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Generalized Dual to Ratio-cum-Product Type Estimators in Double Sampling for Stratification

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SUMMARY

Double sampling for stratification is a statistical technique used in survey sampling to improve the efficiency of estimating population parameters estimates. In the present paper we have proposed a generalized dual to ratio-cum-product type estimator of population mean. The expression for the MSE (Mean Square Error) of the proposed estimator has been derived upto first degree of approximation. The comparison of the proposed estimator with Ige and Tripathi (1987) and Lone *et al.* (2020) estimators in double sampling for stratification indicated that proposed estimator is more efficient than the other estimators given in the literature. Motivated by Tailor and Lone (2014), the generalised version of the proposed estimator is also suggested in the present paper. The empirical study indicated that the proposed estimator is more efficient than the other estimators.

Keywords: Double sampling for stratification, Mean square error.

1. INTRODUCTION

Stratified random sampling presumes the knowledge of stratum size as well as sampling frame for all strata. In many cases strata weights are either unavailable or are outdated. Under these circumstances the researchers usually use double sampling for stratification. In double sampling for stratification, a preliminary sample of size n' is selected by simple random sampling without replacement to estimate strata weights and then a sub-sample of n units, n_h from the h-th stratum, is drawn to collect information on the study variable as well as the auxiliary variable.

Double sampling for stratification is a valuable technique in survey sampling, as it can provide more accurate and efficient estimates of population parameters while reducing the required sample size. The use of this technique should be considered in situations where the cost of measuring a variable is high or when there is significant variation in population characteristics across strata. In double sampling for stratification, dual to ratio and product estimators are two popular methods used to estimate population totals or means.

Overall, dual to ratio and product estimators are important tools in double sampling for stratification, and their use can lead to more precise and efficient estimates of population parameters. Dual to ratio cum product type estimators are a class of estimators used in double sampling for stratification that combine information from both the preliminary and second-phase samples. This class of estimators includes the dual to ratio estimator, the dual to product estimator, and the dual to ratio cum product estimator. The dual to ratio estimator uses the ratio of the second-phase sample mean to the preliminary sample mean as a scaling factor, and the difference between the product of the two sample means and the product of the preliminary sample mean and the estimated ratio as an adjustment factor. The dual to product estimator uses the product of the second-phase sample mean

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and the preliminary sample mean as a scaling factor, and the ratio of the second-phase sample mean to the preliminary sample mean as an adjustment factor. The dual to ratio cum product estimator combines both scaling and adjustment factors.

Ige and Tripathi (1987) studied the classical ratio and product estimators in double sampling for stratification. Singh and Vishwakarma (2007) discussed a general procedure for estimating the populations mean using double sampling for stratification. Tailor *et al.* (2014) suggested ratio and product type exponential estimators of population mean in double sampling for stratification. Following Srivenkataramana (1980) and Bandyopadhyay (1980) transformation, Lone *et al.* (2020) proposed an alternative to Ige and Tripathi (1987) estimators in double sampling for stratification. Singh (1967) and Lone *et al.* (2020) motivated authors to study a dual to ratio-cum-product type estimator in case of double sampling for stratification. The problem of estimating the finite population mean in double sampling for stratification has been discussed by many researchers including Tripathi and Bahl (1991), Chouhan (2012), Jatwa (2014), Tailor and Lone (2014) and Tailor *et al.*(2014), Singh and Nigam (2020a,b,c), Singh and Nigam (2022a,b,c), Vishwakarma and Singh (2012) and Vishwakarma and Zeeshan (2018). Overall, dual to ratio cum product type estimators are important tools in double sampling for stratification and have shown promising results in improving the precision and efficiency of estimation.

2. PROCEDURE, NOTATIONS AND DEFINITIONS

Let us consider a finite population $U = \{U_1, U_2, U_3, ...U_N\}$ of size N in which strata weight $\frac{N_h}{N}, \{h = 1, 2, 3, ...L\}$ are unknown. In these conditions we use double sampling for stratification. Procedure for double sampling for stratification is given below

(a) at first phase of sample S of size n' using simple random sampling without replacement is drawn and auxiliary variates x and z are observed.

(b) the samples is stratified into L strata on the basis of observed variables x and z. Let n'_h denotes the number of units in h^{th} stratum $(h = 1, 2, 3, ..., L)$ such that $n' = \sum_{h=1}^{L} n'_h$.

(c) from each n'_h unit, a sample of size $n_h = v_h n'_h$ is drawn where $0 < v_h < 1$, $\{h = 1, 2, 3, ..., L\}$, is the predetermined probability of selecting a sample of size n_h from each strata of size n'_h and it constitutes a sample S' of size $n = \sum_{h=1}^{L} n_h$. In S' both study variate y and auxiliary variates x and z are observed.

Let y be the study variate and x and z are the two auxiliary variate respectively. Let us define

$\bar{X} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} x_{hi}$:Population mean of the auxiliary variate x

$\bar{Y} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} y_{hi}$:Population mean of the study variate y

$\bar{Z} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} z_{hi}$:Population mean of the auxiliary variate z

$\bar{X}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} x_{hi}$: h^{th} stratum mean for the auxiliary variate x

$\bar{Y}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} y_{hi}$: h^{th} stratum mean for the study variate y

$\bar{Z}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} x_{hi}$: h^{th} stratum mean for the auxiliary variate z

$S_{xh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(x_{hi} - \bar{X}_h\right)^2$: h^{th} stratum population mean square of the auxiliary variate x

$S_{yh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(y_{hi} - \bar{Y}_h\right)^2$: h^{th} stratum population mean square of the study variate y

$S_{zh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(z_{hi} - \bar{Z}_h\right)^2$: h^{th} stratum population mean square of the auxiliary variate z

$\rho_{yxh} = \frac{S_{yxh}}{S_{yh} S_{xh}}$: Correlation coefficient between y and x in the stratum h

$\bar{x}'_h = \frac{1}{n'_h}\sum_{h=1}^{n'_h}\bar{x}_{hi}$: First phase sample mean of the h^{th} stratum for the auxiliary variate x

$\bar{z}'_h = \frac{1}{n'_h}\sum_{h=1}^{n'_h}\bar{z}_{hi}$: First phase sample mean of the h^{th} stratum for the auxiliary variate z

$f = \frac{n'}{N}$: First phase sampling fraction.

$n = \sum_{h=1}^{L} n_h$: size of the sample S'

$w'_h = \frac{n'_h}{n'}$: h^{th} stratum weight in the first phase sample

$\bar{x}' = \frac{1}{n'_h}\sum_{h=1}^{n'_h} w_h \bar{x}'_h$: Unbiased estimator of population mean $\bar{X}$

$\bar{z}' = \frac{1}{n'_h}\sum_{h=1}^{n'_h} w_h \bar{z}'_h$:Unbiased estimator of population mean $\bar{Z}$

Ige and Tripathi (1987) and Lone *et al.* (2020) defined some estimators in double sampling for stratification as

$$t_1 = \bar{y}_{ds}\left(\frac{\bar{x}'}{\bar{x}_{ds}}\right) \tag{2.1}$$

$$t_2 = \bar{y}_{ds}\left(\frac{\bar{z}_{ds}}{\bar{z}'}\right) \tag{2.2}$$

$$t_1^* = \frac{\bar{y}_{ds}}{\bar{x}'}\left[\frac{N\bar{x}' - n\bar{x}_{ds}}{N-n}\right] \tag{2.3}$$

and

$$t_2^* = \frac{\bar{y}_{ds}}{\bar{\mathbf{z}}'^{-1}}\left[\frac{N-n}{N\bar{\mathbf{z}}' - n\bar{z}_{ds}}\right] \tag{2.4}$$

Where $\bar{x}_{ds} = \sum_{h=1}^{L} w_h\bar{x}_h$, $\bar{y}_{ds} = \sum_{h=1}^{L} w_h\bar{y}_h$ and $\bar{z}_{ds} = \sum_{h=1}^{L} w_h\bar{z}_h$

where z is an auxiliary variate which is negatively correlated with the study variate y and notations $\bar{z}_{ds}$ and $\bar{z}'$ have their usual meanings.

The biases and mean squared errors of estimators t_1, t_2, t_1^* and t_2^* up to the first degree of approximation are defined as

$$B(t_1) = \frac{1}{\bar{X}}\left[\sum_{h=1}^{L}\frac{W_h}{n'}\left(\frac{1}{v_h}-1\right)\left\{R_1S_{xh}^2 - S_{yxh}\right\}\right] \tag{2.5}$$

$$B(t_2) = \frac{1}{\bar{Z}}\left[\sum_{h=1}^{L}\frac{W_h}{n'}\left(\frac{1}{v_h}-1\right)S_{yzh}\right] \tag{2.6}$$

$$B(t_1^*) = -\frac{g}{\bar{X}}\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{yxh} \tag{2.7}$$

$$B(t_2^*) = \frac{1}{\bar{Z}}\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[g^2R_2\,S_{zh}^2 + g\,S_{yzh}\right] \tag{2.8}$$

$$MSE(t_1) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2 + R_1^2S_{xh}^2 - 2R_1S_{yxh}\right] \tag{2.9}$$

$$MSE(t_2) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2 + R_2^2S_{zh}^2 + 2R_2S_{yzh}\right] \tag{2.10}$$

$$MSE(t_1^*) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2 + g^2R_1^2S_{xh}^2 - 2\,gR_1S_{yxh}\right] \tag{2.11}$$

and

$$MSE(t_2^*) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2 + g^2R_2^2S_{zh}^2 + 2gR_2S_{yzh}\right] \tag{2.12}$$

Motivated by Koyuncu and Kadilar (2009), Tailor *et al.* (2015) proposed generalized ratio-cum-product type estimators in double sampling for stratification as

$$t_3 = \bar{y}_{ds}\left(\frac{\bar{x}'}{\bar{x}_{ds}}\right)\left(\frac{\bar{z}_{ds}}{\bar{z}'}\right) \tag{2.13}$$

The bias and mean squared error of the estimator t_3 are obtained as

$$B(t_3) = \frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[\frac{1}{\bar{X}}\left(R_1S_{xh}^2 - S_{yxh}\right) + \frac{1}{\bar{Z}}\left(S_{yzh} - R_1S_{xzh}\right)\right] \tag{2.14}$$

and

$$MSE(t_3) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[S_{yh}^2 + R_1^2S_{xh}^2 + R_2^2S_{zh}^2 - 2R_1S_{yxh} + 2R_2S_{yzh} - 2R_1R_2S_{xzh}\right] \tag{2.15}$$

3. PROPOSED ESTIMATOR

Motivated by Lone at al. (2015), generalized dual to ratio-cum-product type estimators in double sampling for stratification is proposed as

$$h_{ds}^{*}=\bar{y}_{ds}\left[\alpha\left(\frac{N\bar{x}'-n\bar{x}_{ds}}{\bar{x}'(N-n)}\right)+(1-\alpha)\left(\frac{\bar{\mathbf{z}}'(N-n)}{N\bar{\mathbf{z}}'-n\bar{z}_{ds}}\right)\right] \quad (3.1)$$

Where α is suitably chosen real constant can be determined such that Mean Square Error of h_{ds}^{*} is minimum.

It is to be noted that:

(i) for $\alpha=1$ in (3.1) h_{ds}^{*} reduces to dual to ratio type estimator suggested by Lone *et al.* (2021) as

$$h_{ds}^{*}=\bar{y}_{ds}\left(\frac{N\bar{x}'-n\bar{x}_{ds}}{\bar{x}'(N-n)}\right)$$

(ii) for $\alpha=0$ in (3.1) h_{ds}^{*} reduces to dual to product type estimator suggested by Lone *et al.* (2021) as

$$h_{ds}^{*}=\bar{y}_{ds}\left(\frac{\bar{\mathbf{z}}'(N-n)}{N\bar{\mathbf{z}}'-n\bar{z}_{ds}}\right)$$

To obtain the biases and mean squared errors of the proposed estimator h_{ds}^{*} we write

$\bar{y}_{ds}=\bar{Y}(1+e_o)$, $\bar{x}_{ds}=\bar{X}(1+e_1)$, $\bar{x}'=\bar{X}(1+e_1')$, $\bar{z}_{ds}=\bar{Z}(1+e_2)$ and $\bar{z}'=\bar{Z}(1+e_2')$

such that $E(e_o)=E(e_1)=E(e_1')=E(e_2)=E(e_2')=0$ and

$$E(e_0^2)=\frac{1}{\bar{Y}^2}\left[S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h S_{yh}^2\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_1^2)=\frac{1}{\bar{X}^2}\left[S_x^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h S_{xh}^2\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_2^2)=\frac{1}{\bar{Z}^2}\left[S_z^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h S_{zh}^2\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_0e_1)=\frac{1}{\bar{Y}\bar{X}}\left[\left(\frac{1-f}{n'}\right)S_{yx}+\frac{1}{n'}\sum_{h=1}^{L}W_h S_{yxh}\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_0e_2)=\frac{1}{\bar{Y}\bar{Z}}\left[\left(\frac{1-f}{n'}\right)S_{yz}+\frac{1}{n'}\sum_{h=1}^{L}W_h S_{yzh}\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_1e_2)=\frac{1}{\bar{X}\bar{Z}}\left[\left(\frac{1-f}{n'}\right)S_{xz}+\frac{1}{n'}\sum_{h=1}^{L}W_h S_{xzh}\left(\frac{1}{v_h}-1\right)\right],$$

$$E(e_0e_1')=\frac{1}{\bar{Y}\bar{X}}\left(\frac{1-f}{n'}\right)S_{yx},\quad E(e_1'^2)=\frac{1}{\bar{X}^2}S_x^2\left(\frac{1-f}{n'}\right),$$

$$E(e_2'^2)=\frac{1}{\bar{Z}^2}S_z^2\left(\frac{1-f}{n'}\right),\quad E(e_1e_1')=\frac{1}{\bar{X}^2}\left(\frac{1-f}{n'}\right)S_x^2,$$

$$E(e_2e_2')=\frac{1}{\bar{Z}^2}S_z^2\left(\frac{1-f}{n'}\right),\quad E(e_1'e_2')=\frac{1}{\bar{X}\bar{Z}}\left(\frac{1-f}{n'}\right)S_{xz}$$

$$E(e_0e_2')=\frac{1}{\bar{Y}\bar{Z}}\left(\frac{1-f}{n'}\right)S_{yz}\ \text{and}\ E(e_1e_2')=\frac{1}{\bar{X}\bar{Z}}\left(\frac{1-f}{n'}\right)S_{xz}.$$

Hence up to the first degree of approximation the bias and mean square error of the proposed estimator h_{ds}^{*} is obtained as.

$$B(h_{ds}^{*})=\bar{Y}\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\left[\frac{g^2S_{zh}^2}{\bar{Z}^2}+\frac{gS_{yzh}}{\bar{Y}\bar{Z}}-\alpha\left\{\frac{gS_{zh}^2}{\bar{Z}^2}+\frac{gS_{yxh}}{\bar{Y}\bar{X}}+\frac{gS_{yzh}}{\bar{Y}\bar{Z}}\right\}\right] \quad (3.2)$$

and

$$MSE(h_{ds}^{*})=S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)\Big[\left(S_{yh}^2+g^2R_2^2S_{zh}^2+2gR_2S_{yzh}\right)+\alpha^2\left(g^2R_1^2S_{xh}^2+g^2R_2^2S_{zh}^2+2g^2R_1R_2S_{xzh}\right)-2\alpha\left(gR_1S_{yxh}+gR_2S_{yzh}+g^2R_1R_2S_{xzh}+g^2R_2^2S_{zh}^2\right)\Big] \quad (3.3)$$

Equation (3.3) can also be written as

$$MSE(h_{ds}^{*})=A+B+g^2R_2^2C+2gR_2D+\alpha^2\left(g^2R_1^2E+g^2R_2^2C+2g^2R_1R_2F\right)-2\alpha\left(gR_1G+gR_2D+g^2R_1R_2F+g^2R_2^2C\right) \quad (3.4)$$

Where

$$A=S_y^2\left(\frac{1-f}{n'}\right),\quad B=\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{yh}^2,$$

$$C=\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{zh}^2,\quad D=\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{yzh},$$

$$E=\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{xh}^2,\quad F=\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{xzh},$$

$$G=\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{v_h}-1\right)S_{yxh}$$

The mean square error of the proposed estimator h_{ds}^{*} is minimum when

$$\alpha=\frac{\eta}{\kappa},$$

Thus the resulting minimum mean square error of the proposed estimator h_{ds}^* is

$$MSE_{Min}\,(h_{ds}^*) = \gamma - \frac{\eta^2}{\kappa},$$

where $\eta = gR_1G + gR_2D + g^2R_1R_2F + g^2R_2^2C$,

$\gamma = A + B + g^2R_2^2C + 2gR_2D$ and

$\kappa = g^2R_1^2E + g^2R_2^2C + 2gR_1R_2F$.

4. EFFICIENCY COMPARISON

The variance of usual unbiased estimator $\bar{y}_{ds}$ in double sampling for stratification is given as

$$V\left(\bar{y}_{ds}\right) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L} W_h S_{yh}^2\left(\frac{1}{v_h} - 1\right). \tag{4.1}$$

From (2.9), (2.10), (2.11), (2.12), (2.15), (3.4) and (4.1), it is concluded that the proposed estimator $\bar{y}_{RPd}^*$ would be more efficient than

(i) $\bar{y}_{ds}$ if

$$g^2R_2^2C + 2gR_2D + \alpha^2\kappa - 2\alpha\eta < 0 \tag{4.2}$$

(ii) t_1 if

$$g^2R_2^2C + 2gR_2D + \alpha^2\kappa - 2\alpha\eta - 2R_1^2E + 2R_1G < 0 \tag{4.3}$$

(iii) t_2 if

$$g^2R_2^2C + 2gR_2D + \alpha^2\kappa - 2\alpha\eta - R_2^2C - 2R_2D < 0 \tag{4.4}$$

(iv) t_1^* if

$$g^2R_2^2C + 2gR_2D + \alpha^2\kappa - 2\alpha\eta - g^2R_1^2E + 2gR_1G < 0 \tag{4.5}$$

(v) t_2^* if

$$g^2R_2^2C + 2gR_2D + \alpha^2\kappa - 2\alpha\eta - g^2R_2^2C - 2gR_2D < 0 \tag{4.6}$$

(vi) t_3 if

$$g^2R_2^2C + 2gR_2D + \alpha^2\kappa - 2\alpha\eta - R_1^2E - R_2^2C + 2R_1G - 2R_2D + 2R_1R_2F < 0 \tag{4.7}$$

where $R_1 = \dfrac{\bar{Y}}{\bar{X}}$, $R_2 = \dfrac{\bar{Y}}{\bar{Z}}$ $g_1 = \dfrac{N}{N-n}$ and $g = \dfrac{n}{N-n}$.

5. GENERALISED DUAL TO RATIO –CUM-PRODUCT TYPE ESTIMATOR

Motivated by Tailor and Lone (2014), generalised dual to ratio –cum-product types estimator in double sampling for stratification can be defined as

$$t_{ds}^* = M\,\bar{y}_{ds}\left[\alpha\left(\frac{N\bar{x}' - n\bar{x}_{ds}}{\bar{x}'(N-n)}\right) + (1-\alpha)\left(\frac{\bar{\mathbf{z}}'(N-n)}{N\bar{\mathbf{z}}' - n\bar{z}_{ds}}\right)\right] \tag{5.1}$$

where $0 \le \alpha \le 1$ and M is a suitably chosen constant to be determined such that MSE of t_{ds}^* is minimum,

Hence up to the first degree of approximation the Bias and mean square error of the proposed estimator t_{ds}^* is obtained as.

$$B\left(t_{ds}^*\right) = \bar{Y}\left[M\left\{1 + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{v_h} - 1\right)\left\{\left(\frac{g^2S_{zh}^2}{\bar{Z}^2} + \frac{gS_{yzh}}{\bar{Y}\bar{Z}}\right) - \alpha\left(\frac{gS_{zh}^2}{\bar{Z}^2} + \frac{gS_{yxh}}{\bar{Y}\bar{X}} + \frac{gS_{yzh}}{\bar{Y}\bar{Z}}\right)\right\}\right\} - 1\right] \tag{5.2}$$

and

$$\begin{aligned}MSE\left(t_{ds}^*\right) = \bar{Y}^2\Big[M^2\Big\{1 &+ \frac{1}{\bar{Y}^2}S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{v_h} - 1\right)\\ &\left(\frac{S_{yh}^2}{\bar{Y}^2} + \frac{3g^2S_{zh}^2}{\bar{Z}^2} + \frac{4gS_{yzh}}{\bar{Y}\bar{Z}}\right) + \alpha^2\frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{v_h} - 1\right)\left(\frac{g^2S_{xh}^2}{\bar{X}^2} + \right.\\ &\left.\frac{g^2S_{zh}^2}{\bar{Z}^2} + \frac{2gS_{yzh}}{\bar{Y}\bar{Z}} + \frac{g^2S_{xzh}}{\bar{X}\bar{Z}}\right) - 2\alpha\frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{v_h} - 1\right)\left(\frac{2g^2S_{zh}^2}{\bar{Z}^2} + \right.\\ &\left.\frac{2gS_{yxh}}{\bar{Y}\bar{X}} + \frac{2gS_{yzh}}{\bar{Y}\bar{Z}} + \frac{g^2S_{xzh}}{\bar{X}\bar{Z}}\right)\Big\} + 1 - 2m\Big\{1 + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{v_h} - 1\right)\\ &\left(\frac{g^2S_{zh}^2}{\bar{Z}^2} + \frac{gS_{yzh}}{\bar{Y}\bar{Z}}\right) - \alpha\frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{v_h} - 1\right)\left(\frac{g^2S_{zh}^2}{\bar{Z}^2} + \frac{gS_{yxh}}{\bar{Y}\bar{X}} + \frac{gS_{yzh}}{\bar{Y}\bar{Z}}\right)\Big\}\Big]\end{aligned} \tag{5.3}$$

Equation (5.3) can also be written as

$$MSE\left(t_{ds}^*\right) = \bar{Y}^2\left(1 + M^2\psi - 2M\pi\right)$$

where

$$\begin{aligned}\psi = \Big\{1 &+ \frac{1}{\bar{Y}^2}S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{v_h} - 1\right)\left(\frac{S_{yh}^2}{\bar{Y}^2} + \right.\\ &\left.\frac{3g^2S_{zh}^2}{\bar{Z}^2} + \frac{4gS_{yzh}}{\bar{Y}\bar{Z}}\right) + \alpha^2\frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{v_h} - 1\right)\left(\frac{g^2S_{xh}^2}{\bar{X}^2} + \right.\\ &\left.\frac{g^2S_{zh}^2}{\bar{Z}^2} + \frac{2gS_{yzh}}{\bar{Y}\bar{Z}} + \frac{g^2S_{xzh}}{\bar{X}\bar{Z}}\right) - 2\alpha\frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{v_h} - 1\right)\\ &\left(\frac{2g^2S_{zh}^2}{\bar{Z}^2} + \frac{2gS_{yxh}}{\bar{Y}\bar{X}} + \frac{2gS_{yzh}}{\bar{Y}\bar{Z}} + \frac{g^2S_{xzh}}{\bar{X}\bar{Z}}\right)\Big\}\end{aligned}$$

and

$$\pi = \left\{1 + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{v_h} - 1\right)\left(\frac{g^2 S_{zh}^2}{\bar{Z}^2} + \frac{gS_{yzh}}{\bar{Y}\,\bar{Z}}\right) - \alpha\frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{v_h} - 1\right)\left(\frac{g^2 S_{zh}^2}{\bar{Z}^2} + \frac{gS_{yxh}}{\bar{Y}\,\bar{X}} + \frac{gS_{yzh}}{\bar{Y}\,\bar{Z}}\right)\right\}$$

The mean square error of the proposed estimator t_{ds}^* is minimum when

$$M = \frac{\pi}{\psi}.$$

Thus the resulting minimum mean square error of the proposed estimator t_{ds}^* is

$$MSE_{\min}(t_{ds}^*) = \bar{Y}^2\left(1 - \rho^2\right) \tag{5.4}$$

where $\rho^2 = \dfrac{\pi^2}{\psi}$

6. EFFICIENCY COMPARISON

From (2.9), (2.10), (2.11), (2.12), (2.15), (3.4), (5.4) and (4.1), it is concluded that the proposed estimator t_{ds}^* would be more efficient than

(i) $\bar{y}_{ds}$ if

$$\rho^2 > 1 - \frac{1}{\bar{Y}^2}(A + B) \tag{5.1}$$

(ii) t_1 if

$$\rho^2 > 1 - \frac{1}{\bar{Y}^2}\left(A + B + R_1^2 E - 2R_1 G\right) \tag{5.2}$$

(iii) t_2 if

$$\rho^2 > 1 - \frac{1}{\bar{Y}^2}\left(A + B + R_2^2 C + 2R_2 D\right) \tag{5.3}$$

(iv) t_1^* if

$$\rho^2 > 1 - \frac{1}{\bar{Y}^2}\left(A + B + R_1^2 g^2 E - 2gR_1 G\right) \tag{5.4}$$

(v) t_2^* if

$$\rho^2 > 1 - \frac{1}{\bar{Y}^2}\left(A + B + R_2^2 g^2 C + 2gR_2 D\right) \tag{5.5}$$

(vi) t_3 if

$$\rho^2 > 1 - \frac{1}{\bar{Y}^2}\left(A + B + R_1^2 E + R_2^2 C - 2R_1 G + 2R_2 D - 2R_1 R_2 F\right) \tag{5.6}$$

(vii) h_{ds}^* if

$$\rho^2 > 1 - \frac{1}{\bar{Y}^2}\left\{A + B + g^2 R_2^2 C + 2gR_2 D + \alpha^2\left(g^2 R_1^2 E + g^2 R_2^2 C + 2g^2 R_1 R_2 F\right) - 2\alpha\left(gR_1 G + gR_2 D + g^2 R_1 R_2 F + g^2 R_2^2 C\right)\right\} \tag{6.7}$$

7. EMPIRICAL STUDY

To show the performance of the proposed estimators in comparison to other considered estimators, a population data set is being used. The description of population is given below.

Population I- [Source: Tailor *et al.* (2014)]

y : Productivity (MT/Hectare) x : Production in ‘000 Tons and z : Area in ‘000 hectare

Constant	Stratum I	Stratum II
N_h	10	10
n_h	4	4
n'_h	7	7
$\bar{Y}_h$	1.70	3.65
$\bar{X}_h$	10.41	289.14
$\bar{Z}_h$	6.32	80.67
S_{yh}	0.50	1.41
S_{xh}	3.53	111.61
S_{zh}	1.19	10.82
S_{yxh}	1.60	144.87
S_{yzh}	-0.05	-7.04
S_{xzh}	1.38	-92.02
S_y^2	2.20	

Table 1 reveals that the proposed dual to ratio-cum-product type estimator h_{ds}^* has highest percent relative efficiency in comparison to usual unbiased estimator $\bar{y}_{ds}$, Ige and Tripathi (1987) ratio and product estimators t_1 and t_2, dual to ratio and product type estimators t_1^* and t_2^* given by Lone *et al.* (2020) and

Table 1. MSE's and Percent Relative Efficiencies of $\bar{y}_{ds}$, t_1 , t_2 , t_1^* , t_2^* , t_3 , h_{ds}^* and t_{ds}^* with respect to $\bar{y}_{ds}$

Estimators	MSE	Percent Relative Efficiency (PRE)
$\bar{y}_{ds}$	0.107	100.00
t_1	0.073	144.99
t_2	0.095	111.86
t_1^*	0.061	175.51
t_2^*	0.096	110.62
t_3	0.067	158.09
h_{ds}^*	**0.059**	**181.35**
t_{ds}^*	**0.056**	**191.07**

Tailor *et al.* (2015) ratio-cum-product type estimator t_3 . Table 1 also reveals that the proposed estimator t_{ds}^* has highest percent relative efficiency in comparison to $\bar{y}_{ds}$, t_1, t_2 , t_1^* , t_2^* , t_3 and h_{ds}^* estimators.

8. CONCLUSION

In this paper we proposed the generalized dual to ratio-cum-product estimators in double sampling for stratification. The proposed estimator provides a flexible and efficient alternative to existing estimators and the recently proposed dual to ratio-cum-product estimators. The conditions under which the proposed estimator h_{ds}^* has less mean square error in comparison to other considered estimators are obtained. The performance of the proposed estimators was evaluated through a numerical study, which demonstrated the superiority over the other commonly used estimators in terms of mean square error. The application of the proposed estimator to a real dataset also showed the effectiveness in practice. Therefore we recommend the use of the proposed estimator in double sampling for stratification, particularly when the population values of the auxiliary variable are unknown or hard to obtain.

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