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Generalized Class of Some Novel Estimators under Ranked Set Sampling

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SUMMARY

In this paper, mean estimators under ranked set sampling are reviewed. In this paper, we have also presented some improved novel classes of estimators for estimating the population mean using auxiliary variable under ranked set sampling. We have derived the expressions for bias and mean squared error of the proposed estimators up to the first order of approximation and the proposed classes of estimators are found to be more efficient than the other estimators in this study. In an attempt to verify the efficiencies of proposed estimators, theoretical results are supported by empirical study.

Keywords: Study variable; Auxiliary variable; Bias, Mean square error; Ranked set sampling.

MSC: 62D05

1. INTRODUCTION

Ranked set sampling (RSS) is an improved sampling method over simple random sampling (SRS). McIntyre (1952) was the first to explain RSS for estimating the population mean. McIntyre (1952) showed that the RSS estimator is an unbiased estimator of the population mean. He also showed that the RSS estimator of the population mean is more efficient than the SRS estimator based on the same sample size. Takahasi and Wakimoto (1968) gave the necessary mathematical theory of RSS. Dell and Clutter (1972) considered the case of perfect and imperfect ranking and showed that the mean under RSS is an unbiased estimator of the population mean.

Samawi and Muttlak (1996) suggested ratio estimators of population mean in RSS and showed that the RSS estimators gave improved results over their SRS counterparts. Ganeshand Ganeslingam (2006) compared RSS with SRS for the estimation of the mean and the ratio. He concluded that RSS gives a better estimate for both the mean and the ratio. Singh *et al.* (2014) suggested a general procedure for estimating the population mean using RSS. Bouza and Al-Omari (2014) and Bouza *et al.* (2018) provided a review of RSS, its modification, and its application. Mandowara and Mehta (2016) introduced modified ratio-cum-product estimators under RSS. For more recent work interested readers may refer to the works of Bhushan and Kumar (2020a, 2020b, and 2021) and Bhushan *et al.* (2022) for a comprehensive study of RSS.

In RSS, we rank randomly selected units from the population merely by observation or prior experience after which only a few of these sampled units are measured. RSS is more cost-friendly than SRS because fewer samples need to be collected and measured. For example, if we want to estimate the contamination level in an area, which is a costly process. We may rank the extent of defoliation i.e. black spot or deprivation of leaves of trees. Then select sampling units based on the ranking of the extent of defoliation and then measure the contamination level of only selected units after ranking.

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2. SAMPLING METHODOLOGY

RSS takes the following steps.

1. Select sampling units from the target population.
2. Randomly partitioned sampling units into disjoint subsets each having a pre-assigned size (usually taken to be ≤4, as it is convenient and minimizes ranking error.
3. Rank each sub-set.
4. Measure one suitable selected unit from each ranked sub-set.

Coming to the mathematical formulation of RSS, if in the RSS scheme, we want to select a sample of size k. We select k random sets each of size k from the target population. Each set is then ranked by observation/inspection/prior information or a convenient/cheap method.

Original observation After Ranking

$$\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1k} \\ x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ x_{k1} & x_{k2} & \cdots & x_{kk} \end{bmatrix} \rightarrow \begin{bmatrix} x_{1(1)} & x_{1(2)} & \cdots & x_{1(k)} \\ x_{2(1)} & x_{2(2)} & \cdots & x_{2(k)} \\ \vdots & \vdots & \vdots & \vdots \\ x_{k(1)} & x_{k(2)} & \cdots & x_{k(k)} \end{bmatrix}$$

Here x_{ij} represents j^{th} the observation in the i^{th} set and is the ordered statistic in the set. After Ranking, select the diagonal units and them. We have now $x_{1(1)}$, $x_{2(2)}$, ……, $x_{k(k)}$ by selecting the smallest ranked unit from the first row, the second smallest ranked unit from the second row, and so on until the largest unit from k^{th} row selected. This will be the Ranked Set Sample (RSS). We can repeat the whole steps r times to obtain an RSS of size n=rk.

$$\bar{X}_{RSS} = \frac{1}{rk}\sum_{l=1}^{r}\sum_{i=1}^{k} x_{i(i)l} \tag{1.1}$$

$$var\left(\bar{X}_{RSS}\right) = \frac{\sigma^2}{n} - \frac{1}{rk^2}\sum_{i=1}^{k}\left(\mu_{(i)} - \mu\right)^2 \tag{1.2}$$

where $\mu_{(i)}$ the mean of the i^{th} is ranked set and is given by

$$\mu_{(i)} = \frac{1}{r}\sum_{l=1}^{r} x_{i(i)l} \qquad , i = 1,2,\ldots,k \tag{1.3}$$

In this paper, we take the situation where we use ranking on an auxiliary variable. Consider a finite population U = $(U_1, U_2, \ldots, U_N)$ based on N identifiable units with a study variable Y and auxiliary variables X. We define

$$\bar{y}_{[n]} = \frac{1}{kr}\sum_{i=1}^{k}\sum_{l=1}^{r} y_{[i]l} \ \ and\ \ \bar{x}_{(n)} = \frac{1}{kr}\sum_{i=1}^{k}\sum_{l=1}^{r} x_{(i)l}$$

as the sample means for the study and auxiliary variables.

$$\hat{C}_y = \frac{1}{\bar{y}_{[n]}}\sqrt{\frac{1}{rk-1}\sum_{i=1}^{k}\sum_{l=1}^{r}\left(y_{[i]l} - \bar{y}_{[n]}\right)^2} \quad and$$

$$\hat{C}_x = \frac{1}{\bar{x}_{(n)}}\sqrt{\frac{1}{rk-1}\sum_{i=1}^{k}\sum_{l=1}^{r}\left(x_{(i)l} - \bar{x}_{(n)}\right)^2}$$

As the sample coefficient of variation for the study and auxiliary variables.

$$\hat{C}_{yx} = \frac{1}{(rk-1)\bar{y}_{[n]}\bar{x}_{(n)}}\sum_{i=1}^{k}\sum_{l=1}^{r}\left(y_{[i]l} - \bar{y}_{[n]}\right)\left(x_{(i)l} - \bar{x}_{(n)}\right)$$

To obtain bias and MSE of the estimators, we define

$\bar{y}_{[n]} = \bar{Y}(1 + \epsilon_0)$,

$\bar{x}_{(n)} = \bar{X}(1 + \epsilon_1)$,

such that

E (ϵ_0)=E (ϵ_1)=0,

E (ϵ_0^2)=$\eta C_y^2 - D_{y[i]}^2$=V_{20},

E (ϵ_1^2)=$\eta C_x^2 - D_{x[i]}^2 = V_{02}$,

E $(\epsilon_0\epsilon_1) = \eta C_{yx} - D_{yx[i]}$=$V_{11}$,

where,

$D_{y[i]}^2 = \frac{1}{k^2 r \bar{y}_{[n]}^2}\sum_{i=1}^{k}\left(\mu_{[iy]} - \bar{y}_{[n]}\right)^2$,

$D_{x[i]}^2 = \frac{1}{k^2 r \bar{x}_{(n)}^2}\sum_{i=1}^{k}\left(\mu_{(ix)} - \bar{x}_{(n)}\right)^2$,

$D_{yx[i]} = \frac{1}{k^2 r \bar{y}_{[n]}\bar{x}_{(n)}}\sum_{i=1}^{k}\left(\mu_{[iy]} - \bar{y}_{[n]}\right)\left(\mu_{(ix)} - \bar{x}_{(n)}\right)$,

$\eta = \frac{1}{kr}$.

where $\mu_{[iy]}$ and $\mu_{(ix)}$ are the means of the i^{th} ranked set and are given by

$\mu_{[iy]} = \frac{1}{r}\sum_{l=1}^{r} y_{i(i)l}$, $\mu_{(ix)} = \frac{1}{r}\sum_{l=1}^{r} x_{i(i)l}$.

3. EXISTING ESTIMATORS

Estimators	MSE
usual mean estimator $t_{RSS} = \bar{y}_{[n]}$	$\text{Var}\,(t_{RSS}) = \bar{Y}^2(\eta C_y^2 - D_{y[i]}^2) = \bar{Y}^2 V_{20}$
Samawi and Muttlak (1996) $t_{r,RSS} = \frac{\bar{y}_{[n]}}{\bar{x}_{(n)}}\bar{X}$	$\text{MSE}\,(t_{r,RSS}) = \bar{Y}^2[\eta(C_y^2 + C_x^2 - 2\rho C_y C_x) - D_{y[i]}^2 - D_{x[i]}^2 + 2D_{yx[i]}]$
Yu and Lam (1997) $t_{reg,RSS} = \bar{y}_{[n]} + \hat{\beta}(\bar{X} - \bar{x}_{(n)})$	$MSE(t_{reg,RSS}) = \bar{Y}^2\left[\eta C_y^2 - D_{y[i]}^2 - \frac{(\eta C_{yx} - D_{yx[i]})^2}{(\eta C_x^2 - D_{x[i]}^2)}\right],$ $where\ \hat{\beta} = \frac{\hat{R}(\eta \hat{C}_{yx} - \hat{D}_{yx[i]})}{(\eta \hat{C}_x^2 - \hat{D}_{x[i]}^2)}\quad and\ \hat{R} = \frac{\bar{y}_{[n]}}{\bar{x}_{(n)}}$
Kadilar *et al.* (2009) $t_{kc} = \frac{k\bar{y}_{[n]}}{\bar{x}_{(n)}}\bar{X}$	$MSE\,(t_{kc}) = \bar{Y}^2[\eta C_x^2 - D_{x[i]}^2 - 2k(\eta C_{yx} - D_{yx[i]}) + k^2(\eta C_y^2 - D_{y[i]}^2) + (k+1)^2]$, where $k = \frac{1+\eta C_{yx} - D_{yx[i]}}{1+\eta C_y^2 - D_{y[i]}^2}$
Al-Omari *et al.* (2009) $t_{ai} = \bar{y}_{[n]}\left(\frac{\bar{X} + q_i}{\bar{x}_{(n)} + q_i}\right), i = 1,3$ where q_i is the ith quartile of variable X.	$MSE(t_{ai}) = \bar{Y}^2\left((\eta C_y^2 - D_{y[i]}^2) + \theta_i^2(\eta C_x^2 - D_{x[i]}^2) - 2\theta_i(\eta C_{xy} - D_{xy[i]})\right)$, i = 1,3 counterparts. Shiva $where\ \theta_i = \frac{\bar{X}}{\bar{X} + q_i}\quad, i = 1,3$
Al-Hadhrami (2009) $t_{sa} = (\bar{y}_{[n]} + \hat{\beta}\ \bar{X} - \bar{x}_{(n)}))\left(\frac{A\bar{X}+B}{A\bar{x}_{(n)}+B}\right)$ Where A & B are either constants or functions of some known population parameters.	$MSE(t_{sa}) = \bar{Y}^2\left[\eta C_y^2 - D_{y[i]}^2 - \frac{(\eta C_{yx} - D_{yx[i]})^2}{(\eta C_x^2 - D_{x[i]}^2)}\right]$
Mandowara and Mehta (2013) $t_{mm1} = \bar{y}_{[n]}\left(\frac{\bar{X} + C_x}{\bar{x}_{(n)} + C_x}\right)$ $t_{mm2} = \bar{y}_{[n]}\left(\frac{\bar{X} + \beta_2(x)}{\bar{x}_{(n)} + \beta_2(x)}\right)$ $t_{mm3} = \bar{y}_{[n]}\left(\frac{\beta_2(x)\bar{X} + C_x}{\beta_2(x)\bar{x}_{(n)} + C_x}\right)$ $t_{mm4} = \bar{y}_{[n]}\left(\frac{C_x\bar{X} + \beta_2(x)}{C_x\bar{x}_{(n)} + \beta_2(x)}\right)$ $t_{mm5} = \bar{y}_{[n]}\left(\frac{\beta_2(x)\bar{X} + C_x}{\beta_2(x)\bar{x}_{(n)} + C_x}\right)^{\alpha_5}$ $t_{mm6} = \bar{y}_{[n]}\left(\frac{C_x\bar{X} + \beta_2(x)}{C_x\bar{x}_{(n)} + \beta_2(x)}\right)^{\alpha_6}$ where $\beta_2(x)$ is the coefficient of kurtosis.	$MSE(t_{mmi}) = \bar{Y}^2\left((\eta C_y^2 - D_{y[i]}^2) + \theta_i^2(\eta C_x^2 - D_{x[i]}^2) - 2\theta_i(\eta C_{xy} - D_{xy[i]})\right)\ i = 1,2,3,4$ $MSE(t_{mmi}) = \bar{Y}^2\left((\eta C_y^2 - D_{y[i]}^2) + \gamma_i^2\theta_i^2(\eta C_x^2 - D_{x[i]}^2) - 2\gamma_i\theta_i(\eta C_{xy} - D_{xy[i]})\right)\ , i = 5,6$ where, $\theta_1 = \frac{\bar{X}}{\bar{X} + C_x}, \theta_2 = \frac{\bar{X}}{\bar{X} + \beta_2(x)}, \theta_3 = \theta_5 = \frac{\beta_2(x)\bar{X}}{\beta_2(x)\bar{X} + C_x}\ and\ \theta_4 = \theta_6 = \frac{C_x\bar{X}}{C_x\bar{X} + \beta_2(x)}$ $\gamma_5 = \frac{\rho_{yx}C_y}{\theta_5 C_x}\ and\ \gamma_6 = \frac{\rho_{yx}C_y}{\theta_6 C_x}.$
Singh *et al.* (2014) $t_s = \lambda_1\bar{y}_{[n]} + \lambda_2\bar{y}_{[n]}\left(\frac{\bar{X}^*}{\theta\bar{x}_{(n)}^{\ *} + (1-\theta)\bar{X}^*}\right)^g$ where g is a real constant assuming values 1 and -1, λ_1 and λ_2 are scalars. Also, $\bar{X}^* = a\bar{X} + b$ and $\bar{x}_{(n)}^{\ *} = a\bar{x} + b$, a(≠0) and b symbolize either real values or function of available.	$MSE(t_s) = \bar{Y}^2\left[1 - \frac{(B - 2CD + AD^2)}{AB - C^2}\right]$ where, $A = 1 + V_{20},\ \theta = \frac{a\bar{X}}{a\bar{X}+b},$ $B = 1 + V_{20} + g\,(2g+1)\theta^2\alpha^2 V_{02} - 4g\alpha\theta V_{11},$ $C = 1 + V_{20} + \frac{g(g+1)}{2}\theta^2\alpha^2 V_{02} - 2g\alpha\theta V_{11},$ $D = 1 + \frac{g(g+1)}{2}\theta^2\alpha^2 V_{02} - g\alpha\theta V_{11}.$

Jeelani *et al.* (2014) $t_j = (\bar{y}_{[n]} + \hat{\beta}(\bar{X} - \bar{x}_{(n)}))\left(\frac{A\bar{X}+B}{A\bar{x}_{(n)}+B}\right)$ $t_{j1} = (\bar{y}_{[n]} + \hat{\beta}(\bar{X} - \bar{x}_{(n)}))\left(\frac{\bar{X}+Q_d}{\bar{x}_{(n)}+Q_d}\right)$ $t_{j1} = (\bar{y}_{[n]} + \hat{\beta}(\bar{X} - \bar{x}_{(n)}))\left(\frac{\bar{X}+M_d}{\bar{x}_{(n)}+M_d}\right)$ $t_{j2} = (\bar{y}_{[n]} + \hat{\beta}(\bar{X} - \bar{x}_{(n)}))\left(\frac{M_d\bar{X}+Q_d}{M_d\bar{x}_{(n)}+Q_d}\right)$ $t_{j2} = (\bar{y}_{[n]} + \hat{\beta}(\bar{X} - \bar{x}_{(n)}))\left(\frac{Q_d\bar{X}+M_d}{Q_d\bar{x}_{(n)}+M_d}\right)$ where A & B are either constants of function of some known population parameters. Q_d is quartile deviation and M_d is median of auxiliary variable.	$MSE(t_{ji}) = \bar{Y}^2\left(\left(\eta C_y^2 - D_{y[i]}^2\right) + D_i^2\left(\eta C_x^2 - D_{x[i]}^2\right) - 2D_i\left(\eta C_{xy} - D_{xy[i]}\right)\right)$ i=1, 2, 3, 4 where $D_i = \frac{B\beta + A\bar{Y} + A\beta\bar{X}}{A\bar{X}+B}$. Min MSE $(t_{ji}) = \bar{Y}^2\left[\eta C_y^2 - D_{y[i]}^2 - \frac{(\eta C_{yx} - D_{yx[i]})^2}{(\eta C_x^2 - D_{x[i]}^2)}\right]$
Brar and Malik (2014) $t_{bm} = \bar{y}_{[n]}\left(\frac{\bar{X}+B}{\bar{x}_{(n)}+B}\right)$ $t_{bm1} = \bar{y}_{[n]}\left(\frac{\bar{X}+C_x}{\bar{x}_{(n)}+C_x}\right)$ $t_{bm2} = \bar{y}_{[n]}\left(\frac{\bar{X}+\rho}{\bar{x}_{(n)}+\rho}\right)$ $t_{bm3} = \bar{y}_{[n]}\left(\frac{\bar{X}+q_i}{\bar{x}_{(n)}+q_i}\right)$	$MSE(t_{bmi}) = \bar{Y}^2\left(\left(\eta C_y^2 - D_{y[i]}^2\right) + G_i^2\left(\eta C_x^2 - D_{x[i]}^2\right) - 2G_i\left(\eta C_{xy} - D_{xy[i]}\right)\right)$, i=1,2,3 Min MSE $(t_{bmi}) = \bar{Y}^2\left[\eta C_y^2 - D_{y[i]}^2 - \frac{(\eta C_{yx} - D_{yx[i]})^2}{(\eta C_x^2 - D_{x[i]}^2)}\right]$
Mandowara and Mehta (2016) $t_{mm7} = \bar{y}_{[n]}\left[\emptyset\left(\frac{C_x\bar{X}+\beta_2(x)}{C_x\bar{x}_{(n)}+\beta_2(x)}\right) + (1-\emptyset)\left(\frac{C_x\bar{x}_{(n)}+\beta_2(x)}{C_x\bar{X}+\beta_2(x)}\right)\right]$	MSE$(t_{mm7}) = \bar{Y}^2\left(\eta C_y^2(1-\rho_{yx}) - \{D_{y[i]} - \frac{(t-g)}{2t}D_{x[i]}\}^2\right)$ where $t = \frac{C_x\bar{X}}{C_x\bar{X}+\beta_2(x)}$ and $\emptyset = \frac{t+g}{2t}$.
Saini and Kumar (2016) $t_{ski} = \bar{y}_{[n]}\left(\frac{\bar{X}-\bar{x}_{(n)}+Q_i}{\bar{X}+\bar{x}_{(n)}+Q_i}\right), i = 1,3$ where Q_i is the ith quartile of variable X.	$MSE(t_{ski}) = \bar{Y}^2\left(\left(\eta C_y^2 - D_{y[i]}^2\right) + I_i^2\left(\eta C_x^2 - D_{x[i]}^2\right) - 2I_i\left(\eta C_{xy} - D_{xy[i]}\right)\right), i = 1,3$ $where\ I_i = \frac{\bar{X}+Q_i}{(2\bar{X}+Q_i)^2}$
Khan and Shabbir (2016) $t_{ks} = (k_1\bar{y}_{[n]} + k_2(\bar{X}-\bar{x}_{(n)}))\left[\alpha\exp\left(\frac{\bar{X}^*-\bar{x}_{(n)}^*}{\bar{X}^*+\bar{x}_{(n)}^*}\right) + (1-\alpha)\frac{\bar{X}^*}{\bar{x}_{(n)}^*}\right]$ where α is a constant and k_1 and k_2 are duly opted scalars.	$MSE(t_{ks}) = \bar{Y}^2(k_1-1)^2 + k_1^2A + k_2^2B + 2k_1(k_1-1)C + 2k_2(k_1-1)D + 2k_1k_2E$ $where\ A = \bar{Y}^2\left(V_{20} + \left(1-\frac{\alpha}{2}\right)^2\theta^2V_{02} - 2\left(1-\frac{\alpha}{2}\right)\theta V_{11}\right)$, $B = \bar{X}^2V_{02}$, $C = \bar{Y}^2\left(\left(1-\frac{5\alpha}{8}\right)^2\theta^2V_{02} - \left(1-\frac{\alpha}{2}\right)\theta V_{11}\right)$, $D = \bar{X}\bar{Y}\left(1-\frac{\alpha}{2}\right)\theta V_{02}$, $E = \bar{X}\bar{Y}\left[\left(1-\frac{\alpha}{2}\right)\theta V_{02} - V_{11}\right]$, $k_1 = \frac{B(\bar{Y}^2+C) - D(D+E)}{B(\bar{Y}^2+2C+A)-(D+E)^2}$, $k_2 = \frac{D(A+C) - E(\bar{Y}^2+C)}{B(\bar{Y}^2+2C+A)-(D+E)^2}$.
Jeelani *et al.* (2017) $t_{bj} = \zeta\frac{\bar{y}_{[n]}}{\bar{x}_{(n)}}\bar{X}$ where, $\zeta = \left(\frac{\bar{X}}{\bar{X}+1}\right)D_i$.	MSE $(t_{bj}) = \bar{Y}^2\left(\left(\eta C_y^2 - D_{y[i]}^2\right) + \zeta_i^2\left(\eta C_x^2 - D_{x[i]}^2\right) - 2\zeta_i\left(\eta C_{xy} - D_{xy[i]}\right)\right)$ i=1, 2… 9 Min MSE $(t_{bj}) = \bar{Y}^2\left[\eta C_y^2 - D_{y[i]}^2 - \frac{(\eta C_{yx} - D_{yx[i]})^2}{(\eta C_x^2 - D_{x[i]}^2)}\right]$

Khan and Ismail (2019) $t_k = \bar{y}_{[n]}\left(\frac{A\bar{X}+B}{A\bar{x}_{(n)}+B}\right)$ $t_{k1} = \bar{y}_{[n]}\left(\frac{\bar{X}+\beta_1(x)}{\bar{x}_{(n)}+\beta_1(x)}\right)$ $t_{k2} = \bar{y}_{[n]}\left(\frac{\beta_2(x)\bar{X}+\beta_1(x)}{\beta_2(x)\bar{x}_{(n)}+\beta_1(x)}\right)$ $t_{k3} = \bar{y}_{[n]}\left(\frac{C_x\bar{X}+\beta_1(x)}{C_x\bar{x}_{(n)}+\beta_1(x)}\right)$	$MSE(t_{ki}) = \bar{Y}^2\left(\left(\eta C_y^2 - D_{y[i]}^2\right) + G_i^2\left(\eta C_x^2 - D_{x[i]}^2\right) - 2G_i\left(\eta C_{xy} - D_{xy[i]}\right)\right)$ Min MSE (t_{ki})=$\bar{Y}^2\left[\eta C_y^2 - D_{y[i]}^2 - \frac{(\eta C_{yx}-D_{yx[i]})^2}{(\eta C_x^2 - D_{x[i]}^2)}\right]$
Bhushan and Kumar (2020a) $t_{b1} = \alpha_1\bar{y}_{[n]} + \beta_1(\bar{X} - \bar{x}_{(n)})$ $t_{b2} = \alpha_2\bar{y}_{[n]}\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right)^{\beta_2}$ $t_{b3} = \alpha_3\bar{y}_{[n]}\left(\frac{\bar{X}}{\bar{x}_{(n)}+\beta_3(\bar{X}-\bar{x}_{(n)})}\right)$	MSE (t_{bi}) = $\bar{Y}^2\left[1 - \frac{Q_i^2}{P_i}\right]$ i =1, 2, 3 $where\ P_1 = P_3 = 1 + V_{20} - \frac{V_{11}^2}{V_{02}}\ and\ P_2 = 1 + V_{20} + V_{11} - \frac{2V_{11}^2}{V_{02}}$ $Q_1 = Q_3 = 1\ and\ Q_2 = 1 + \frac{V_{11}}{2} - \frac{V_{11}^2}{2V_{02}}.$
Bhushan and Kumar (2020b) $t_{b4} = \bar{y}_{[n]}\left(1 + log\frac{\bar{X}}{\bar{x}_{(n)}}\right)^{\lambda_1}$ $t_{b5} = \bar{y}_{[n]}\left(1 + \lambda_2 log\frac{\bar{X}}{\bar{x}_{(n)}}\right)$ $t_{b6} = \left[\bar{y}_{[n]} + \lambda_3(\bar{X} - \bar{x}_{(n)})\right]\left(1 + log\frac{\bar{X}}{\bar{x}_{(n)}}\right)$	MSE (t_{bi}) = $\bar{Y}^2\left[\eta C_y^2 - D_{y[i]}^2 - \frac{(\eta C_{yx}-D_{yx[i]})^2}{(\eta C_x^2 - D_{x[i]}^2)}\right]$ i=4, 5, 6
Mehta *et al.* (2020) $t_m = \bar{y}_{[n]}\left[\delta\left(\frac{a\bar{X}+b}{a\bar{x}_{(n)}+b}\right)^p + (1-\delta)\left(\frac{a\bar{X}+b}{a\bar{x}_{(n)}+b}\right)\right]$	MSE (t_m) = $\bar{Y}^2\left[\eta C_y^2 - D_{y[i]}^2 - \frac{(\eta C_{yx}-D_{yx[i]})^2}{(\eta C_x^2 - D_{x[i]}^2)}\right]$
Bhushan and Kumar (2021) $t_{b7} = \alpha_4\bar{y}_{[n]}\left(1 + log\frac{\bar{X}}{\bar{x}_{(n)}}\right)^{\lambda_4}$ $t_{b8} = \alpha_5\bar{y}_{[n]}\left(1 + \lambda_5 log\frac{\bar{X}}{\bar{x}_{(n)}}\right)$	MSE (t_{bi})=$\bar{Y}^2\left[1 - \frac{Q_i^2}{P_i}\right]$, i=7, 8 $where\ P_7 = 1 + V_{20} + 2V_{11} - \frac{2V_{11}^2}{V_{02}}\ and\ P_8 = 1 + V_{20} - V_{11} + \frac{5V_{11}^2}{V_{02}}$ $Q_7 = 1 + V_{11} - \frac{V_{11}^2}{2V_{02}}\ and\ Q_8 = 1 - V_{11} - \frac{V_{11}^2}{V_{02}}$
Bhushan *et al.*(2022) $t_{b9} = \eta_1[\bar{y}_{[n]} + \beta_4(\bar{X} - \bar{x}_{(n)})]$ $+\psi_4\bar{y}_{[n]}\left(1 + log\frac{\bar{X}}{\bar{x}_{(n)}}\right)^{\xi_4}$ $t_{b10} = \eta_2\bar{y}_{[n]}\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right)^{\beta_5} + \psi_5\bar{y}_{[n]}\left(1 + log\frac{\bar{X}}{\bar{x}_{(n)}}\right)^{\xi_5}$	$\min MSE(t_{bi}) = \bar{Y}^2\left[1 - \frac{(A_iE_i^2 + B_iD_i^2 - 2C_iD_iE_i)}{A_iB_i - C_i^2}\right]$ $A_1 = 1 + V_{20} + \left(\frac{\beta_4}{R}\right)^2 V_{02} + 2\left(\frac{\beta_4}{R}\right)V_{11},$ $B_1 = 1 + V_{20} + (2\xi_4^2 - 2\xi_4)V_{02} + 4\xi_4V_{11},$ $C_1 = 1 + V_{20} + \left(\frac{\xi_4^2}{2} - \xi_4 + \frac{\beta_4\xi_4}{R}\right)V_{02} + \left(\frac{\beta_4}{R} + 2\xi_4\right)V_{11},$ $D_1 = 1,$ $E_1 = 1 + V_{20} + \left(\frac{\xi_4^2}{2} - \xi_4\right)V_{02} + \xi_4V_{11},$ $A_2 = 1 + V_{20} + (2\beta_5^2 + \beta_4)V_{02} - 4\beta_4V_{11},$ $B_2 = 1 + V_{20} + (2\xi_5^2 - 2\xi_5)V_{02} + 4\xi_5V_{11},$ $C_2 = 1 + V_{20} + \left(\frac{\xi_5^2}{2} - \xi_5 + \frac{\beta_4(\beta_4+1)}{R} - \beta_4\xi_5\right)V_{02} + (2\beta_4 - 2\xi_5)V_{11},$ $D_2 = 1 + \frac{\beta_4(\beta_4+1)}{R}V_{02} - \beta_4V_{11},$ $E_2 = 1 + \left(\frac{\xi_5^2}{2} - \xi_5\right)V_{02} + \xi_5V_{11}.$

4. PROPOSED CLASSES OF ESTIMATORS

Having studied the estimators in section 3, we proposed two classes of estimators for mean based on information on a single auxiliary variable.

$$t_p = \frac{w_1 \bar{y}_{[n]}}{2}\left(\frac{\bar{X}}{\bar{x}_{(n)}} + \frac{\bar{x}_{(n)}}{\bar{X}}\right) + w_2 exp\left(\frac{\bar{X}-\bar{x}_{(n)}}{\bar{X}+\bar{x}_{(n)}}\right)\left[\alpha\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right) + (1-\alpha)\left(1+log\frac{\bar{x}_{(n)}}{\bar{X}}\right)\right] \tag{4.1}$$

$$t_g = w_3\left[\frac{\bar{y}_{[n]}}{2}\left(\frac{\bar{X}}{\bar{x}_{(n)}} + \frac{\bar{x}_{(n)}}{\bar{X}}\right) + \beta(\bar{X}-\bar{x}_{(n)})\right] + w_4 exp\left(\frac{\bar{X}-\bar{x}_{(n)}}{\bar{X}+\bar{x}_{(n)}}\right)\left[\gamma\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right) + (1-\gamma)\left(1+log\frac{\bar{x}_{(n)}}{\bar{X}}\right)\right] \tag{4.2}$$

Where β (Population regression coefficient of y on x) is assumed to be known.

Expressing the estimator t_p given in equation (4.1) in terms of ϵ's we get

$$t_p = w_1\bar{Y}(1+\epsilon_0)\left(\frac{\bar{X}}{\bar{X}(1+\epsilon_1)} + \frac{\bar{X}(1+\epsilon_1)}{\bar{X}}\right) + w_2\, exp\left(\frac{-\epsilon_1}{2+\epsilon_1}\right)\left[\alpha\frac{\bar{X}}{\bar{X}(1+\epsilon_1)} + (1-\alpha)\left(1+log\frac{\bar{X}(1+\epsilon_1)}{\bar{X}}\right)\right] \tag{4.3}$$

$$t_p = w_1\bar{Y}\left(1+\epsilon_0+\frac{\epsilon_1^2}{2}\right) + w_2\left(1-\frac{\epsilon_1}{2}+\frac{3{\epsilon_1}^2}{8}\right)\left(1+\epsilon_1-2\alpha\epsilon_1-\frac{\epsilon_1^2}{2}+\frac{3\alpha\epsilon_1^2}{2}\right) \tag{4.4}$$

$$t_p - \bar{Y} = (w_1-1)\bar{Y} + w_1\bar{Y}(\epsilon_0+\frac{\epsilon_1^2}{2}) + w_2\left(1+\frac{\epsilon_1}{2}-2\alpha\epsilon_1-\frac{5{\epsilon_1}^2}{8}+\frac{5\alpha{\epsilon_1}^2}{2}\right) \tag{4.5}$$

$$Bias(t_p) = \bar{Y}(w_1-1) + w_1\frac{V_{02}}{2} + w_2\left[1-\frac{5V_{02}}{8}+\frac{5\alpha V_{02}}{2}\right] \tag{4.6}$$

Squaring both sides and taking expectations of equation (4.5), we get

$MSE(t_p) = \bar{Y}^2 + \bar{Y}^2{w_1}^2$ (1 +

$$MSE(t_p) = \bar{Y}^2 + \bar{Y}^2{w_1}^2\,(1+V_{20}+V_{02}) + {w_2}^2(1-V_{02}+4\alpha^2V_{02}+3\alpha V_{02}) - 2w_1\bar{Y}^2\left(1+\frac{V_{02}}{2}\right) - 2w_2\bar{Y}\left(1-\frac{5V_{02}}{8}+\frac{5\alpha V_{02}}{2}\right) + 2w_1w_2\bar{Y}(1-\frac{V_{02}}{8}+\frac{5\alpha V_{02}}{2}+\frac{V_{11}}{2}-2\alpha V_{11}) \tag{4.7}$$

$$MSE(t_p) = A_1 + {w_1}^2B_1 + {w_2}^2C_1 - 2w_1D_1 - 2w_2E_1 + 2w_1w_2F_1 \tag{4.8}$$

where $A_1=\bar{Y}^2$,

$B_1=\bar{Y}^2(1+V_{20}+V_{02})$,

$C_1 = 1-V_{02}+4\alpha^2V_{02}+3\alpha V_{02}$,

$D_1=\bar{Y}^2(1+\frac{V_{02}}{2})$,

$E_1=\bar{Y}\left(1-\frac{5V_{02}}{8}+\frac{5\alpha V_{02}}{2}\right)$,

$F_1=\bar{Y}\left(1-\frac{V_{02}}{8}+\frac{5\alpha V_{02}}{2}+\frac{V_{11}}{2}-2\alpha V_{11}\right)$.

To find out the minimum MSE for the estimator t_p, we partially differentiate equation (4.8) with respect to w_1 & and w_2 equating to zero we get

$$w_1{}^* = \frac{C_1D_1-E_1F_1}{F_1^2-B_1C_1} \tag{4.9}$$

$$w_2{}^* = \frac{D_1F_1-B_1C_1}{F_1^2-B_1C_1} \tag{4.10}$$

Putting the optimum values of w_1 & w_2 in the equation (4.8) we get a minimum MSE of the estimator t_p as

$$\min MSE = \left[A_1 + \frac{C_1D_1^2+B_1E_1^2-2D_1E_1F_1}{F_1^2-B_1C_1}\right] \tag{4.11}$$

Expressing the estimator t_p given in equation (4.2) in terms of ϵ's we get

$$t_g = w_3\bar{Y}(1+\epsilon_0)\left[\left(\frac{\bar{X}}{\bar{X}(1+\epsilon_1)}+\frac{\bar{X}(1+\epsilon_1)}{\bar{X}}\right) - \beta\bar{X}\epsilon_1\right] + w_4\, exp\left(\frac{-\epsilon_1}{2+\epsilon_1}\right)\left[\gamma\frac{\bar{X}}{\bar{X}(1+\epsilon_1)} + (1-\gamma)\left(1+log\frac{\bar{X}(1+\epsilon_1)}{\bar{X}}\right)\right] \tag{4.12}$$

$$t_g = w_3\bar{Y}(1+\epsilon_0)\left(1+\frac{\epsilon_1^2}{2}-\beta\bar{X}\epsilon_1\right)+ w_4\left(1-\frac{\epsilon_1}{2}+\frac{3\epsilon_1{}^2}{8}\right)\left(1+\epsilon_1-2\gamma\epsilon_1-\frac{\epsilon_1^2}{2}+\frac{3\gamma\epsilon_1^2}{2}\right) \tag{4.13}$$

$$t_g-\bar{Y} = (w_3-1)\bar{Y}+w_3\bar{Y}(\epsilon_0+\frac{\epsilon_1^2}{2}-\beta\delta\epsilon_1)+ w_4\left(1+\frac{\epsilon_1}{2}-2\gamma\epsilon_1-\frac{5\epsilon_1{}^2}{8}+\frac{5\gamma\epsilon_1{}^2}{2}\right) \tag{4.14}$$

$$Bias(t_g) = \bar{Y}(w_1-1)+w_3\frac{V_{02}}{2}+ w_4\left[1-\frac{5V_{02}}{8}+\frac{5\gamma V_{02}}{2}\right] \tag{4.15}$$

Squaring on both sides and taking expectations of equation (4.14),we get

$$MSE(t_g) = \bar{Y}^2+\bar{Y}^2w_3{}^2\ \ (1+V_{20}+V_{02}+ \delta^2\beta^2V_{02}-2\delta\beta V_{11})+w_4{}^2(1-V_{02}+ 4\gamma^2V_{02}+3\gamma V_{02})-2w_3\bar{Y}^2\left(1+\frac{V_{02}}{2}\right)- 2w_4\bar{Y}\left(1-\frac{5V_{02}}{8}+\frac{5\gamma V_{02}}{2}\right)+2w_3w_4\bar{Y}(1- \frac{V_{02}}{8}+\frac{5\gamma V_{02}}{2}+\frac{V_{11}}{2}-2\gamma V_{11}-\frac{\delta\beta V_{02}}{2}+2\gamma\delta\beta V_{02}) \tag{4.16}$$

$$MSE(t_g) = A_2+w_3{}^2B_2+w_4{}^2C_2-2w_3D_2-2w_4E_2+2w_3w_4F_2 \tag{4.17}$$

where $A_2=\bar{Y}^2, \delta=\dfrac{\bar{X}}{\bar{Y}}$,

$B_2=\bar{Y}^2\left(1+V_{20}+V_{02}+\delta^2\beta^2V_{02}-2\delta\beta V_{11}\right)$,

$C_2=1-V_{02}+4\alpha^2V_{02}+3\gamma V_{02}$,

$D_2=\bar{Y}^2\left(1+\dfrac{V_{02}}{2}\right)$,

$E_2=\bar{Y}\left(1-\dfrac{5V_{02}}{8}+\dfrac{5\gamma V_{02}}{2}\right)$,

$F_2=\bar{Y}\left(1-\dfrac{V_{02}}{8}+\dfrac{5\gamma V_{02}}{2}+\dfrac{V_{11}}{2}-2\gamma V_{11}-\dfrac{\delta\beta V_{02}}{2}+2\gamma\delta\beta V_{02}\right)$.

To find out the minimum MSE for the estimator t_g, we partially differentiate equation (4.17) wrt w_3 & w_4 and equating to zero we get

$$w_3{}^* = \frac{C_2D_2-E_2F_2}{F_2^2-B_2C_2} \tag{4.18}$$

$$w_4{}^* = \frac{D_2F_2-B_2C_2}{F_2^2-B_2C_2} \tag{4.19}$$

Putting the optimum values of w_3 & w_4 in the equation (4.17) we get a minimum MSE of the estimator t_g as

$$\min MSE = \left[A_2+\frac{C_2D_2^2+B_2E_2^2-2D_2E_2F_2}{F_2^2-B_2C_2}\right] \tag{4.20}$$

Proposed class of estimators t_p reduce into following estimators for suitably chosen values of α given below

$$t_{p1} = \frac{w_1\bar{y}_{[n]}}{2}\left(\frac{\bar{X}}{\bar{x}_{[n]}}+\frac{\bar{x}_{[n]}}{\bar{X}}\right)+w_2exp\left(\frac{\bar{X}-\bar{x}_{[n]}}{\bar{X}+\bar{x}_{[n]}}\right)\left(\frac{\bar{X}}{\bar{x}_{[n]}}\right) \tag{4.21}$$

$$t_{p2} = \frac{w_1\bar{y}_{[n]}}{2}\left(\frac{\bar{X}}{\bar{x}_{[n]}}+\frac{\bar{x}_{[n]}}{\bar{X}}\right)+w_2exp\left(\frac{\bar{X}-\bar{x}_{[n]}}{\bar{X}+\bar{x}_{[n]}}\right)\left(1+log\frac{\bar{x}_{[n]}}{\bar{X}}\right) \tag{4.22}$$

Proposed class of estimators t_g reduce into some estimators for suitably chosen values of γ given below

$$t_{g1} = w_3\left[\frac{\bar{y}_{[n]}}{2}\left(\frac{\bar{X}}{\bar{x}_{[n]}}+\frac{\bar{x}_{[n]}}{\bar{X}}\right)+\beta\left(\bar{X}-\bar{x}_{[n]}\right)\right]+ w_4exp\left(\frac{\bar{X}-\bar{x}_{[n]}}{\bar{X}+\bar{x}_{[n]}}\right)\left(\frac{\bar{X}}{\bar{x}_{[n]}}\right) \tag{4.23}$$

$$t_{g2} = w_3\left[\frac{\bar{y}_{[n]}}{2}\left(\frac{\bar{X}}{\bar{x}_{[n]}}+\frac{\bar{x}_{[n]}}{\bar{X}}\right)+\beta\left(\bar{X}-\bar{x}_{[n]}\right)\right]+ w_4exp\left(\frac{\bar{X}-\bar{x}_{[n]}}{\bar{X}+\bar{x}_{[n]}}\right)\left(1+log\frac{\bar{x}_{[n]}}{\bar{X}}\right) \tag{4.24}$$

5. EMPIRICAL STUDY

In this section, we compare the performance of the proposed estimators with the other estimators considered in this paper by considering real population as follows:

Population: Source: Sarjinder Singh (2003)

Y: Real estate farms loans,

X: Non real estate farms loans,

From the above population, we took ranked set samples with size k=3 and number of cycles r=4, 5,6,10. For these samples, we have calculated MSE and PRE for different estimators.

Table 1. The Mean Square Errors (MSE) and Percentage Relative Efficiencies(PRE) of the Estimators

Estimators	K=3, r=4, n=rk=12	K=3, r=5, n=rk=15	K=3, r=6, n=rk=18	K=3, r=10, N=rk=30
t_{RSS}	11552.3791 (100.00)	19752.407 (100.00)	11065.05554 (100.00)	5748.64008 (100.00)
$t_{r,RSS}$	7301.3221 (158.2231)	7170.007 (275.48)	6329.035581 (174.83)	4647.76857 (123.68)
$t_{reg,RSS}$	5992.717 (192.7736)	5631.803 (350.72)	6323.455597 (174.98)	3157.62779 (182.05)
$t_{kc,RSS}$	7259.53 (159.1339)	7167.836 (275.57)	6205.87 (178.29)	4641.274 (123.85)
t_{a1}	6817.129 (169.4610)	6448.028 (306.3325)	6329.395 (174.8201)	4218.503 (136.2720)
t_{a2}	6735.218 (171.5219)	8263.605 (239.0289)	7800.237 (141.8553)	3317.904 (173.2611)
t_{sa}	5992.717 (192.7736)	5631.803 (350.72)	6323.455597 (174.98)	3157.62779 (182.05)
t_{mm1}	12414.7 (93.0540)	11703.5 (168.77)	11171.5 (99.04)	10293.4 (55.84)
t_{mm2}	12400.0 (93.1643)	11671.2 (169.24)	11164.6 (99.10)	10286.5 (55.88)
t_{mm3}	12476.7 (92.5916)	11780.9 (167.66)	11226.8 (98.55)	10348.6 (55.54)
t_{mm4}	12408.1 (93.1035)	11688.1 (168.99)	11168.8 (99.07)	10290.7 (55.86)
t_{mm5}	10075.5 (114.6581)	9056.6 (218.09)	7036.4 (157.25)	6141.9 (93.59)
t_{mm6}	10075.5 (114.6581)	9056.6 (218.09)	7036.4 (157.25)	6141.9 (93.59)
t_s	5751.964 (200.8423)	5465.116 (361.4270)	6204.521 (178.3385)	3057.791 (187.9997)
t_j	5992.717 (192.7736)	5631.803 (350.72)	6323.455597 (174.98)	3157.62779 (182.05)
t_{bm}	5992.717 (192.7736)	5631.803 (350.72)	6323.455597 (174.98)	3157.62779 (182.05)
t_{mm7}	10075.45 (114.6586)	9022.914 (218.91)	7035.19 (157.28)	6139.934 (93.62)
t_{ks}	5762.58 (200.47)	5439.48 (363.1304)	6161.571 (179.5817)	3074.605 (186.9716)
t_{sk1}	11547.68 (100.0406)	19741.73 (100.0540)	11062.27 (100.0251)	5746.05 (100.0450)
t_{sk2}	11548.44 (100.0341)	19743.44 (100.0454)	11062.71 (100.0212)	5746.464 (100.0378)
t_{bj}	5992.717 (192.7736)	5631.803 (350.72)	6323.455597 (174.98)	3157.62779 (182.05)
t_k	5992.717 (192.7736)	5631.803 (350.72)	6323.455597 (174.98)	3157.62779 (182.05)
t_m	5992.717 (192.7736)	5631.803 (350.72)	6323.455597 (174.98)	3157.62779 (182.05)
t_{b1}	5851.507 (197.4257)	5556.58 (355.4777)	6191.674 (178.7086)	3111.565 (184.75)
t_{b2}	5782.112 (199.7951)	5645.19 (349.8980)	6188.335 (178.8050)	3079.154 (186.6954)
t_{b3}	5851.507 (197.4257)	5556.58 (355.4777)	6191.674 (178.7086)	3111.565 (184.7507)
t_{b4}	5992.717 (192.7736)	5631.803 (350.72)	6323.455597 (174.98)	3157.62779 (182.05)
t_{b5}	5992.717 (192.7736)	5631.803 (350.72)	6323.455597 (174.98)	3157.62779 (182.05)
t_{b6}	5992.717 (192.7736)	5631.803 (350.72)	6323.455597 (174.98)	3157.62779 (182.05)
t_{b7}	5496.791 (210.1658)	4955.27 (398.6141)	6069.172 (182.3157)	2972.893 (193.3685)
t_{b8}	5401.93 (213.8565)	4975.83 (396.9670)	6435.81 (171.9294)	2936.17 (195.7870)
t_{b9}	5494.257 (210.2628)	4951.552 (398.9134)	6068.893 (182.3241)	2972.039 (193.4241)
t_{b10}	5992.717 (192.7736)	4955.27 (398.6141)	6068.975 (182.3216)	2972.675 (193.3827)
t_{p1}	**1754.144 (658.5764)**	**1767.1517 (1117.7538)**	**1047.9041 (1055.9225)**	**1145.029 (502.0519)**
t_{p2}	2639.148 (437.7313)	3492.449 (565.5746)	1737.42 (636.8670)	1897.889 (302.8965)
t_{g1}	4171.549 (276.9325)	4126.412 (478.6823)	3801.043 (291.1057)	2373.397 (242.2114)
t_{g2}	**2023.974 (570.7770)**	**2948.397 (669.9371)**	**1051.637 (1052.1744)**	**1225.071 (469.2495)**

The formula for Percent Relative Efficiency (PRE) is

$$\text{PRE (estimators)} = \frac{MSE(t_{RSS})}{MSE(estimator)} \times 100$$

From Table 1, we can conclude that the proposed estimators performs better than existing estimators.

6. CONCLUSION

In this article estimators of population mean under ranked set sampling are investigated. Bias and Mean square error equations are derived. We have proposed estimators for the population mean in Ranked set sampling using the information of auxiliary variables. The expressions for Bias and MSE of the suggested estimators have been derived up to the first order of approximation. Empirical study for comparing the efficiency of the proposed estimators with other estimators have been used.

The results have been shown in the Tables 1. The Table shows that the proposed estimators turn out to be more efficient as compared to the other estimators. The proposed estimators are found to be rather improved in terms of lesser MSE and greater PRE as compared to the existing estimators. It is also observed from the empirical study that the MSE of the proposed estimators decreases as the values of the sample size increase whereas the PRE of the suggested estimators increases as the values of the sample size increase. The estimator t_{p1} is found to be most efficient among the suggested estimators.

Based on our empirical study, we can conclude that our proposed estimators can be preferred over the other estimators taken in this paper in several real situations like agriculture sciences, mathematical sciences, biological sciences, poultry, business, economics, commerce, social sciences etc.

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