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Favourable Allocation Models for Symmetric Distributions in Ranked Set Sampling

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SUMMARY

Kaur *et al.* (2000) suggested an optimal allocation model for symmetric distribution in ranked set sampling (RSS). This model is not much applicable due to its dependency upon extreme or mid order statistic only. In this paper, an attempt to make a "favourable" and near optimal allocation has been made by allocating each rank order at least once and considering the opposite behavior of Neyman allocation for symmetric distribution. The case of perfect ranking is considered. The utility of the proposed model in terms of relative precision has been shown for some symmetric distributions. Favourable model outperforms both for equal and Neyman's allocations and quite close to the optimal model for each set size. The model will provide a practical approach in situations where RSS is likely to lead to an improvement over simple random sampling and the underlying distribution is symmetric.

Keywords: KPT model; Neyman's allocation; Order statistic; Ranked set sample; Relative precision; Symmetric distribution.

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1. INTRODUCTION

Considerable work has been done in various aspects of ranked set sampling (RSS) since its inception from McIntyre (1952) due to its importance in term of relative precision (RP) over SRS as well as avoiding actual measurements to each selected units. One of the important aspects is the allocation models for set sizes. Unequal allocation model is preferred over equal allocation while using RSS. Some useful allocation models are available for the case of skewed distribution in which standard deviations of order statistic increases with the increase of rank of order statistics. For example, well known Neyman's optimum allocation model of stratified random sampling is there in which the sample units are allocated into ranks in proportion to the standard deviation of each rank. Kaur *et al.* (1994, 1997) proposed '*t*' and (*s, t*)' models which were considered as 'Near' optimal allocation models and compared them with Neyman's optimal allocation and equal allocation models. Some limitation was found for these models for actual application; therefore, Tiwari and Chandra (2011) suggested the systematic allocation model for skewed distributions in RSS. Recently, Bhoj and Chandra (2019) proposed a simple allocation models which is mainly recommended for the highly skewed distributions. In comparison, limited allocation models are available for symmetric distributions.

In symmetric distributions, the optimal allocation strategy is precisely the opposite of the Neyman's strategy (Kaur *et al.*, 2000). This means, for symmetric distributions, one should measure more units corresponding to those rank orders having the smallest variances. For the symmetric distributions, Kaur, Patil and Taillie (2000) suggested an optimal allocation model (for convenience, we call it as 'KPT' model) and compared with equal and Neyman's method in terms of the precision of the estimator of population mean. Yanagawa and Chen (1980) suggested a minimum variance linear unbiased median-mean estimator of population mean for a family of symmetric distribution.

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Shirahata (1982) examined more general procedures that are unbiased for symmetric distributions. The method of selecting ranked set sample may be seen in literature (say, Tiwari and Chandra, 2011).

For RSS with equal allocation or balanced RSS, let $Y_{(i:k)j}$, $i = 1,2,....k; j = 1,2,....m$, denote the measured unit for the i^{th} rank order in the j^{th} cycle. For fixed i, $Y_{(i:k)j}$, $j = 1,2,....m$, are *i.i.d.* with mean $\mu_{(i:k)}$ and variance. $\sigma^2_{(i:k)}$.. Under RSS with equal allocation, an unbiased estimator of population mean μ is the ranked set sample mean-

$$\overline{Y}_{(k)eql} = \frac{1}{km}\sum_{i=1}^{k}\sum_{j=1}^{m} Y_{(i:k)j} \tag{1.1}$$

The variance of $\overline{Y}_{(k)eql}$ is

$$Var(\overline{Y}_{(k)eql}) = \frac{1}{k^2 m}\sum_{i=1}^{k}\sigma^2_{(i:k)} \tag{1.2}$$

In RSS with unequal allocation or unbalanced RSS, suppose $m_i \geq 1$ units are measured corresponding to the i^{th} rank, $i = 1,2,....k$ and suppose $T_i = \sum_{j=1}^{m_i} Y_{(i:k)j}$. Then, under RSS with unequal allocation, an unbiased estimator of μ and its variance are given by

$$\overline{Y}_{(k)ueql} = \frac{1}{k}\sum_{i=1}^{k}\frac{T_i}{m_i} \tag{1.3}$$

and

$$Var(\overline{Y}_{(k)ueql}) = \frac{1}{k^2}\sum_{i=1}^{k}\frac{\sigma^2_{(i:k)}}{m_i} \tag{1.4}$$

Balanced RSS is more precise method than SRS (Dell and Clutter, 1968) and unbalanced RSS is always gives better performance than balanced RSS when an appropriate unequal allocation is made (Bhoj and Chandra, 2019). In Neyman's allocation approach with sample size $n = \sum_{i=1}^{k} m_i$, we have

$$m_i = \frac{n\sigma_{(i:k)}}{\sum_{i=1}^{k}\sigma_{(i:k)}} \tag{1.5}$$

The Section 2 describes the optimal allocation (KPT) model of Kaur *et al.* (2000). Section 3 provides the proposed favourable and simple allocation model for both classes of symmetric distributions to overcome the drawbacks in KPT model. This model is opposite to the Neyman's allocation model and has an advantage over KPT model in the sense that measurements are made upon each rank orders. Section 4 contains some examples from the two classes of symmetric distributions to demonstrate the utility of the proposed model.

2. KPT ALLOCATION MODELS FOR SYMMETRIC DISTRIBUTIONS

For the case of symmetric distributions, the Neyman's allocation provides marginal RP which remains very close to that of equal allocation. To overcome this difficulty, an allocation model for symmetric distribution was proposed by Kaur *et al.* (2000). This model measures either only mid or extreme rank orders. This model is however optimal in terms of RP but not sufficient. It seems to be unreliable in practice as it is based upon at most two (when *k* is even) rank orders only.

In skewed case, the standard deviations (SD) of order statistics are either increases or decreases if the distribution is positively or negatively skewed distributions respectively. However, for the case of symmetric distributions, the SD of first and largest, second and second largest order statistics and so on are same. This pattern needs opposite to the skewed distributions while minimizing the variance of mean or other such parameter under interest. Therefore, the strategy for optimal allocation in symmetric distributions is quite opposite of the Neyman's allocation strategy (Kaur *et al.*, 2000).

There are two kinds of symmetric distributions, namely, mound and U-shaped distributions. In mound shaped distribution, $\sigma^2_{(i:k)}$ is increasing in *i* for $1 \leq i \leq M$ and $\sigma^2_{(i:k)}$ is decreasing in *i* for $M \leq i \leq k$, where $M = \frac{k+1}{2}$ is the unique middle rank order when *k* is odd. In U-shaped distribution, $\sigma^2_{(i:k)}$ is decreasing in *i* for $1 \leq i \leq M$ and $\sigma^2_{(i:k)}$ is increasing in *i* for $M \leq i \leq k$.

Kaur *et al.* (2000) derived the expressions of asymptotic RP for both kinds of symmetric distributions. They ignored the rank orders with large variances and measured only the rank order having the smallest variances for mound distributions. It is the optimal allocation model for finding the optimal variance of the best linear unbiased estimator (BLUE) of μ. Their optimal variance of the estimator for large *n* is

$$\sigma^2_m(KPT) = \frac{\sigma^2_{(1:k)}}{n}. \tag{1.6}$$

After comparing the asymptotic variance in (1.6) with the variance under SRS, the asymptotic RP is

$$RP_m(KPT) = \frac{\sigma^2}{\sigma^2_{(1:k)}} \quad (1.7)$$

While in the case of U-shaped symmetric distributions they measured only the rank order having the largest variances and derived the asymptotic variance of the BLUE of μ as

$$\sigma_u^2(KPT) = \begin{cases} \dfrac{\sigma^2_{\left(\frac{k}{2}:k\right)}}{n}, \text{ if } k \text{ is even} \\ \dfrac{\sigma^2_{\left(\frac{k+1}{2}:k\right)}}{n}, \text{ if } k \text{ is odd} \end{cases} \cdot \quad (1.8)$$

The asymptotic RP compared with SRS is

$$RP_u(KPT) = \begin{cases} \dfrac{\sigma^2}{\sigma^2_{\left(\frac{k}{2}:k\right)}}, \text{ if } k \text{ is even} \\ \dfrac{\sigma^2}{\sigma^2_{\left(\frac{k+1}{2}:k\right)}}, \text{if } k \text{ is odd} \end{cases} . \quad (1.9)$$

In addition, with ignoring most of the rank order, other important limitation is that the allocations factor(s) are found fractional and hence requires several adjustments to make them integers.

3. FAVOURABLE ALLOCATION MODELS FOR SYMMETRIC DISTRIBUTIONS

Bhoj and Chandra (2019) proposed an allocation model for skewed distribution having heavy right tail under RSS. In the present paper (i) the use of the fact that for symmetric distributions, the optimal allocation strategy is precisely the opposite of the Neyman strategy has been used, and (ii) all rank order statistics while proposing the unbiased estimator of the population mean has been considered. Therefore, opposite model of Bhoj and Chandra (2019) has been attempted for symmetric distributions. The significance of this model is found in Bhoj and Chandra (2019). In this paper, we propose "favourable" allocation models for symmetric distributions. Here the word "favourable" is considered in the sense that the proposed allocation model is near to optimal allocation model proposed by Kaur *et al.*, 2000 for the case of symmetric distributions and helpful for attempting the case of data which follows the symmetric distributions. For example, for the survey like selecting number of trees having different age groups from any natural forest consisting of large number of trees, the height distribution is likely to follow the symmetric distribution. The selection of trees through this proposed fabourable allocation model is preferable as each order statistic is selected in the sample and the precision of this model is close to the optimal allocation model of Kaur *et al.*, 2000.

As discussed earlier, the criterion used to arrive at favourable allocation is based on the opposite of Neyman's allocation and each order statistic is to be selected. The main drawback of optimal allocation model is that either this depends on extreme or mid order statistics while other order statistics were ignored. This drawback has been overcome in the proposed model by considering all order statistics for estimation of mean. The proposed allocation model is simple and accurate for unbalanced RSS for two kinds of symmetric distributions. This model may be considered as a near optimal allocation model for selecting the sample for the purpose of estimating population mean of symmetric distributions.

For symmetric distributions, one knows that

$$\mu = \begin{cases} \frac{1}{2}\left(\mu_{(i:k)} + \mu_{(k-i+1:k)}\right), \text{for } 1 \le i < M \\ \mu_{(i:k)}, \text{for } i = M \ \& \ k \text{ is odd} \end{cases}$$

and

Here, following theorem for finding the proposed BLUE of μ is useful.

Theorem 3.1. Let $X_1, X_2, ..., X_n$ are n independent random variables with a common mean μ and with variances $\sigma_1^2, \sigma_2^2, ..., \sigma_n^2$. The linear combination, with $A_1 + A_2 + ... + A_n = 1$, that has the smallest variance and is obtained by taking A_i inversely proportional to σ_i^2. The resulting minimum variance is

$$\frac{1}{\dfrac{1}{\sigma_1^2} + \dfrac{1}{\sigma_2^2} + ... + \dfrac{1}{\sigma_n^2}} \quad (3.1)$$

The proposed favourable allocation models for both types of symmetric distributions under unbalanced RSS are given in the following subsections.

3.1 *Mound Shaped Distribution*

For mound shaped distribution, the proposed allocation model is

$$m_i = m_{k-i+1} = \begin{cases} k^2+1-\dfrac{k(k-1)}{2}, \text{for } i=1, \text{for every } k \\ \dfrac{k+2}{2}-i, \quad \text{for } 2 \le i < M \ \& \ k \text{ is even} \\ \dfrac{k+3}{2}-i, \quad \text{for } 2 \le i \le M \ \& \ k \text{ is odd} \end{cases} \quad (3.2)$$

Corresponding sample size (n) is

$$n = \begin{cases} 2\left(\left(k^2+1-\dfrac{k(k-1)}{2}\right)+\dfrac{k-2}{2}+\dfrac{k-4}{2}+...+2+1\right), \text{ if } k \text{ is even} \\ 2\left(\left(k^2+1-\dfrac{k(k-1)}{2}\right)+\dfrac{k-1}{2}+\dfrac{k-3}{2}+...+2\right)+1, \text{ if } k \text{ is odd} \end{cases}$$

or

$$n = \begin{cases} \dfrac{5k^2+2k+8}{4}, \text{ if } k \text{ is even} \\ \dfrac{5k^2+4k+3}{4}, \text{ if } k \text{ is odd} \end{cases} \quad (3.3)$$

Denoting the measured units by $Y_{(i:k)j}$, $i=1,2,....k; j=1,2,....m_i$, an unbiased estimator of μ based on i^{th} and $(k-i+1)^{th}$ order statistics under allocation model (3.2) is

$$\overline{Y}_{(i:k-i+1)m} = \begin{cases} \dfrac{1}{2}\left(\dfrac{T_1+T_k}{k^2+1-\dfrac{k(k-1)}{2}}\right), \text{for } i=1 \\ \dfrac{1}{2}\left(\dfrac{T_i+T_{k-i+1}}{\dfrac{k+2}{2}-i}\right), \text{for } 2 \le i < M \ \& \ k \text{ is even} \\ \dfrac{1}{2}\left(\dfrac{T_i+T_{(k-i+1)}}{\dfrac{k+3}{2}-i}\right), \text{for } 2 \le i < M \ \& \ k \text{ is odd} \\ T_M, \text{ for } i=M \ \& \ k \text{ is odd} \end{cases} \quad (3.4)$$

where $T_i = \sum_{j=1}^{m_i} Y_{(i:k)j}$ and subscript 'm' denotes for mound shaped distribution.

The variance of $\overline{Y}_{(i:k-i+1)m}$ is

$$Var\left(\overline{Y}_{(i:k-i+1)m}\right) = \begin{cases} \dfrac{\sigma^2_{(1:k)}}{2\left(k^2+1-\dfrac{k(k-1)}{2}\right)}, \text{for } i=1 \\ \dfrac{\sigma^2_{(i:k)}}{2\left(\dfrac{k+2}{2}-i\right)}, \text{for } 2 \le i < M \ \& \ k \text{ is even} \\ \dfrac{\sigma^2_{(i:k)}}{2\left(\dfrac{k+3}{2}-i\right)}, \text{for } 2 \le i < M \ \& \ k \text{ is odd} \\ \sigma^2_{(M:k)}, \text{for } i=M \ \& \ k \text{ is odd} \end{cases}$$

(3.5)

Here, the BLUE of $\overline{Y}_{(i:k-i+1)m}$ $(i=1,2,...,M)$ for μ can be written as

$$a^{/}\hat{\mu}_m = \sum_{i=1}^{M} a_i \overline{Y}_{(i:k-i+1)m} \text{ with } \sum_{i=1}^{M} a_i = 1 \quad (3.6)$$

where $a^{/} = (a_1, a_2, ..., a_M)$ and $\hat{\mu}^{/}_m = \left(\overline{Y}_{(1:k)m}, \overline{Y}_{(2:k-1)m}, ..., \overline{Y}_{(M:M)m}\right)$. All $\overline{Y}_{(i:k-i+1)m}$ $(i=1, 2, ..., M)$ are the independent random variables with common mean μ and different variances given in Eq. (3.5). Using (3.1) in (3.6) with (3.5), we get

$$Var(a^{/}\hat{\mu}_m) = \begin{cases} \dfrac{1}{\dfrac{k^2+k+2}{\sigma^2_{(1:k)}}+\sum_{i=2}^{k/2}\left(\dfrac{k+2-2i}{\sigma^2_{(i:k)}}\right)}, \text{if } k \text{ is even} \\ \dfrac{1}{\dfrac{k^2+k+2}{\sigma^2_{(1:k)}}+\sum_{i=2}^{(k-1)/2}\left(\dfrac{k+3-2i}{\sigma^2_{(i:k)}}\right)+\dfrac{1}{\sigma^2_{(M:k)}}}, \text{if } k \text{ is odd} \end{cases}$$

(3.7)

The RP of $a^{/}\hat{\mu}_m$ compare with SRS with the same sample size n as given in Eq. (3.3) is

$$RP_m(fav) = \begin{cases} \dfrac{4\sigma^2\left[\dfrac{k^2+k+2}{\sigma^2_{(1:k)}}+\sum_{i=2}^{k/2}\left(\dfrac{k+2-2i}{\sigma^2_{(i:k)}}\right)\right]}{5k^2+2k+8}, \text{if } k \text{ is even} \\ \dfrac{4\sigma^2\left(\dfrac{k^2+k+2}{\sigma^2_{(1:k)}}+\sum_{i=2}^{(k-1)/2}\left(\dfrac{k+3-2i}{\sigma^2_{(i:k)}}\right)+\dfrac{1}{\sigma^2_{(M:k)}}\right)}{5k^2+4k+3}, \text{ if } k \text{ is odd} \end{cases}$$

(3.8)

U-shaped Distribution

The proposed favourable allocation model for the U-shaped distribution is

$$m_i = m_{k-i+1} = \begin{cases} i, \quad \text{for } 1 \le i < M \ \& \text{ for every } k \text{ except at } i=k/2 \text{ with even } k \\ k^2+1-\dfrac{k(k-1)}{2}, \quad \text{for } i=k/2 \ \& \ k \text{ is even or } i=M \ \& \ k \text{ is odd} \end{cases}$$

(3.9)

The resulting sample size is

$$n=\begin{cases}2\left(1+2+...upto\,\frac{k-2}{2}\,terms\right)+2\left(k^2+1-\frac{k(k-1)}{2}\right), \text{if } k \text{ is even}\\ 2\left(1+2+....\frac{k-1}{2}\,terms\right)+\left(k^2+1-\frac{k(k-1)}{2}\right), if\ k \text{ is odd}\end{cases}$$

Or

$$n=\begin{cases}\frac{5k^2+2k+8}{4}, if\ k \text{ is even}\\ \frac{3k^2+2k+3}{4}, \text{if } k \text{ is odd}\end{cases}. \tag{3.10}$$

For U-shaped symmetric distribution with allocation model (3.9), an unbiased estimator of μ based on i^{th} and $(k-i+1)^{th}$ order statistics is given by

$$\overline{Y}_{(i:k-i+1)u}=\begin{cases}\frac{1}{2}\left(\frac{T_i+T_{k-i+1}}{i}\right), \text{for } 1\le i<M \text{ \& for every } k \text{ except at } i=k/2 \text{ with even } k\\ \frac{1}{2}\left(\frac{T_i+T_{k-i+1}}{k^2+1-\frac{k(k-1)}{2}}\right), \text{for } i=k/2 \text{ \& } k \text{ is even}\\ \frac{T_M}{k^2+1-\frac{k(k-1)}{2}}, \text{for } i=M \text{ \& } k \text{ is odd}\end{cases} \tag{3.11}$$

Where, the subscript 'u' denotes for the U-shaped distribution.

The variance of $\overline{Y}_{(i:k-i+1)u}$ is

$$Var\left(\overline{Y}_{(i:k-i+1)u}\right)=\begin{cases}\frac{\sigma^2_{(i:k)}}{2i}, \text{for } 1\le i<M \text{ \& for any } k \text{ except at } i=k/2 \text{ with even } k\\ \frac{\sigma^2_{(\frac{k}{2}:k)}}{k^2+k+2}, \text{for } i=k/2 \text{ \& } k \text{ is even}\\ \frac{\sigma^2_{(M:k)}}{k^2+1-\frac{k(k-1)}{2}}, \text{for } i=M \text{ \& } k \text{ is odd}\end{cases}. \tag{3.12}$$

The BLUE of $\overline{Y}_{(i:k-i+1)u}\ (i=1,2,...,M)$ for μ can be written as:

$$b^{/}\hat{\mu}_u=\sum_{i=1}^{M}b_i\overline{Y}_{(i:k-i+1)u}, \text{ with } \sum_{i=1}^{M}b_i=1 \tag{3.13}$$

where $b^{/}=(b_1,b_2,...,b_M)$ and $\hat{\mu}_u^{/}=\left(\overline{Y}_{(1:k)u},\overline{Y}_{(2:k-1)u},....,\overline{Y}_{(M:M)u}\right)$. Using Eq. (3.1) in Eq. (3.13) with Eq. (3.12), the variance of $b^{/}\hat{\mu}_u$ is

$$Var\left(b^{/}\hat{\mu}_u\right)=\begin{cases}\frac{1}{\sum_{i=1}^{\frac{k-2}{2}}\left(\frac{2i}{\sigma^2_{(i:k)}}\right)+\frac{k^2+k+2}{\sigma^2_{(\frac{k}{2}:k)}}}, \text{if } k \text{ is even}\\ \frac{1}{\left(\sum_{i=1}^{(k-1)/2}\frac{2i}{\sigma^2_{(i:k)}}+\frac{k^2+1-\frac{k(k-1)}{2}}{\sigma^2_{(M:k)}}\right)}, \text{if } k \text{ is odd}\end{cases} \tag{3.14}$$

The RP of $b^{/}\hat{\mu}_u$ compared with SRS with the sample size of Eq. (3.10) is

$$RP_u(fav)=\begin{cases}\frac{4\sigma^2}{5k^2+2k+8}\left(\sum_{i=1}^{\frac{k-2}{2}}\left(\frac{2i}{\sigma^2_{(i:k)}}\right)+\frac{k^2+k+2}{\sigma^2_{(\frac{k}{2}:k)}}\right), \text{if } k \text{ is even}\\ \frac{4\sigma^2}{3k^2+2k+3}\left(\sum_{i=1}^{(k-1)/2}\frac{2i}{\sigma^2_{(i:k)}}+\frac{k^2+1-\frac{k(k-1)}{2}}{\sigma^2_{(M:k)}}\right), if\ k \text{ is odd}\end{cases}. \tag{3.15}$$

4. NUMERICAL COMPARISONS

The numerical values of RP_{eql}, RP_{Ney}, $RP_m(KPT)$ (or $RP_u(KPT)$) and $RP_m(fav)$ (or $RP_u(fav)$) for some symmetric distributions are compared for the set size *k =2, 3 ...10*. Under mound shaped distributions, we have considered only uniform distribution and under U-shaped distribution, standard normal, standard logistic, standard double exponential and standard special distributions are considered. For the values of variances of order statistics $\sigma^2_{(i:k)}$ we refer to Harter and Balakrishnan (1996). The computed different RPs for mound shaped distribution is given in Table 1, however,

Table 1. Computed values of RP_{eql} (1), RP_{Ney} (2), $RP_m(KPT)$ (3) and $RP_m(fav)$ (4) for one mound shaped distribution for *k = 2(1)10*

Set size (*k*)	U(0,1)			
	1	2	3	4
2	1.5000	1.5000	1.5000	1.5000
3	2.0000	2.0096	2.2222	2.1852
4	2.5000	2.5255	3.1249	3.0381
5	3.0000	3.0458	4.1999	3.9792
6	3.5000	3.5692	5.4445	5.1614
7	4.0000	4.0951	6.8573	6.3409
8	4.4999	4.6227	8.4374	7.8494
9	4.9998	5.1518	10.1854	9.2614
10	5.5000	5.6824	12.1007	11.0959

Table 2. Computed values of RP_{eql} (1), RP_{Ney} (2), $RP_u(KPT)$ (3) and $RP_u(fav)$ (4) for some U-shaped distributions for *k = 2(1)10*

Set size (*k*)	N(0,1)				Logistic				Double exponential				Standard Special			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
2	1.4669	1.4669	1.4669	1.4669	1.4367	1.4367	1.4367	1.4367	0.6787	0.6787	0.6787	0.6787	1.4008	1.4008	1.4008	1.4008
3	1.9137	1.9187	2.2288	2.1307	1.8380	1.8578	2.5506	2.3422	0.8649	0.8909	1.5652	1.3744	1.7520	1.7933	2.9264	2.6003
4	2.3469	2.3610	2.7742	2.7125	2.2164	2.2685	3.1638	3.0423	1.0191	1.0824	1.9204	1.8182	2.0723	2.1766	3.6168	3.4364
5	2.7702	2.7968	3.4864	3.3223	2.5783	2.6717	4.1651	3.8335	1.1637	1.2708	2.8475	2.4946	2.3712	2.5523	4.9596	4.4530
6	3.1857	3.2276	4.0616	3.9565	2.9275	3.0692	4.8468	4.6411	1.3014	1.4566	3.2967	3.0854	2.6538	2.9222	5.7650	5.4568
7	3.5949	3.6546	4.7517	4.5315	3.2667	3.4622	5.7958	5.3584	1.4338	1.6402	4.2436	3.7241	2.9236	3.2872	7.0185	6.3543
8	3.9990	4.0785	5.3422	5.2009	3.5975	3.8515	6.5125	6.2368	1.5620	1.8219	4.7549	4.4333	3.1830	3.6481	7.8833	7.4687
9	4.3986	4.4998	6.0219	5.7488	3.9212	4.2377	7.4322	6.8954	1.6868	2.0022	5.7120	5.0189	3.4335	4.0055	9.0876	8.2737
10	4.7945	4.9191	6.6230	6.4480	4.2388	4.6214	8.1706	7.8309	1.8087	2.1810	6.2680	5.8342	3.7447	4.4885	10.2796	9.7796

the same for U shaped distributions are provided in Table 2. From Table 1 and 2, it is seen that for *k=2* all four allocation methods for symmetric distribution are equivalent. All RPs are increasing with the increase of *k*.

The performance of equal and Neyman's allocation is almost same but the proposed favourable allocation model outperforms both. Moreover, the proposed favourable allocation model is quite close to the KPT optimal model.

From these results, it is concluded that when the measurements of units are costly and time consuming and the underlying distribution is symmetric, SRS become poor in comparison of RSS. The efficiency of the procedure can be increased by applying appropriate allocation model of available order statistics. This paper showed that, despite using all order statistics while estimating population mean, the performance of proposed model is very near to the KPT optimal allocation model. Therefore, the proposed favourable allocation model may be considered as a near optimal allocation model for selecting the sample while estimating population mean of symmetric distributions.

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