Available online at www.isas.org.in/jisas
JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 76(3) 2022 115–120

A Note on the Estimation of Population Mean in Double Sampling for Stratification

Hilal A. Lone[1], Rajesh Tailor[2] and Med Ram Verma[3]

[1]*Govt. Degree College, Baramulla, J&K, India*
[2]*Vikram University, Ujjain*
[3]*ICAR-Indian Veterinary Research Institute, Izatnagar*

Received 13 June 2022; Revised 08 November 2022; Accepted 15 December 2022

SUMMARY

In the present paper we have proposed dual to ratio-cum-product type estimator for estimation of the population mean in double sampling for stratification. The motivation of proposed estimator is based on Singh (1967) and Lone *et al.* (2020). We have derived expressions for bias and MSE for the proposed estimator. The mean square error of the proposed estimator is compared with usual unbiased estimator of population mean in double sampling for stratification, Ige and Tripathi (1987) estimators, Tailor *et al.* (2015) estimator and Lone *et al.* (2020) estimators. We have obtained the conditions under which proposed estimator is more efficient than other estimators. The paper concludes with a numerical illustration

Keywords: Double sampling for stratification, Bias, Mean squared error.

1. INTRODUCTION

Stratified random sampling presumes the knowledge of stratum size as well as sampling frame for all strata. In many situations strata weights are not available or strata weights are outdated. This situation leads investigator to use double sampling for stratification. In double sampling for stratification, a preliminary sample of size n' is selected by simple random sampling without replacement to estimate strata weights and then a sub-sample of n units, n_h from the h^{th} stratum, is drawn to collect information on the study variate as well as the auxiliary variate.

Ige and Tripathi (1987) studied the classical ratio and product estimators in double sampling for stratification. Singh and Vishwakarma (2007) discussed a general procedure for estimating the populations mean using double sampling for stratification. Tailor *et al.* (2014) suggested ratio and product type exponential estimators of population mean in double sampling for stratification. Following Srivenkataramana (1980) and Bandyopadhyay (1980) transformation, Lone *et al.* (2020) proposed an alternative to Ige and Tripathi (1987) estimators in double sampling for stratification. Singh (1967) and Lone *et al.* (2020) motivated authors to study a dual to ratio-cum-product type estimator in case of double sampling for stratification. The problem of estimating the finite population mean in double sampling for stratification has been discussed by many researchers including Tripathi and Bahl (1991), Chouhan (2012), Jatwa (2014), Tailor and Lone (2014) and Tailor *et al.*(2014).

Corresponding author: Med Ram Verma
E-mail address: drmrverma@gmail.com

Procedure, Notations and Definitions

Let us consider a finite population $U = \{U_1, U_2, U_3, ...U_N\}$ of size N in which strata weight $\frac{N_h}{N}, \{h = 1, 2, 3, ...L\}$ are unknown. In these conditions we use double sampling for stratification. Procedure for double sampling for stratification is given below

(a) at first phase of sample S of size n' using simple random sampling without replacement is drawn and auxiliary variates x and z are observed.

(b) the samples is stratified into L strata on the basis of observed variables x and z. Let n'_h denotes the number of units in h^{th} stratum $(h = 1, 2, 3, ..., L)$ such that $n' = \sum_{h=1}^{L} n'_h$.

(c) from each n'_h unit, a sample of size $n_h = v_h n'_h$ is drawn where $0 < v_h < 1$, $\{h = 1, 2, 3, ..., L\}$, is the predetermined probability of selecting a sample of size n_h from each strata of size n'_h and it constitutes a sample S' of size $n = \sum_{h=1}^{L} n_h$. In S' both study variate y and auxiliary variates x and z are observed.

Let y be the study variate and x and z are the two auxiliary variate respectively. Let us define

Notations	**Descriptions**
$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi}$	Population mean of the auxiliary variate x
$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi}$	Population mean of the study variate y
$\bar{Z} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} z_{hi}$	Population mean of the auxiliary variate z
$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$	h^{th} stratum mean for the auxiliary variate x
$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$	h^{th} stratum mean for the study variate y
$\bar{Z}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$	h^{th} stratum mean for the auxiliary variate z
$S_{xh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(x_{hi} - \bar{X}_h\right)^2$	h^{th} stratum population mean square of the auxiliary variate x,
$S_{yh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)^2$	h^{th} stratum population mean square of the study variate y,
$S_{zh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(z_{hi} - \bar{Z}_h\right)^2$	h^{th} stratum population mean square of the auxiliary variate z,
$\rho_{yxh} = \frac{S_{yxh}}{S_{yh}S_{xh}}$	Correlation coefficient between y and x in the stratum h,
$\bar{x}'_h = \frac{1}{n'_h}\sum_{h=1}^{n'_h} \bar{x}_{hi}$	First phase sample mean of the h^{th} stratum for the auxiliary variate x ,
$\bar{z}'_h = \frac{1}{n'_h}\sum_{h=1}^{n'_h} \bar{z}_{hi}$	First phase sample mean of the h^{th} stratum for the auxiliary variate z ,
$f = \frac{n'}{N}$	First phase sampling fraction.
$n = \sum_{h=1}^{L} n_h$	size of the sample S'
$w'_h = \frac{n'_h}{n'}$	h^{th} stratum weight in the first phase sample ,
$\bar{x}' = \frac{1}{n'_h}\sum_{h=1}^{n'_h} w'_h \bar{x}'_h$	Unbiased estimator of population mean $\bar{X}$,
$\bar{z}' = \frac{1}{n'_h}\sum_{h=1}^{n'_h} w'_h \bar{z}'_h$	Unbiased estimator of population mean $\bar{Z}$.

Ige and Tripathi (1987) defined classical ratio and product estimators in double sampling for stratification as

$$\bar{y}_{Rd} = \bar{y}_{ds}\left(\frac{\bar{x}'}{\bar{x}_{ds}}\right). \tag{1.1}$$

and

$$\bar{y}_{Pd} = \bar{y}_{ds}\left(\frac{\bar{z}_{ds}}{\bar{z}'}\right). \tag{1.2}$$

Where $\bar{x}_{ds} = \sum_{h=1}^{L} w_h \bar{x}_h$, $\bar{y}_{ds} = \sum_{h=1}^{L} w_h \bar{y}_h$ and $\bar{z}_{ds} = \sum_{h=1}^{L} w_h \bar{z}_h$

Lone *et al.* (2020) proposed an alternative to Ige and Tripathi (1989) estimators in double sampling for stratification as

$$\bar{y}^*_{Rd} = \frac{\bar{y}_{ds}}{\bar{x}'}\left[\frac{N\bar{x}' - n\bar{x}_{ds}}{N - n}\right], \tag{1.3}$$

and

$$\bar{y}^*_{Pd} = \frac{\bar{y}_{ds}}{\mathbf{z}'^{-1}}\left[\frac{N - n}{N\bar{z}' - n\bar{z}_{ds}}\right]. \tag{1.4}$$

where z is an auxiliary variate which is negatively correlated with the study variate y and notations $\bar{z}_{ds}$ and $\bar{z}'$ have their usual meanings.

The biases and mean squared errors of estimators $\bar{y}_{Rd}$, $\bar{y}_{Pd}$, $\bar{y}^*_{Rd}$ and $\bar{y}^*_{Pd}$ up to the first degree of approximation are defined as

$$B\left(\bar{y}_{Rd}\right)=\frac{1}{\bar{X}}\left[\sum_{h=1}^{L}\frac{W_h}{n'}\left(\frac{1}{v_h}-1\right)\left\{R_1 S_{xh}^2-S_{yxh}\right\}\right], \qquad (1.5)$$

$$B\left(\bar{y}_{Pd}\right)=\frac{1}{\bar{Z}}\left[\sum_{h=1}^{L}\frac{W_h}{n'}\left(\frac{1}{v_h}-1\right)S_{yzh}\right], \qquad (1.6)$$

$$B\left(\bar{y}_{Rd}^*\right)=-\frac{g}{\bar{X}}\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{yxh}\,, \qquad (1.7)$$

$$B\left(\bar{y}_{Pd}^*\right)=\frac{1}{\bar{Z}}\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[g^2 R_2\, S_{zh}^2+g\, S_{yzh}\right], \qquad (1.8)$$

$$MSE\left(\bar{y}_{Rd}\right)=S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+R_1^2 S_{xh}^2-2R_1 S_{yxh}\right], \qquad (1.9)$$

$$MSE\left(\bar{y}_{Pd}\right)=S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+R_2^2 S_{zh}^2+2R_2 S_{yzh}\right], \qquad (1.10)$$

$$MSE\left(\bar{y}_{Rd}^*\right)=S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2 R_1^2 S_{xh}^2-2\,gR_1 S_{yxh}\right], \qquad (1.11)$$

and

$$MSE\left(\bar{y}_{Pd}^*\right)=S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+g^2 R_2^2 S_{zh}^2+2gR_2 S_{yzh}\right]. \qquad (1.12)$$

Motivated by Koyuncu and Kadilar (2009), Tailor *et al.* (2015) proposed generalized ratio-cum-product type estimators in double sampling for stratification as

$$\bar{y}_{RP}=\bar{y}_{ds}\left(\frac{\bar{x}'}{\bar{x}_{ds}}\right)\left(\frac{\bar{z}_{ds}}{\bar{z}'}\right) \qquad (1.13)$$

The bias and mean squared error of the estimator $\bar{y}_{RP}$ are obtained as

$$B\left(\bar{y}_{RP}\right)=\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[\frac{1}{\bar{X}}\left(R_1 S_{xh}^2-S_{yxh}\right)+\frac{1}{\bar{Z}}\left(S_{yzh}-R_1 S_{xzh}\right)\right] \qquad (1.14)$$

and

$$MSE\left(\bar{y}_{RP}\right)=S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2+R_1^2 S_{xh}^2+R_2^2 S_{zh}^2-2R_1 S_{yxh}+2R_2 S_{yzh}-2R_1 R_2 S_{xzh}\right] \qquad (1.15)$$

2. PROPOSED ESTIMATORS

Motivated by Singh (1967) and Lone *et al.* (2020), we propose the following dual to ratio-cum-product type estimator in double sampling for stratification as

$$\bar{y}_{RPd}^*=\bar{y}_{ds}\left(\frac{\bar{x}_{ds}^*}{\bar{x}'}\right)\left(\frac{\bar{\mathbf{z}}'}{\bar{z}_{ds}^*}\right)$$

or
$$\bar{y}_{RPd}^*=\frac{\bar{y}_{ds}}{\bar{x}'}\left(\frac{N\bar{x}'-n\bar{x}_{ds}}{N-n}\right)\left(\frac{\bar{\mathbf{z}}'(N-n)}{N\bar{z}'-n\bar{z}_{ds}}\right) \qquad (2.1)$$

Where $\bar{x}_{ds}^*=\dfrac{N\bar{x}'-n\bar{x}_{ds}}{N-n}$ and $\bar{z}_{ds}^*=\dfrac{N\bar{z}'-n\bar{z}_{ds}}{N-n}$

To obtain the biases and mean squared errors of the proposed estimator $\bar{y}_{RPd}^*$ we write

$\bar{y}_{ds}=\bar{Y}\left(1+e_o\right)$, $\bar{x}_{ds}=\bar{X}\left(1+e_1\right)$, $\bar{x}'=\bar{X}\left(1+e_1'\right)$, $\bar{z}_{ds}=\bar{Z}\left(1+e_2\right)$ and $\bar{z}'=\bar{Z}\left(1+e_2'\right)$

such that $E\left(e_o\right)=E(e_1)=E\left(e_1'\right)=E(e_2)=E\left(e_2'\right)=0$ and

$$E\left(e_0^2\right)=\frac{1}{\bar{Y}^2}\left[S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\,S_{yh}^2\left(\frac{1}{v_h}-1\right)\right],$$

$$E\left(e_1^2\right)=\frac{1}{\bar{X}^2}\left[S_x^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\,S_{xh}^2\left(\frac{1}{v_h}-1\right)\right],$$

$$E\left(e_2^2\right)=\frac{1}{\bar{Z}^2}\left[S_z^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\,S_{zh}^2\left(\frac{1}{v_h}-1\right)\right],$$

$$E\left(e_0 e_1\right)=\frac{1}{\bar{Y}\bar{X}}\left[\left(\frac{1-f}{n'}\right)S_{yx}+\frac{1}{n'}\sum_{h=1}^{L}W_h\,S_{yxh}\left(\frac{1}{v_h}-1\right)\right],$$

$$E\left(e_0 e_2\right)=\frac{1}{\bar{Y}\bar{Z}}\left[\left(\frac{1-f}{n'}\right)S_{yz}+\frac{1}{n'}\sum_{h=1}^{L}W_h\,S_{yzh}\left(\frac{1}{v_h}-1\right)\right],$$

$$E\left(e_1 e_2\right)=\frac{1}{\bar{X}\bar{Z}}\left[\left(\frac{1-f}{n'}\right)S_{xz}+\frac{1}{n'}\sum_{h=1}^{L}W_h\,S_{xzh}\left(\frac{1}{v_h}-1\right)\right],$$

$$E\left(e_0 e_1'\right)=\frac{1}{\bar{Y}\bar{X}}\left(\frac{1-f}{n'}\right)S_{yx}\,,\quad E\left(e_1'^2\right)=\frac{1}{\bar{X}^2}S_x^2\left(\frac{1-f}{n'}\right),$$

$$E\left(e_2'^2\right)=\frac{1}{\bar{Z}^2}S_z^2\left(\frac{1-f}{n'}\right),\quad E\left(e_1 e_1'\right)=\frac{1}{\bar{X}^2}\left(\frac{1-f}{n'}\right)S_x^2,$$

$$E\left(e_2 e_2'\right)=\frac{1}{\bar{Z}^2}S_z^2\left(\frac{1-f}{n'}\right),\quad E\left(e_1' e_2'\right)=\frac{1}{\bar{X}\bar{Z}}\left(\frac{1-f}{n'}\right)S_{xz}$$

$$E\left(e_0 e_2'\right)=\frac{1}{\bar{Y}\bar{Z}}\left(\frac{1-f}{n'}\right)S_{yz} \text{ and } E\left(e_1 e_2'\right)=\frac{1}{\bar{X}\bar{Z}}\left(\frac{1-f}{n'}\right)S_{xz}\,.$$

Expressing proposed estimator in terms of $e_i\,'s$, we have

$$\bar{y}^*_{RPd} = \frac{\bar{Y}(1+e_o)}{\bar{X}(1+e_1')}\left(\frac{N\bar{X}(1+e_1') - n\bar{X}(1+e_1)}{N-n}\right)$$
$$\left(\frac{\bar{Z}(1+e_2')}{N\bar{Z}(1+e_2') - n\bar{Z}(1+e_2)/(N-n)}\right)$$

$$\bar{y}^*_{RPd} = \bar{Y}(1+e_o)\left(\frac{1+g_1e_1' - ge_1}{1+e_1'}\right)\left(\frac{(1+e_2')}{1+g_1e_2' - ge_2}\right)$$

$$\bar{y}^*_{RPd} = \bar{Y}(1+e_o)(1+g_1e_1' - ge_1)(1+e_1')^{-1}(1+e_2')$$
$$(1+g_1e_2' - ge_2)^{-1}$$

$$\bar{y}^*_{RPd} - \bar{Y} = \bar{Y}\big[e_o - ge_1 + ge_1' + ge_2 - ge_2' + ge_2e_2' + g^2e_1'e_2 - g^2e_1'e_2' - g^2e_1e_2 + g^2e_1e_2' - ge_1'^2 + ge_1e_1' + ge_oe_2 - ge_oe_2' + ge_oe_1' - ge_oe_1 - g_1e_2'^2 + g_1^2e_2'^2 + g^2e_2^2 - 2gg_1e_2e_2'\big] \quad (2.2)$$

Taking expectations on both sides of **(2.2),** we have

$$B(\bar{y}^*_{RPd}) = \bar{Y}\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[\frac{g^2S^2_{zh}}{\bar{Z}^2} - \frac{g^2S_{xzh}}{\bar{X}\bar{Z}} + \frac{gS_{yzh}}{\bar{Y}\bar{Z}} - \frac{gS_{yxh}}{\bar{Y}\bar{X}}\right] \quad (2.3)$$

Squaring and then taking expectations on both sides of (2.2), we have

$$MSE(\bar{y}^*_{RPd}) = \bar{Y}^2E(e_o - ge_1 + ge_1' + ge_2 - ge_2')^2$$

Hence up to the first degree of approximation the mean square error of the proposed estimator $\bar{y}^*_{RPd}$ is obtained as.

$$MSE(\bar{y}^*_{RPd}) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\big[S^2_{yh} + g^2R_1^2S^2_{xh} + g^2R_2^2S^2_{zh} - 2gR_1S_{yxh} + 2gR_2S_{yzh} - 2g^2R_1R_2S_{xzh}\big] \quad (2.4)$$

3. EFFICIENCY COMPARISONS

The variance of usual unbiased estimator $\bar{y}_{ds}$ in double sampling for stratification is given as

$$V(\bar{y}_{ds}) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h\,S^2_{yh}\left(\frac{1}{v_h}-1\right). \quad (3.1)$$

From (1.9), (1.10), (1.11), (1.12), (1.15), (2.4) and (3.1), it is concluded that the proposed estimator $\bar{y}^*_{RPd}$ would be more efficient than

(i) $\bar{y}_{ds}$ if

$$\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\big[g^2R_1^2S^2_{xh} + g^2R_2^2S^2_{zh} - 2gR_1S_{yxh} + 2gR_2S_{yzh} - 2g^2R_1R_2S_{xzh}\big] < 0 \quad (3.2)$$

(ii) $\bar{y}_{Rd}$ if

$$\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\big[R_1^2(g^2-1)S^2_{xh} + g^2R_2^2S^2_{zh} - 2R_1(g-1)S_{yxh} + 2gR_2S_{yzh} - 2g^2R_1R_2S_{xzh}\big] < 0 \quad (3.3)$$

(iii) $\bar{y}_{Pd}$ if

$$\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\big[R_1^2g^2S^2_{xh} + (g^2-1)R_2^2S^2_{zh} - 2R_1gS_{yxh} + 2(g-1)R_2S_{yzh} - 2g^2R_1R_2S_{xzh}\big] < 0 \quad (3.4)$$

(iv) $\bar{y}^*_{Rd}$ if

$$\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\big[g^2R_2^2S^2_{zh} + 2gR_2S_{yzh} - 2g^2R_1R_2S_{xzh}\big] < 0 \quad (3.5)$$

(v) $\bar{y}^*_{Pd}$ if

$$\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\big[g^2R_1^2S^2_{xh} - 2gR_1S_{yxh} - 2g^2R_1R_2S_{xzh} < 0\big] \quad (3.6)$$

(vi) $\bar{y}_{RP}$ if

$$\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\big[(g^2-1)R_1^2S^2_{xh} + (g^2-1)R_2^2S^2_{zh} - 2(g-1)R_1S_{yxh} + 2(g-1)R_2S_{yzh} - 2(g^2-1)R_1R_2S_{xzh} < 0 \quad (3.7)$$

where $R_1 = \frac{\bar{Y}}{\bar{X}}$, $R_2 = \frac{\bar{Y}}{\bar{Z}}$ $g_1 = \frac{N}{N-n}$ and $g = \frac{n}{N-n}$.

4. EMPIRICAL STUDY

To show the performance of the proposed estimator $\bar{y}^*_{RPd}$ in comparison to other considered estimators, a population data set is being used. The description of population is given below.

Population I- [Source: Tailor *et al.* (2014)]

y : Productivity (MT/Hectare) x : Production in ‘000 Tons and z : Area in ‘000 hectare

Table 4.1 shows that the proposed estimator $\bar{y}^*_{RPd}$ has **201.60** percent relative efficiency which is higher than $\bar{y}_{ds}$, $\bar{y}_{Rd}$, $\bar{y}_{Pd}$, $\bar{y}^*_{Rd}$, $\bar{y}^*_{Pd}$ and $\bar{y}_{RP}$. Table 4.2 shows that the proposed estimator $\bar{y}^*_{RPd}$ has **-0.0179** Bias , which is less than the other existing estimators.

Constant	Stratum I	Stratum II
N_h	10	10
n_h	4	4
n'_h	7	7
$\bar{Y}_h$	1.70	3.65
$\bar{X}_h$	10.41	289.14
$\bar{Z}_h$	6.32	80.67
S_{yh}	0.50	1.41
S_{xh}	3.53	111.61
S_{zh}	1.19	10.82
S_{yxh}	1.60	144.87
S_{yzh}	-0.05	-7.04
S_{xzh}	1.38	-92.02
S_y^2	2.20	

Table 4.1. MSE's and Percent Relative Efficiencies of $\bar{y}_d$, $\bar{y}_{Rd}$, $\bar{y}_{Pd}$, $\bar{y}^*_{Rd}$, $\bar{y}^*_{Pd}$, $\bar{y}_{RP}$ and $\bar{y}^*_{RPd}$ with respect to $\bar{y}_d$

Estimators	MSE	Percent Relative Efficiency (PRE)
$\bar{y}_{ds}$	0.107	100.00
$\bar{y}_{Rd}$	0.073	144.99
$\bar{y}_{Pd}$	0.095	111.86
$\bar{y}^*_{Rd}$	0.061	175.51
$\bar{y}^*_{Pd}$	0.096	110.62
$\bar{y}_{RP}$	0.067	158.09
$\bar{y}^*_{RPd}$	**0.053**	**201.60**

Table 4.2. Bias's of $\bar{y}_{Rd}$, $\bar{y}_{Pd}$, $\bar{y}^*_{Rd}$, $\bar{y}^*_{Pd}$, $\bar{y}_{RP}$ and $\bar{y}^*_{RPd}$

Estimators	BIAS
$\bar{y}_{Rd}$	0.01378
$\bar{y}_{Pd}$	-0.0044
$\bar{y}^*_{Rd}$	-0.0174
$\bar{y}^*_{Pd}$	-0.0023
$\bar{y}_{RP}$	**0.01041**
$\bar{y}^*_{RPd}$	-0.0179

5. CONCLUSION

The conditions under which the proposed estimator $\bar{y}^*_{RPd}$ has less mean squared error in comparison to other considered estimators are obtained. From the numerical study it is revealed that the proposed dual to ratio-cum-product type estimator $\bar{y}^*_{RPd}$ has less mean square error in comparison to usual unbiased estimator $\bar{y}_{ds}$, Ige and Tripathi (1987) ratio and product estimators $\bar{y}_{Rd}$ and $\bar{y}_{Pd}$, dual to ratio and product type estimators $\bar{y}^*_{Rd}$ and $\bar{y}^*_{Pd}$ given by Lone *et al.* (2020) and Tailor *et al.* (2015) ratio-cum-product type estimator $\bar{y}_{RP}$ .Thus the proposed estimator $\bar{y}^*_{RPd}$ are recommended for use in practice for estimating the finite population mean under certain conditions.

ACKNOWLEDGEMENTS

The authors are highly thankful to learned reviewer and the Editor for their valuable comments on the original version of the paper.

REFERENCES

Bandyopadhyay, S. (1980). Improved ratio and product estimators. Sankhya series, **42(2)**, 45-49.

Chouhan, S. (2012). Improved estimation of parameters using auxiliary information in sample surveys. Ph.D. Thesis, Vikram University, Ujjain, M.P. India.

Ige, A.F. and Tripathi, T.P. (1987). On doubling for stratification and use of auxiliary information. J. Ind. Soc. Agricul. Stat., **39**,191-201.

Jatwa, N.K. (2014). Estimation of population parameters in presence of auxiliary information. Ph.D. Thesis, Vikram University, Ujjain, M.P. India.

Koyuncu, N. and Kadilar, C. (2009). Family of estimators of population mean using two auxiliary variable in stratified random sampling. Comm. Statist. Theo. Meth., **139(8)**, 2552-2558.

Lone, H.A., Tailor, R. and Verma, M.R. (2020).An Alternative to ratio and product type estimators of finite population mean in double sampling for stratification. J. Ind. Soc. Agricul. Stat.,**74(1)**, 63-68.

Singh, H.P. and Vishwakarma, G.K. (2007). A general procedure for estimating the mean using double sampling for stratification. Model Assisted Statistics and Applications, **2(4)**, 225-237.

Singh, M.P. (1967). Radio cum product method of estimation, Metrika, **12**, 34-42.

Srivenkataramana, T. (1980). A dual to ratio estimator in sample surveys. Biometr., Jour., **67(1)**, 199-204.

Tailor R. and Lone, H.A. (2014). Ratio –cum- product estimator of finite population mean in double sampling for stratification. J. of Rel. & Statist. Stds., **7(1)**, 93-101.

Tailor, R., Chouhan, S. and Kim, J.M. (2014). Ratio and product type exponentials estimators of population mean in double sampling for stratification. Comm. for Statist. Appl. & Meths., **21(1)**, 1-9.

Tailor, R., Lone, H.A. and Pandey, R. (2015). Generalized Ratio-Cum-Product Type Estimator of Finite Population Mean in Double Sampling for Stratification. Comm. for Statist. Appl. & Meths., **22(3)**, 255–264.

Tripathi, T.P. and Bahl, S. (1991). Estimation of mean using double sampling for stratification and multivariate auxiliary information. Comm. Statist. Theo. & Meth., **20**, 2589-2602.