https://doi.org/10.56093/JISAS.V78I3.10
Available online at http://isas.org.in/isa/jisas
JOURNAL OF THE INDIAN SOCIETY OF
AGRICULTURAL STATISTICS 78(3) 2024 267–276

Ranked Set Sampling Model for Response Estimation of Developmental Programs with Exponential Impacts

Neeraj Tiwari[1], Girish Chandra[2] and Shailja Bhari[1]
[1]Soban Singh Jeena University, Almora
[2]University of Delhi, Delhi

Received 02 January 2025; Revised 03 February 2025; Accepted 04 February 2025

SUMMARY

Government and non-government organizations regularly initiate developmental programs in successive phases. Each phase plays a significant role in the development process. These programs aim to enhance the socio-economic conditions of communities and individuals by addressing issues such as poverty, health, education, women empowerment, and infrastructure. Every year, many such programs are implemented in several areas for various purposes, including health awareness programs, women empowerment programs, cleanliness programs, vaccination campaigns, and educational programs to improve health and public participation. Chandra *et al.* (2018a) considered the linear impact of programs across successive phases and used a multiplicative model that linked with predefined survey and response variables. They employed the model to estimate the population mean of the response variable using the Ranked Set Sampling (RSS) on the survey variable, which led to the linear impact valuation of developmental programs. In this paper, we have suggested an exponential impact to reflect more realistic growth patterns in numerous development processes. We have proposed an estimator of the response variable under RSS based on the survey variable with exponential impact and compared to an estimator of RSS with Linear impact in terms of relative precision (RP). The pattern of various RPs is explored using the real-life example of Education for all towards quality with equity.

Keywords: Ranked Set Sampling; Exponential Impact; Impact estimation; Relative precision; Multiplicative model.

1. INTRODUCTION

The implementation of developmental programs is necessary for the growth of any community or an individual. These programs typically follow distinct phases including planning, implementation, monitoring, and evaluation, which ensures systematic and effective development. These programs improve the deserved impact of the relevant component with interest. Developmental strategy includes the program initiatives like the annihilation of polio in children up to age five, eradication of poverty in the community, cancer awareness plans, mid-day meal for boosting Gross Enrollment Ratio (GER), women empowerment, skill development and employment, environmental conservation and so on. These can be implemented across different time phases, such as, half-yearly, yearly, 5-yearly, or much more. The time depends upon the volume, scope, and geographical spread of the region or units where the program is to be implemented.

The units of the various phases have varying impacts on the implementation of programs in successive phases. Here, these different phases could be viewed as different strata. All units within a stratum may have a similar impact on the desired component, but they are different from units in another stratum. Sometimes the development program may face a lagged impact on the units of the desired component, which would lead to an inaccurate result. For improvement purposes, weights would be assigned to impacts under different phases, which may depend on the impact pattern. Chandra *et al.* (2018a) introduced two types of variables for the assessment of developmental programs, one is an implementation of the variable with a lagged impact, and the second is a non-implementation of the variable without a lagged impact.

Corresponding author: Shailja Bhari
E-mail address: shailjabhari36@gmail.com

The desired characteristic of the unit without program implementation is termed the *Survey Variable*, S. The program's implementation impacts S acquiring different values depending upon the nature of the impact. The change in values of the component under the impact of the program is different from the original value of S and then is termed as the *Response Variable*, R. This can better be explained with an example from Wikipedia, 2024: The government of India launched one of the development programs '*Mid-Day Meal Scheme'* under the name of "National Programme of Nutritional Support to Primary Education" on 15th August 1995 in Tamil Nadu, later denoted as *National Programme of Mid-day Meal in Schools.* This scheme was implemented in all the states of India by 2002 and has been serving 120 million schools and Education Guarantee Scheme centers to date. *National Food Security Act, 2013* enclosed this *Mid-Day Meal Scheme,* which was later re-named as '*PM-POSHAN'* in September 2021, which was further implemented throughout the country in different years. The response to this program is monitored at multiple levels (such as National, State, District, Municipal, Block, Village, and School) by various committees within different time intervals (such as quarterly, monthly, daily, and fortnight). The S value depends upon the proportional advantage attained by the person included in a particular category such as age, family, income, geographical region, etc.

The value of S is independent of the impact of the program being implemented, and therefore, it remains constant or unaffected by the program's impact. The numerical value utilized for S to reflect the impact of the program which may be obtained as the response variable R. In this sense, it is discovered that R is a growth function on the subject of the number of phases, since the impact of the program increases with every successive phase.

The term 'impact' for any specific program's phase was described by Pandey (2010). He explained the impact as the proportional advantage accomplished with and without the program in the context of multiplicative impact. Many authors have focused on different types of impact patterns.

In this paper, we consider the program's implementation in successive phases, with the program's impact is exponentially proportional to each phase. The impact follows the multiplicative model with exponential series in successive phases for generating further advanced ideas. The component of R will be estimated based on the impact of successive phases. We aim to estimate R over k program phases while preserving the known nature of S.

Different sampling techniques are widely utilized in research estimation processes. Cochran (1977) discussed that the nature of the population being studied and the specific objectives of the research are fundamental considerations in determining the most appropriate estimation methods. Simple Random Sampling (SRS) and Stratified Random Sampling have been usually used in development programs. In this paper, RSS has been used with the multiplicative model based on an exponential impact, which shows the rapid growth/decline of the program.

RSS was introduced by McIntyre (1952). It is the sample selection technique that also estimates the population parameters. When the actual measurements of the variable of interest are costly or time-consuming, then the units are ranked according to their value through visual inspection or other methods without actual measurements. It potentially leads to higher precision while reducing the cost. RSS is the best choice for this present paper. The units may easily be ranked on visual assessment.

The utility of RSS has been explored in various areas, especially environment-related cases. McIntyre (1952, 1978) used RSS in estimating pasture and forage yield. Johnson *et al.* (1993) examined forests, grassland, and other vegetation resources. Nahhas *et al.* (2002) estimated bone mineral density in a human population whereas Chen *et al.* (2005, 2006, and 2007) conducted health and nutritional examination. Chen *et al.* (2004) surveyed single-family home sales prices. Husby *et al.* (2005) checked crop production and Kowalczyk (2005) conducted market and consumer surveys. Chandra *et al.* (2011) presented the RSS in the first phase of Adaptive Cluster Sampling (ACS) to obtain precise estimates for rare and endangered species. Singh *et al.* (2014), Khan *et al.* (2016), Khan and Shabbir (2016), Zamanzade and Mahdizadeh (2017), Ozturk (2019), Bhoj and Chandra (2018, 2019, 2022), Tiwari *et al.* (2015, 2019), Latpate *et al.* (2021), Omari and Abdallah (2023), Koshti and Kamalja (2024), Taconeli (2024) gave some advanced techniques in the development of RSS.

In Section 2, the assumptions for a program with successive impacts over different phases, and estimation of R using S have been discussed. The relationship between means and variances of R and S under RSS procedure with linear impact has also been suggested. In Section 3, an estimator of the population mean of R based on RSS procedure with exponential impact on S has been suggested. Additionally, we determined the relative precision (RP) of the proposed estimator using exponential impact to an estimator based on RSS along linear impact. In Section 4, we explained the RPs for a real-world development program. Section 5 describes the computation of RP in two scenarios. Section 6 summarizes the results of the paper with discussion.

2. ESTIMATION OF MEAN OF R USING RSS WITH LINEAR IMPACT

RSS is an improved technique for the estimation of population parameters in a situation when an actual measurement of units is expensive but ranking of the units based on some methods which is easy and has negligible cost like visual inspection etc. This method has been used in many real-life situations in addition to substantial theoretical developments in the past several decades. Chandra *et al.* (2018a, 2018b) used the RSS approach for the first time to examine the impact of R through S on a developmental program implemented over the phases. A description of this procedure has been briefed with a linear impact over the phases, which is as follows:

Start by selecting a simple random sample of k units from the population of the survey variable S. Rank these k units based on some related variable (not necessarily the survey variable S itself). This ranking is done without actual measurements. Identify the unit with the lowest rank (rank 1) from this first sample. Repeat the random sampling process to select another simple random sample of size k from the population and rank these k units as before. Now, identify the unit with the second-lowest rank (rank 2) in this second sample. This completes one cycle of the RSS procedure, where k units have been selected based on their ranks from k simple random samples. The cycle may be replicated m times to get a balanced ranked set sample of size $n = km.$ With the consideration that there is a perfect ranking within each cycle, one unit is held by each rank order statistics or phase, and the m ranked units from each phase are involved in the measurement of S. To reduce the complexity of the calculations, $m = 1$ was chosen.

The k^2 ordered observations in the k samples on Scan be displayed as

$$S_{(11)}, S_{(12)}, \ldots, S_{(1k)}$$
$$S_{(21)}, S_{(22)}, \ldots, S_{(2k)}$$
$$\vdots$$
$$S_{(k1)}, S_{(k2)}, \ldots, S_{(kk)}$$

The observations $S_{(11)}, S_{(22)}, \ldots, S_{(kk)}$ are actual measurements for S.

Chandra *et al.* (2018a) suggested the RSS approach, with the linear impact of a development program through successive phases, to estimate the responses (impact) of the program. A particular development program is implemented over k phases, and the impacts following a linear model throughout the phases. The successive impacts follow an arithmetic progression, that is, the impact of the i^{th} phase out of k phases is defined by

$$I_{[i:k]} = a + (i-1)d \tag{1}$$

where a (first phase value) and d (common difference between two successive phase values) are real positive numbers.

The impact develops as a program proceeds across its successive phases. Therefore, each phase builds on the impact of the previous phase. Impacts are in ascending order from the first phase (lowest phase) to the k^{th} phase (highest phase). This shows that the effect of the program grows over time as it is implemented broadly. There are three main variables taken under study namely response variable R, survey variable S, and impact variable I. The following multiplicative model was used

$$\mathbf{R} = \mathbf{SI} + \boldsymbol{\varepsilon} \tag{2}$$

where, $\mathbf{S}, \mathbf{R},$ and $\mathbf{I}$ represent the vector of $S, R,$ and I respectively for all successive phases of the program and $\boldsymbol{\varepsilon}$ is a vector of random error with mean 0 and unknown variance σ_e^2 and is independent of $S.$

Let $S_{(i:k)} \left(\equiv S_{(ii)}\right)$ and $R_{(i:k)}, i = 1, \ldots, k$ denote the values of S and R, respectively for the unit taken for measurement belonging to i^{th} rank (phase) order. Under this model, Chandra *et al.* (2018a) suggested that

$$R_{(i:k)} = S_{(i:k)} \times I_{[i:k]} + \varepsilon_{(i:k)} \tag{3}$$

Under the consideration for the i^{th} phase, $\mu_{R(i:k)}$ and $\mu_{S(i:k)}$ represents the population mean and $\sigma^2_{R(i:k)}$ and $\sigma^2_{S(i:k)}$ represents the population variances of *R* and *S* respectively. The peculiarities corresponding to the i^{th} phase of both the variables are independently and identically distributed with respective means $\mu_{R(i:k)}$, $\mu_{S(i:k)}$ and variances $\sigma^2_{R(i:k)}$, $\sigma^2_{S(i:k)}$ respectively for fixed i.

Under the RSS procedure, the unbiased estimator of μ_R with its variance is given by

$$\bar{R}_{(k)RSS} = \frac{1}{k}\sum_{i=1}^{k} CS_{(i:k)} \tag{4}$$

$$Var\left(\bar{R}_{(k)RSS}\right) = \frac{C^2}{k}\frac{\sum_{i=1}^{k}\sigma^2_{S(i:k)}}{k} \tag{5}$$

where $C = \frac{\left(2a + (k-1)d\right)}{2}$ denotes the average impact of all k phases.

Table 1. An empirical example showing the values of $S, R,$ and I with $d = 0.2$

Phase No. i	*R*	*S*	*I*
1	5	5	1
2	12	10	1.2
3	21	15	1.4
4	32	20	1.6
5	45	25	1.8
6	60	30	2
7	77	35	2.2
8	96	40	2.4
9	117	45	2.6
10	140	50	2.8
11	165	55	3.0
12	192	60	3.2
13	221	65	3.4
14	252	70	3.6
15	285	75	3.8

This method is illustrative with a hypothetical example (Table 1). Suppose, one 15-phased program is implemented with linear impact, governed by the common difference $d = 0.2$. The observed values of *S* are taken and consequently, values of *R* are computed using (3). The impact value for the first phase of *R* and *S* is the same and *i=1* implied that *a=1*. Thereby, the estimator $\bar{R}_{(k)RSS}$ and its variance $Var\left(\bar{R}_{(k)RSS}\right)$ using (4) and (5) can be obtained. Chandra *et al.* (2018a) showed that the RSS procedure performs better than the SRS procedure in terms of RP.

In this paper, it is assumed that the impact of the subsequent phases are very high and have the exponential growth instead of linear growth.

3. ESTIMATION OF MEAN OF *R* USING RSS WITH EXPONENTIAL IMPACT

We consider that the impact of the i^{th} phase out of k phases can be defined by an exponential function and is defined by

$$I_{[i:k]} = \begin{cases} ae^{b(i-1)}, i = 2,3,\ldots,k \\ 1, \quad i = 1 \end{cases} \tag{6}$$

where a(initial value) and b (constant in an exponential equation) are real positive numbers with e^b = constant multiplier (growth/decay factor)

A program grows along its phases and its impact tends to deepen with time. The progressive impact is common in developmental programs. The impact is typically moderate and confined when the program is initially implemented. They gain momentum gradually and its effect becomes vital at the peak stage. The implementation phase of the program has the lowest impact and the culmination phase (k^{th} phase) has the highest impact. Here, we extract the response variable R through survey variable S using (3) and impact variable I using (6).One more illustration of the developmental program with exponential impact is shown in Table 2.

For the first phase (*i* = 1), *a* = 1 shows that the impact value of *R* and *S* is the same. The hypothetical scores obtained in Table 2 show the presence and absence of the program for the well-defined category. In reality, it is difficult to measure variable S as measuring every unit is costly and time-consuming.

Table 2. Example showing the values of S, R and I with $d = 0.2$

Phase No. i	S	a	b	I	R
1	5	1.0	0.2	1	5
2	10	1.5	0.4	2	22
3	15	2.0	0.6	7	100
4	20	2.5	0.8	28	551
5	25	3.0	1.0	164	4095
6	30	3.5	1.2	1412	42360
7	35	4.0	1.4	17788	622589
8	40	4.5	1.6	329087	13163480
9	45	5.0	1.8	8970374	403666824
10	50	5.5	2.0	361129830	18056491513
11	55	6.0	2.2	21509477077	1183021239223
12	60	6.5	2.4	1897962725137	113877763508192
13	65	7.0	2.6	248362415717594	16143557021643600
14	70	7.5	2.8	48237012744274700	3376590892099230000
15	75	8.0	3.0	13914199532164000000	1043564964912300000000

Therefore, we estimate the mean of the response variable R by applying the RSS technique to the survey variable S.

Under this model, the proposed estimator of μ_R is given by:

$$\bar{R}_{(k)}e = \frac{1}{k}\sum_{i=1}^{k} DS_{(i:k)} \tag{7}$$

where $D = \dfrac{a\left(1-e^{bk}\right)}{k\left(1-e^{b}\right)}$ denotes the average impact for all k phases. (8)

Using the property of RSS that

$\sum_{i=1}^{k} E\left(S_{(i:k)}\right) = k\mu_S$, it is verified that $E\left(\bar{R}_{(k)}e\right) = \mu_R$

The variance of $\bar{R}_{(k)}e$ is

$$Var\left(\bar{R}_{(k)}e\right) = \frac{D^2}{k}\frac{\sum_{i=1}^{k}\sigma^2_{S(i:k)}}{k} \tag{9}$$

The RP of the proposed estimator $\bar{R}_{(k)}e$ with $\bar{R}_{(k)RSS}$ using (5) and (9) is

$$RP = \frac{Var\left(\bar{R}_{(k)RSS}\right)}{Var\left(\bar{R}_{(k)}e\right)} = \frac{\dfrac{C^2}{k}\dfrac{\sum_{i=1}^{k}\sigma^2_{S(i:k)}}{k}}{\dfrac{D^2}{k}\dfrac{\sum_{i=1}^{k}\sigma^2_{S(i:k)}}{k}} = \frac{C^2}{D^2}\frac{\sum_{i=1}^{k}\sigma^2_{S(i:k)}}{\sum_{i=1}^{k}\sigma^2_{S(i:k)}} = \frac{C^2}{D^2}$$

Substituting the values of C and D, we get

$$RP = \frac{a^2 + (k-1)ad + \dfrac{(k-1)^2}{4}d^2}{\dfrac{a^2\left(1-2b^k+b^{2k}\right)}{k^2\left(1-2b+b^2\right)}}$$

$$RP = \frac{\left[a^2 + (k-1)ad + \dfrac{(k-1)^2}{4}d^2\right]\left[k^2\left(1-2e^b+e^{2b}\right)\right]}{\left[a^2\left(1-2e^{bk}+e^{2bk}\right)\right]} \tag{10}$$

Taking the log of (8),

$$\log D = \log\frac{a\left(1-e^{bk}\right)}{k\left(1-e^{b}\right)}$$

$$\log D = \log a\left(1-e^{bk}\right) - \log k\left(1-e^{b}\right) \tag{11}$$

The exponential function (12) is used for the expansion, which are as follows,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \tag{12}$$

$$\log\left(1-e^x\right) = \log\left[1-\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\right)\right]$$

$$\log\left(1-e^x\right) = \log\left[-x-\frac{x^2}{2!}-\frac{x^3}{3!}-\ldots\right] \tag{13}$$

From (11), we get,

$$\log a\left(1-e^{bk}\right) = \log a + \log\left(1-e^{bk}\right) \tag{14}$$

$$\log k\left(1-e^{b}\right) = \log k + \log\left(1-e^{b}\right) \tag{15}$$

Using (13), we get

$$\log\left(1-e^{bk}\right) = log\left(-bk-\frac{b^2k^2}{2!}-\frac{b^3k^3}{3!}-\ldots\right) \tag{16}$$

$$\log\left(1-e^{b}\right) = log\left(-b-\frac{b^2}{2!}-\frac{b^3}{3!}-\ldots\right) \tag{17}$$

Substituting (16) and (17) in (14) and (15) respectively,

$$\log a\left(1-e^{bk}\right) = \log a + log\left(-bk-\frac{b^2k^2}{2!}-\frac{b^3k^3}{3!}-\ldots\right)$$

$$= \log a\left(-bk-\frac{b^2k^2}{2!}-\frac{b^3k^3}{3!}-\ldots\right) \tag{18}$$

$$\log k\left(1-e^{b}\right)=\log k+log\left(-b-\frac{b^2}{2!}-\frac{b^3}{3!}-\ldots\right)$$

$$=\log k\left(-b-\frac{b^2}{2!}-\frac{b^3}{3!}-\ldots\right) \quad (19)$$

Substituting (18) and (19) in (11),

$$\log D=\log a\left(-bk-\frac{b^2k^2}{2!}-\frac{b^3k^3}{3!}-\ldots\right)-\log k\left(-b-\frac{b^2}{2!}-\frac{b^3}{3!}-\ldots\right)$$

$$\log D=\log\frac{a\left(-bk-\frac{b^2k^2}{2!}-\frac{b^3k^3}{3!}-\ldots\right)}{k\left(-b-\frac{b^2}{2!}-\frac{b^3}{3!}-\ldots\right)} \quad (20)$$

Taking antilog of (20),

$$D=\frac{a\left(-bk-\frac{b^2k^2}{2!}-\frac{b^3k^3}{3!}-\ldots\right)}{k\left(-b-\frac{b^2}{2!}-\frac{b^3}{3!}-\ldots\right)}$$

Omitting high order powers of 3 and above for easy calculations,

$$D=\frac{a\left(-bk-\frac{b^2k^2}{2!}\right)}{k\left(-b-\frac{b^2}{2!}\right)}=\frac{a\left(bk+\frac{b^2k^2}{2!}\right)}{k\left(b+\frac{b^2}{2!}\right)}=\frac{a\left(bk+\frac{b^2k^2}{2}\right)}{k\left(b+\frac{b^2}{2}\right)}$$

$$=\frac{a\left(2bk+b^2k^2\right)}{k\left(2b+b^2\right)}=\frac{a\left(2bk+k^2b^2\right)}{\left(2bk+kb^2\right)} \quad (21)$$

If $k^2b^2 \geq kb^2$ in (21), then $\left(2bk+k^2b^2\right) \geq \left(2bk+kb^2\right)$.

The term $\left(2bk+k^2b^2\right)$ increases with the increase in the value of k. However, for a certain range, $\left(2bk+k^2b^2\right)$ would be similar to $\left(2bk+kb^2\right)$. Then, the following result could be observed for the equation (21).

$$D \approx a \quad (22)$$

Substituting (22) in (10),

$$RP=\frac{C^2}{D^2}=\frac{a^2+(k-1)ad+\frac{(k-1)^2}{4}d^2}{a}$$

Assuming the initial value $(a=1)$, then

$$RP=\frac{1+(k-1)d+\frac{(k-1)^2}{4}d^2}{1} \quad (23)$$

$$=1+(k-1)d+\frac{(k-1)^2}{4}d^2$$

= 1 + some positive term> 1

This shows that the RP of the proposed estimator based on an exponential impact $\bar{R}_{(k)}e$ to an estimator based on linear impact $\bar{R}_{(k)RSS}$ is greater than one. Therefore, we observe that an exponential impact performs better than a linear impact, empirically, it is discussed in section 5.

4. NUMERICAL BASED ON A REAL-DATA SET

Example based on Education for all towards quality with equity

India has numerous real-life examples that have been studied and reported across various fields. "Education for all towards quality with equity", India is one of the reports which consists of the developmental program that instigated the target of providing free and compulsory education to all children of India. The Gross Enrollment Ratio (GER) of SC students in higher secondary education was reported by the government for the period 2006 to 2010 in this program report. The values provided by schools across the country laid the foundation of GER. In real terms, it becomes extremely time-consuming and difficult to obtain the information from all the schools in India on time. The GER performance for boys and girls of class XI-XII between the ages 16-17 are given in Table 3.

Table 3. GER in Higher Secondary Education (2006 to 2010)

Phase no.	GER of SC students	
	Boys	Girls
2006 $(i=1)$	29.2	21.8
2007 $(i=2)$	30.1	25.3
2008 $(i=3)$	30.9	26.6
2009 $(i=4)$	37.4	33.5
2010 $(i=5)$	40.3	36.1

The impact values of each successive phase can be computed using (6). We transformed all GER values in the form of impacts by dividing the value by its preceding value from Table 3. Then the transformed values are equated to the average impact value $D = \frac{a\left(1-e^{bk}\right)}{k\left(1-e^{b}\right)}$ to get the value of b, shown in Table 4.

Table 4. GER Transformation into Exponential Impact

Phase no. (i)	GER of SC students	
	Boys	**Girls**
1	1.00	1.00
2	1.03	1.16
3	1.03	1.05
4	1.21	1.26
5	1.08	1.08
Average (D)	1.07	1.11
Value of b	0.033	0.051

After equating, we get two values of b, one is positive and another is zero. The value zero is not possible, therefore we take only the positive value of b.

In the same way, using equation (1), the impact values of the program can be computed in each successive phase. Again, from Table 3,we transformed all GER values into impacts by dividing the value by the first phase impact value. Then the average of transformed values may be equated to the average impact value, $C = \frac{\left(2a+\left(k-1\right)d\right)}{2}$ to obtain the value of d, shown in Table 5.

Table 5. GER Transformation into Linear Impact

Phase no. (i)	GER of SC students	
	Boys	**Girls**
1	1.00	1.00
2	1.03	1.16
3	1.06	1.22
4	1.28	1.54
5	1.38	1.66
Average (C)	1.15	1.31
Value of d	0.075	0.155

Now, using the above impact of the program we survey the schools using the RSS procedure for exponential impact and linear impact. We use the simulated samples for the values of S which have been randomly generated for five phases from the range 29.20 to 40.30 for boys and 21.80 to 36.10 for girls. In

Table 6. Ranked Set Sample of Size 15 with $k=5, m=3$ for GER (Boy)

Cycle	Set I (for $S_{(11)}$)	Set II (for $S_{(22)}$)	Set III (for $S_{(33)}$)	Set IV (for $S_{(44)}$)	Set V (for $S_{(55)}$)
I	**36.45**	38.36	35.01	36.86	30.24
	31.79	**35.12**	34.87	40.29	39.77
	31.66	39.88	**38.60**	40.20	31.44
	30.20	33.64	39.03	**36.89**	34.01
	35.81	31.18	36.81	36.11	**36.06**
II	**35.38**	31.28	35.65	31.25	35.47
	30.89	**39.18**	35.36	31.79	36.10
	32.73	39.63	**32.34**	30.89	35.50
	37.94	36.88	39.63	**33.03**	35.77
	40.21	31.93	30.95	34.81	**36.81**
III	**37.53**	34.16	34.47	32.05	37.12
	31.69	**37.86**	30.69	31.9	29.99
	34.89	33.86	**30.45**	32.21	35.59
	37.48	38.53	32.85	**39.60**	37.06
	32.12	36.59	30.49	39.22	**37.83**

Table 7. Ranked Set Sample of Size 15 with $k=5, m=3$ for GER (Girl)

Cycle	Set I (for $S_{(11)}$)	Set II (for $S_{(22)}$)	Set III (for $S_{(33)}$)	Set IV (for $S_{(44)}$)	Set V (for $S_{(55)}$)
I	**26.05**	27.9	30.61	29.77	22.47
	29.82	**33.59**	28.97	25.33	30.16
	25.21	29.83	**22.91**	23.19	33.74
	32.64	34.15	22.42	**34.09**	29.72
	28.71	34.46	30.97	30.97	**35.21**
II	**23.03**	29.21	28.87	23.21	34.99
	29.14	**32.52**	23.52	28.52	33.84
	26.49	27.92	**34.23**	35.81	29.04
	23.96	23.3	23.19	**27.77**	24.32
	31.27	23.68	32.81	22.27	**32.57**
III	**26.85**	34.86	27.78	29.23	33.62
	33.32	**34.07**	24.61	33.93	22.27
	27.55	34.05	**33.03**	28.62	25.29
	25.51	33.36	22.19	**26.24**	35.75
	28.58	29.74	25.63	22.70	**28.68**

practice, the number of schools for measuring the Gross Enrollment Ratio (GER) for this particular category is selected randomly every year. For this example, we have taken $m=3$ with sample size $mk=15$. Units taken for measurements are shown by the **bold and red** figures in Table 6 and 7.

In this example, the values of $\sigma^2_{S(i:k)}$ corresponding to the five rank order statistics are computed as 0.556, 1.361, 10.228, 15.1142 and 2.2609 for boys and 1.832, 23.998, 3.611, 1.589 and 0.153 for girls, respectively. The proposed estimator of the mean of the population by the exponential impact was computed with $D=0.033$ (Boys) and $D=$ 0.051 (Girls) and are given as $\overline{R}_{(k)}e=37.220$ for boys and $\overline{R}_{(k)}e=32.462$ for girls. The estimator for the population mean by the linear impact was computed with $C=0.075$ (Boys) and $C=0.155$ (Girls) and are given as $\overline{R}_{(k)RSS}=40.038$ for boys and $\overline{R}_{(k)RSS}=38.462$ for girls. Using (10), we computed values of RP with $a=1$ and $k=5$.

Table 8. The computed values of RP based on the values of b for boys and girls

Boys			Girls		
d	***b***	**RP**	***d***	***b***	**RP**
0.075	0.033	1.155	0.155	0.051	1.393

It has been observed from Table 8 that the value of RP is greater than one for different values of b. While comparing the variances of successive phases with exponential impact against linear impact using RSS for estimating responses in developmental programs, we found that the exponential impact is more efficient than the linear impact.

5. COMPUTATION OF RELATIVEPRECISION (RP)

The RP is directly connected to b and d. If one increases or decreases, the other is affected accordingly. To investigate the relationship among these three, we have divided them into two cases, as follows:

Case 1: Relation between RP and b for fixed d

In order to examine the relation between RP and b for a fixed value of d (Table 9), we considered a few values of k and b while fixing d at 0.05 (taken from the above example), then calculated the RP using (10). We observed that with a negative value of b, the RP value increases drastically as the number of phases k increases. However, the RP value drops as b increases at a particular value of k. It is also observed that after a certain value of b (0.05), RP's value begins to decrease as the k value increases. This interprets that when k increases, the RP rapidly increases with negative b and decreases gradually with positive b. The RP widely depends on the growth/decay factor (e^b) of exponential impact. The RP decreases when the factor increases and vice versa. Based on this fact, the computed values of RP are shown in Table 9.

Table 9. The values of Relative Precision in terms of b and k for $d=0.05$

b \ *k*	2	3	4	5	6	7	8
-200	4.203	9.923	18.490	30.250	45.563	64.803	88.360
-150	4.203	9.923	18.490	30.250	45.563	64.803	88.360
-100	4.203	9.923	18.490	30.250	45.563	64.803	88.360
-50	4.203	9.923	18.490	30.250	45.563	64.803	88.360
-30	4.202	9.922	18.490	30.250	45.562	64.802	88.360
-20	4.202	9.922	18.490	30.250	45.562	64.802	88.360
-10	4.202	9.922	18.488	30.247	45.558	64.797	88.352
-9	4.201	9.920	18.485	30.243	45.551	64.787	88.338
-8	4.200	9.916	18.478	30.230	45.532	64.759	88.301
-7	4.195	9.904	18.456	30.195	45.479	64.684	88.199
-6	4.182	9.873	18.398	30.100	45.337	64.482	87.922
-4	4.053	9.562	17.819	29.152	43.909	62.450	85.153
-2	3.260	7.455	13.833	22.618	34.065	48.449	66.062
-1	2.246	4.391	7.666	12.252	18.296	25.941	35.330
-0.5	1.628	2.545	3.829	5.558	7.812	10.667	14.195
-0.1	1.158	1.338	1.541	1.769	2.027	2.316	2.639
0.01	1.040	1.081	1.121	1.162	1.204	1.245	1.287
0.02	1.030	1.059	1.088	1.116	1.144	1.171	1.198
0.03	1.019	1.038	1.055	1.071	1.086	1.101	1.114
0.04	1.009	1.017	1.023	1.028	1.031	1.034	1.035
0.05	0.999	0.996	0.992	0.986	0.979	0.970	0.960
0.1	0.948	0.897	0.845	0.795	0.746	0.697	0.651
0.4	0.677	0.446	0.286	0.179	0.110	0.066	0.039
0.5	0.599	0.344	0.191	0.102	0.053	0.026	0.013
0.8	0.404	0.148	0.050	0.016	0.005	0.001	0.000
0.9	0.351	0.110	0.031	0.008	0.002	0.000	0.000
1	0.304	0.080	0.019	0.004	0.001	0.000	0.000
2	0.060	0.003	0.000	0.000	0.000	0.000	0.000
3	0.009	0.000	0.000	0.000	0.000	0.000	0.000
4	0.001	0.000	0.000	0.000	0.000	0.000	0.000

Case 2: RP based on the values of d and k

By (23), we see that the RP is directly proportional to d and k. Table 10 shows that when the values of d and k increase, then the RP also increases rapidly.

Table 10. The RP values based on d

k	*d*	RP
2	0.05	1.051
3	0.1	1.210
4	1	6.250
5	2	25.000
6	3	72.250
7	5	256.000
8	7	650.250
9	9	1369.000
10	10	2116.000
20	20	36481.000
30	30	190096.000
40	40	609961.000
50	50	1503076.000
80	60	5621641.000
100	70	12013156.000
150	80	35533521.000
200	90	80209936.000
250	100	155027401.000

6. CONCLUSION AND DISCUSSION

In this paper, we proposed the RSS procedure with an exponential impact on the survey variable to obtain an improved estimator for the population mean of the response variable. The proposed estimator is compared to the estimator based on the RSS procedure with linear impact. It is observed that there is a gain in the relative precision of the proposed estimator based on the RSS procedure with an exponential impact, however, it depends on the growth/decay factor of this impact. The relative precisions are also shown for a real-life experimental data on GER, which shows the efficiency of exponential impact over linear one.

Organizations implement developmental programs in phases across different geographical regions or a particular region or community and are interested in knowing the impact of the program in the form of a response variable. This study covers cases in which (i) measuring the units of survey variable of the developmental program are very costly or time-consuming and (ii) the program impact is known in general and follows an exponential trend in particular. In such situations, RSS is a cost-effective and precise method of sample selection that provides a better estimate of the characteristics under study. RSS provides valuable information across different phases or stages of the program. The proposed exponential impact procedure seems to be superior to the linear impact procedure for developmental programs. This paper also shows empirical patterns of exponential series of impacts when the impact of the proceeding phase is known. This may make it easier to attain the optimum result of the program. The limitation of the proposed model is that when the number of phases k increases, so does the relative precision, and then decreases swiftly as the positive value of b increases (around 0.05). Such developmental programs are suitable for the pandemic and epidemic situations (such as COVID-19 and influenza) when the awareness of a disease, precautions guidelines program, and vaccination campaigns are at a high peak for some time. When the effect of a disease slows down, then these programs also come to an end.

ACKNOWLEDGEMENTS

The authors would like to express their gratitude to the anonymous reviewers for providing comments and suggestions which led to the improvement in the manuscript.

REFERENCES

Al-Omari, A.I. and Abdallah, M.S. (2023). Estimation of the distribution function using moving extreme and MiniMax ranked set sampling. *Communications in Statistics-Simulation and Computation*, **52**(5), 1909-1925.

Bhoj, D.S. and Chandra, G. (2022). Ranked set sampling with varied order statistics for skew distributions. *Model Assisted Statistics and Applications,* **17**(3), 161-166.

Bhoj, D.S. and G. Chandra. (2020). Simple unequal allocation procedure for ranked set sampling with skew distributions. *Journal of Modern Applied Statistical Methods,* **18**(2), 2811-15.

Chandra, G., Bhoj, D.S. and Pandey, R. (2018b). Simple unbalanced ranked set sampling for mean estimation of response variable of developmental programs. *Journal of Modern Applied Statistical Methods,* **17**(1), 28.

Chandra, G., Pandey, R., Bhoj, D.S., Nautiyal, R., Ashraf, J. and Verma, M.R. (2018a). Ranked set sampling approach for estimating the response of developmental programs with linear impacts under successive phases. *Pakistan Journal of Statistics*, **34**(2).

Chen, H., Stasny, E.A. and Wolfe, D.A. (2007). Improved procedures for estimation of disease prevalence using ranked set sampling.

Biometrical Journal: Journal of Mathematical Methods in Biosciences, **49**(4), 530-538.

Chen, H., Stasny, E.A. and Wolfe, D.A. (2005). Ranked set sampling for efficient estimation of a population proportion. *Statistics in Medicine*, **24**(21), 3319-29.

Chen, H., Stasny, E.A. and Wolfe, D.A. (2006). An empirical assessment of ranking accuracy in ranked set sampling, *Computational Statistics and Data Analysis*, **51**(2), 1411-1419.

Chen, Z., Bai, Z.D. and Sinha, B.K. (2004). *Ranked set sampling: theory and applications*. New York: Springer-Verlag.

Cochran, W.G. (1977). Sampling techniques, Wiley Publication.

Husby, C.E., Stansy, E.A. and Wolfe, D.A. (2005). An application of ranked set sampling for mean and median estimation using USDA crop production data. *Journal of Agricultural, Biological and Environmental Statistics*, **10**(3), 354-373.

Johnson, G.D., Paul, G.P. and Sinha, A.K. (1993). Ranked set sampling for vegetation research. *AbstractaBotanica*, **17***,* 87-102.

Khan, L. and Shabbir, J. (2016). Improved ratio-type estimators of population mean in ranked set sampling using two concomitant variables. *Pakistan Journal of Statistics and Operational Research,* **12**(3), 507-518.

Khan, L., Shabbir, J. and Kadilar, C. (2016). Efficient classes of ratio-type estimators of population mean under stratified median ranked set sampling. *Pakistan Journal of Statistics*, **32**(6), 475-496.

Koshti, R.D. andKamalja, K.K. (2004). A review on concomitants of order statistics and its application in parameter estimation under ranked set sampling. *Journal of the Korean Statistical Society*, **53**(1), 65-99.

Kowalczyk, B. (2005). Alternative sampling designs some applications of qualitative data in survey sampling. *Statistics in Transition*, **7**(2),427-443.

Latpate, R.V., Kshirsagar, J.K., Gupta, V.K. and Chandra, G. (2021). Advanced sampling methods, Springer, Singapore.

McIntyre, G.A. (1952). A method for unbiased selective sampling using ranked sets. *Australian Journal of Agricultural Research,* **3***,* 385-390.

McIntyre, G.A. (1978). Statistical aspects of vegetation sampling. In: Mannetje, L.T. (ed.), *Measurement of Grassland Vegetation and Animal Production Bulletin* 52, *Commonwealth Bureau of Pastures and Field Crops*. Hurley, Berkshire, U.K., 8-21.

Nahhas, R.W., Wolfe, D.A. and Chen, H. (2002). Ranked set sampling: cost and optimal set size. *Biometrics,* **58***,* 964-971.

Ozturk, O. (2019). Two-stage cluster samples with ranked set sampling designs. *Annals of the Institute of Statistical Mathematics*, **71**, 63-91.

Pandey, R. (2010). Samples allocation decisions for program evaluation: a case of multiplicative effect on units due to developmental initiatives in phased manner. *Folia ForestaliaPolonica*, Series A, **52**(2), 83-88.

Singh, H.P., Tailor, R. and Singh, S. (2014). General procedure for estimating the population mean using ranked set sampling. *Journal of Statistical Computation and Simulation*, **84**(5), 931-945.

Taconeli, C.A. (2024). Dual-rank ranked set sampling. *Journal of Statistical Computation and Simulation*, **94**(1), 29-49.

Tiwari, N. and Chandra, G. (2011). A systematic procedure for unequal allocation for skewed Distributions in Ranked Set Sampling. *Journal of the Indian Society of Agricultural Statistics*, **65**(3), 331-338.

Tiwari, N., Chandra, G. and Nautiyal, R. (2019). Ranked set sampling: A review with new initiative on extreme ranked set sampling. *Statistics and Applications,* **17**(1),121-139.

Tiwari, N., Pandey, G.S. and Chandra, G. (2015). Estimation of location and scale parameters of normal distribution using ranked set sampling. *Statistics in Forestry: Methods and Applications. Coimbatore: Bonfringe Publication*, 78-84.

Wikipedia: the free encyclopedia (2024). Midday Meal Scheme. From https://www.wikipedia.org/wiki/Midday_Meal_Scheme.

Zamanzade, E. and Mahdizadeh, M. (2017). A more efficient proportion estimator in ranked set sampling. *Statistics & Probability Letters*, **129**, 28-33.