	[410.58,2296.82]	[*,1.33]	[*, 0.35]
Positions	3	[638.76,3032.74]	[*,1.75]	[*,0.26]
Error	6	[-4927.88,3460.32] =[0,3460.32]		

* Not defined.

positive values, we consider results from the paper by Aslam, M. (2019).

A clinical psychologist is interested to perform NANONA to compare three methods to minimize the hostility levels among the university students. He applied the HLT test to measure the data from various twelve students and a high HLT score shows the great hostility levels. While measuring HLT scores, the clinical psychologist is uncertain in some scores. Under this situation, he recorded data in the neutrosophic interval. The NANOVA table is given in Table 8.

Table 8. NANOVA for comparing three methods

Source	Df	NSS	NMS	F	p-value
Between samples	[2,2]	[366.2, 377.2]	[183.08, 188.58]	[5.33, 5.40]	[0.0288, 0.0297]
Error	[9,9]	[308.7, 314.5]	[34.31, 34.94]		
Total	[11,11]				

Since, max{p-value = 0.0297} ≤ 0.05,we reject the null hypothesis that the means of three methodsare equal.

4.3 Confidence Intervals for the Pairwise Mean Differences

To make the pairwise comparisons of average weight loss due to treatments, applications and positions, we have calculated the 95% confidence intervals that are presented in Table 9.

The 95% neutrosophic confidence intervals for difference in treatment means A and B and difference in treatment means A and D are [13.5590,77.9410] and [2.9540,67.5460], respectively. Since these confidence intervals do not include zero, there is statistical evidence at the 5% significance level that the average weight loss due to treatment A and treatment B and treatment A and treatment D are significantly different. In all other remaining pairwise comparisons of treatments, i.e., (A,C), (B,C), (B,D), and (C,D) and also pairwise comparisons of applications and positions, their effects are not significantly different as their 95% neutrosophic confidence intervals for difference include zero.

Table 9. 95% Neutrosophic Confidence Intervals for the pairwise Neutrosophic mean differences. [Weight loss (0.1mg)]

Treatments	A-B	A-C	A-D	B-C	B-D	C-D
Lower Limit	13.5590	-8.4085	2.9540	-53.7010	-42.3385	-20.8060
Upper Limit	77.9410	56.4085	67.5460	10.2010	21.3385	43.3060
Application	A1-A2	A1-A3	A1-A4	A2-A3	A2-A4	A3-A4
Lower Limit	-29.8460	-44.7435	-22.7760	-46.9710	-25.0035	-10.4010
Upper Limit	34.3460	19.7435	41.2760	17.4710	39.0035	53.9010
Position	P1-P2	P1-P3	P1-P4	P2-P3	P2-P4	P3-P4
Lower Limit	-58.4210	-40.4935	-40.2410	-14.5060	-14.2535	-31.8260
Upper Limit	5.9210	23.4935	23.7410	50.0060	50.2535	32.3260

5. CONCLUDING REMARKS

Neutrosophic logic, neutrosophic math and neutrosophic statistics are important tools to understand and analyze the incomplete, vague and imprecise information/data. There is a huge scope for fuzzy data derived from experimental design set up. The work can easily be extended to other areas of designing too. We have shown the application of neutrosophic Latin square design by considering the experimental data that was collected for testing the abrasion resistance of rubber-coated fabric in a Martindale wear tester. In this experiment, four types of material (A-D) were tested. The response was measured by the loss in weight in 0.1 milligrams (mg) over a standard period of time. Four applications and four positions were used as blocking factors. From the pairwise comparisons at the 5% significance level, only average weight loss due to treatment A and treatment B and treatment A and treatment D differed significantly. In all other cases, their effects are not found significantly different at 5% significance level.

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