https://doi.org/10.56093/JISAS.V78I1.1
Available online at http://isas.org.in/isa/jisas
JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 78(1) 2024 1–7

Lactation Curves of Mastitic Vrindavani Cattle: A Statistical Approach

Shashank Kshandakar[1], Med Ram Verma[2], Yash Pal Singh[1] and Sanjay Kumar[1]

[1]*ICAR-Indian Veterinary Research Institute, Izatnagar*
[2]*ICAR-Indian Agricultural Statistics Research Institute, New Delhi*

Received 25 November 2022; Revised 14 January 2024; Accepted 30 January 2024

SUMMARY

Mastitis is one of the important animal disease which causes huge economic loss. The present study was aimed to find the best lactation curve model that describes the milk production pattern of Vrindavani cattle suffered from mastitis. In the study, test day milk yield data of 161 mastitic Vrindavani cattle (incidence of the disease occurring within the three months of parturition) was collected from Cattle and Buffalo Farm, LPM section, ICAR-IVRI over 5 years (2009-2014). Eight lactation curve models [Inverse quadratic polynomial model (ND), Incomplete Gamma function (WD), Linear decline model (CL), Wilmink model (WL) Mixed log model (ML), Mitscherlich x Exponential (ME), Morant and Gnanasakthy model (MG) and Ali & Schaeffer lactation curve model (AS)] were fitted. The best model was selected based on different goodness of fit criteria and the Durbin-Watson test was used to examine the presence of autocorrelation present in the data set. Kolmogorov-Smirnova test and Shapiro-Wilk were used to test the normality of the residuals. Based on the various goodness of fit test it was observed that the Mixed Log (ML) model was the best fitted model to describe the lactation pattern of mastitic Vrindavani cattle.

Keywords: Goodness of fit; Lactation curve; Mastitis; Vrindavani cattle.

1. INTRODUCTION

Mastitis is one of the most widespread diseases characterized as an endemic disease affecting dairy herds worldwide. Mastitis defined as the inflammation of the mammary gland and is characterized by a range of physical and chemical changes of the milk with significant pathological changes in the udder glandular tissues (Halasa *et al.,* 2007). Mastitis is a multi-etiological and costliest production disease inflicting major economic losses to the dairy industry (medical treatment, extra labor, discarded milk, early cow replacement costs and reduced milk production) with significant public health risk.

In the present scenario, we cannot imagine mastitis-free dairy farms globally and the average incidence of mastitis varies from 5-20%. The Incidence of mastitis in a farm is the indicator of various managemental practices that affects the farm economy. The adverse climatic conditions and poor managemental practices cause metabolic disturbances, oxidative stress and immune suppression that leads to increased susceptibility towards diseases.

The major factors affecting the prevalence of mastitis included flock size, climatic conditions, distinctive managemental practices, the literacy level of the animal owners and feeding system. The continuing presence of the disease may be attributed to poor practices which include unhygienic conditions, improper milking practices, faulty milking equipments, lack of veterinary medicines and poor housing. The incidence of mastitis depends on breed, season, stress and farm managemental practices etc.

2. VRINDAVANI CATTLE

Vrindavani cattle are recently developed crossbred cattle strain of India developed at ICAR-IVRI. Vrindavani cattle has the 50-75% exotic inheritance of Holstein-Friesian, Brown Swiss, Jersey and 25-50% indigenous inheritance of Hariana cattle. Singh *et al.* (2011) studied various important parameters of Vrindavani cattle and reported that the mean lactation

Corresponding author: Med Ram Verma
E-mail address: medramverma@rediffmail.com

milk yield was 3219.75±41.09, 305-day milk yield was 3047.42±33.8 and peak yield was 16.58±0.16 kg. Kshandakar *et al.* (2018a) studied the various important lactation characters of healthy Vrindavani cattle and reported that the average milk production per lactation was 3257.57±0.13 kg and peak yield was 17.98±0.21 kg. The estimated value of coefficient of heritability (± standard error) from half-sib data for Vrindavani cattle milk production is 0.28578 (± 0.29712).

3. LACTATION CURVES

Lactation curve is defined as the graphical presentation of the milk production pattern of dairy animals from the day of calving to the day of termination of milk production. Lactation curves are used for the prediction of daily, monthly or total milk production of dairy animals. One model is not universally accepted to describe the production pattern and the shape of lactation curves has been varying according to the species of animal, environment and trait.

Brody *et al.* (1923) introduced the first lactation curve model (gamma function) to forecast milk yield over the lactation cycle of the cow. One year later, Brody *et al.*(1924) modified his model with two exponential functions to describe the whole lactation pattern of Holstein-Friesian cows. Curve fitting models have different forms including Inverse quadratic polynomial model (Nelder, 1966), Parabolic exponential (Sikka, 1950), Incomplete gamma (Wood, 1967), Polynomial (Ali and Schaeffer 1987), Exponential (Wilmink, 1987), Cubic splines (Green and Silverman, 1993), Legendre polynomial (Kirkpatrick *et al.*, 1994) and log-quadratic (Adediran *et al.,* 2012). Murphy *et al.* (2014) introduced the Auto-regressive neural network models and concluded that the developed model was found to more accurately forecast milk yield when compared with static neural network models. However, Mechanistic models offer more biological details (Ruelle *et al.,* 2016). Thirunavukkarasu and Rajarathinam (2018) predicted the total milk production using the Stochastic model (Window based Fuzzy time series and Holt-Winters non-seasonal model). Kshandakar *et al.* (2017) studied the effect of metabolic diseases on lactation curves of Vrindavani cattle.

However, scanty literature is available that explores the impact of mastitis on the lactation pattern of dairy animals. Keeping in view, the objective of this study was to find the best lactation curve model that describes the milk production pattern of mastitic Vrindavani cattle.

4. MATERIAL AND METHODS

The data regarding daily test day milk yield, disease conditions, parity of animals and season of calving was recorded from individual history sheets over 5 years (2009-2014) maintained at Cattle and Buffalo Farm of LPM Section ICAR-IVRI, Izatnagar (Uttar Pradesh). During the study period, the average incidence of Mastitis in this farm was 9.63 % and data were collected for the animals suffered from mastitis in the first trimester (3 month after parturition).

5. MODELING THE SHAPE OF LACTATION CURVE

In the present study, 8 Lactation curve models (5 models with three parameters, 2 models with four parameters and 1 model with five parameters) were fitted on average test day milk yield data of Vrindavani cattle.

1. Lactation curve model based on 3 Parameter

A. Inverse quadratic polynomial model (Nelder, 1966)

$$Y_t^{-1} = t / \left(a + bt + ct^2\right)$$

B. Incomplete Gamma function (Wood, 1967)

$$Y_t = at^b e^{-ct}$$

C. Linear decline model (Cobby and Le Du., 1978)

$$Y_t = a - bt - ae^{-ct}$$

D. Wilmink lactation curve model (Wilmink, 1987)

$$Y_t = a + be^{-kt} + ct$$

E. Mixed log model (Guo and Swalve, 1997)

$$Y_t = a + bt^{1/2} + c\ln(t)$$

where "Y_t" is the production at time t, "a" is the scale factor or milk yield at the beginning of lactation, "b" is the rate of change from initial production to peak yield and "c" is the rate of change from peak yield to the end of lactation. The factor "k" was related to the time of peak lactation and derived from the preliminary analysis made on average production.

2. Lactation curve model based on 4 Parameter

F. Mitscherlich x Exponential (Rook *et al.*, 1993)

$$Y_t = a\left(1 - be^{-ct}\right) - dt$$

Where " Y_t " is the production at time t, "a" is the scale factor or milk yield at the beginning of lactation, "b" is the rate of change from initial production to peak yield, "c" is the rate of change from peak yield to the end of lactation and "d" is a parameter related to maximum milk yield.

G. Morant and Gnanasakthy model (Morant and Gnanasakthy, 1989)

$$Y_t = \exp\left(a - bt + ct^{1/2} + d / t\right)$$

where, " Y_t " is the production at time t, "a" is the logarithm of expected yield at mid of lactation, "b" is the rate of change at mid of lactation, "c" is the rate of change of persistency and "d" is the rate of increase in yield at the beginning of lactation

3. Lactation curve model based on 5 Parameter

H. Ali and Schaeffer lactation curve model (Ali and Schaeffer, 1987)

$$Y_t = \mathbf{a} + \mathbf{b}\delta + \mathbf{c}\delta^2 + \mathbf{d}\theta_i + \mathbf{e}\theta_i^2$$

where " Y_t " is the production at time t, "a" is the scale factor or milk yield at the beginning of lactation, "b" is the rate of change from initial production to peak yield at a decreasing rate, "c" is the rate of change from initial production to peak yield at an increasing rate, "d" is the rate of change from peak yield to the end of lactation at a decreasing rate and "e" is the rate of change from peak yield to the end of lactation at increasing rate

where as δ (δ =t/305) and θ_i (θ_i=ln(305/t) are function of time

6. STATISTICAL ANALYSIS

In this study, we used the iteration procedure (Levenberg–Marquardt algorithm) for lactation curve fitting and parameter estimation. In the Iteration procedure, we apply the process of applying a function repeatedly, using the output from one iteration as the input to the next which are used to produce approximate numerical solutions to certain mathematical problems. The estimated value of parameters was finally used to predict milk yield. The models were fitted by using the PROC NLIN statement of the statistical package SAS 9.3 version (SAS Institute Inc. 2011. Cary, NC, USA). "How well some specified model fits the data" is explained by goodness of fit *i.e.,* model selection criterion. In this research work the goodness of fit of a model was accessed by;

A. Coefficient of determination (R^2)

$$\mathbf{R^2} = \mathbf{1} - \frac{\text{Residual Sum of Squares}}{\text{Total Sum of Squares}}$$

The value of coefficient of determination (R^2) ranges between 0-1. R^2 of 1 indicates that the regression line perfectly fits the data. The adjusted coefficient of determination (R^2_{adj}) is more comparable than R^2 for model that involves different numbers of parameters. A model with large R^2_{adj} is more fitted.

B. Adjusted coefficient of determination (R^2_{adj})

$$\mathbf{R^2}_{adj} = \mathbf{1} - \frac{\text{MSPE}}{\text{MS(Corrected Total)}}$$

$$R^2_{adj} = 1 - \frac{(1 - R^2)(P - 1)}{n - 1}$$

C. Root Mean Square Error (RMSE);

$$\text{RMSE} = \sqrt{\frac{\sum(Y_i - \overline{Y_i})^2}{n\text{-}p}}$$

D. Mean Absolute Error (MAE);

$$\text{MAE} = \frac{\sum\left|Y_i - \overline{Y_i}\right|}{n}$$

where, Y_i is the milk yield, i =1, 2, 3, 4…, "n" is the number of experimental observations, "P" is the number of parameters, predicted value. The small value of RMSE and MAE indicates better-fitted model.

E. Akaike's Information Criteria (AIC)

$$\text{AIC} = 2P - 2\ln(L)$$

$$\text{AIC} = n \log_e \text{MSE} + 2P$$

If the number of parameters are different than corrected Akaike's Information Criteria (AIC_c) is used instead of Akaike's Information Criteria (AIC)

F. Corrected Akaike's Information Criteria (AIC_c)

$$AIC_c = AIC + \frac{2P(P+1)}{n - P - 1}$$

where, "P" is the number of parameters in the model and "L" is the maximized value of the likelihood function for the model. The preferred model is the one that has a minimum AIC value

G. Bayesian Information Criterion (BIC)

$$BIC = n \log_e MSE + P \log_e (n)$$

The preferred model is the one which have minimum BIC value

Examination of Residuals (Errors)

Residuals or errors are defined as the difference between the observed and predicted value of the response. For modeling purposes, there are two assumptions

(a) The errors are independently and identically distributed i.e., $\boldsymbol{\varepsilon} \sim N(0, 1)$

(b) The errors have constant variance.

In the present study, the assumption may be tested by using the Durbin-Watson Test (presence of autocorrelation) and Shapiro-Wilk Test (normality of the residuals).

7. RESULTS AND DISCUSSION

The study aimed to investigate the best-fitted lactation curve models that describe the lactation pattern of mastitic Vrindavani cattle. The eight models [Inverse quadratic polynomial model (ND), Incomplete Gamma function (WD), Linear decline model (CL), Wilmink model (WL) Mixed log model (ML), Mitscherlich x Exponential (ME), Morant and Gnanasakthy model (MG) and Ali and Schaeffer lactation curve model (AS)] were fitted on DTDMY (Daily test day milk yield) data of 161 mastitic Vrindavani cattle maintained at cattle and buffalo farm (LPM section, IVRI, Izatnagar). The total milk production, peak production and lactation length varied significantly from healthy to mastitic condition ($P < 0.01$). Similar to the present finding, Sinha *et al.* (2014) conducted a study to assess the incidence and economics of subclinical form of bovine mastitis in the central region of India and concluded that the losses due to mastitis in monetary terms were estimated to be INR 1390 per lactation, among which around 49% was owing to loss of value from milk and 37% on account of veterinary expenses. Singh and Singh (1994) estimated the average decrease in milk yield due to clinical and subclinical mastitis to be 50% and 17.5%, respectively and the overall annual economic loss in India due to mastitis was INR 16,072 million. Dohare *et al.,* (2021) studied the impact of mastitis and concluded that the overall prevalence of mastitis at the organized farm was 24.69% and the average monthly production loss due to mastitis was 36.76 kg. Kumar *et al.* (2021) observed that overall prevalence of subclinical mastitis in peri-urban cross-bred cattle was 36.74% (Prevalence of subclinical mastitis was highest (55.77%) in cross-bred cattle in mid-lactation and Parity 3-5) which varied with parity and stage of milking. The summary of recorded milk production is mentioned below (Table 1).

Table 1. Total milk production (305 days), lactation length and peak production of Vrindavani Cattle in healthy and disease condition

Vrindavani cattle	305-day milk production (kg)		Lactation length (days)		Peak production (kg)	
	Mean	SE	Mean	SE	Mean	SE
Healthy Cattle	3257.57	0.13	299.58	0.76	17.98	0.21
Mastitic Cattle	2300.84	0.14	242.14	9.70	11.18	0.50

7.1 Lactation curve of Vrindavani cattle suffered from mastitis

The ML model was best fitted model to describe the DTDMY (Daily test day milk yield) data of mastitic Vrindavani cattle ($R^2_{(adj)} = 0.9930$, MAE = 0.1576, RMSE = 0.0397, AICc = -1962.0 and BIC = -1950.9). AS was 2nd best fitted model ($R^2_{(adj)} = 0.9929$, MAE = 0.1571, RMSE = 0.0586, AICc = -1720.1 and BIC = -1701.6) followed by CL model ($R^2_{(adj)} = 0.9879$, MAE = 0.1842, RMSE = 0.0639, AICc = -1671.6 and BIC = -1660.5) and ND model was least fitted to describe the DTDMY (Daily test day milk yield) data of mastitic Vrindavani cattle ($R^2_{(adj)} = 0.9487$, MAE = 0.4251, RMSE = 0.1140, AICc = -1318.5 and BIC = -1307.3). The value of parameters of different lactation curves are mentioned in Table 2. The pattern of fortnight test day milk record of mastitic Vrindavani cattle and predicted milk production (by fitted model) are graphically represented. (Fig. 1). Kshandakar *et al.* (2018a) observed that for healthy Multiparous Vrindavani cattle and Vrindavani cattle calving in the winter season Mixed log model was best fitted to DTDMY records. Kshandakar *et al.* (2018a) also concluded that Mitscherlich cum Exponential model was best fitted to DTDMY records of Primiparous Vrindavani cattle and Vrindavani cattle calving in summer and rainy season. Incomplete gamma function was the least fitted model to describe the lactation behavior (Kshandakar *et al.* (2018a).

Similar to the present finding, Dongre *et al.* (2012) concluded that the Mixed log function was the best-fitted lactation curve models (R^2 = 0.8875 and RMSE = 0.08 Kg) that describe the pattern of milk production of first lactating Sahiwal cows maintained at NDRI, Karnal. Previously, Pandey (2007) compared four lactation curve models on weekly and monthly milk yield data of Vrindavani cattle and concluded that Gamma function was best fitted than other three (Quadratic, Parabolic Exponential, Inverse polynomials) models. Similar to the present study, Kshandakar *et al.* (2018b) model the effect of disease on the lactation curve and concluded that the AS model is best fitted to describe the lactation pattern of mastitic Murrah buffaloes.

The peak production and total milk production were predicted by different functions (Table 3). The peak production and total milk production predicted by the different functions are almost close to observed milk production but the ND model over-predicted the peak production.

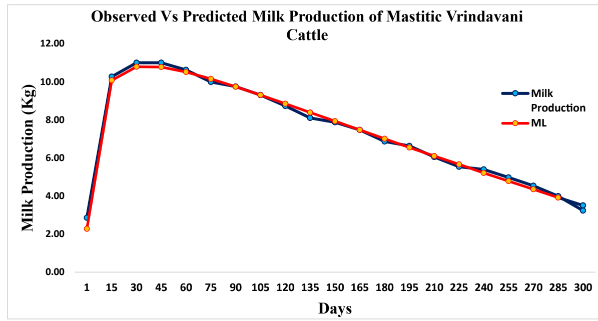

Fig. 1. Observed Vs predicted (by best-fitted model) milk production of mastitic Vrindavani cattle

Table 2. Estimated value of parameters of different lactation curve models of Vrindavani cattle suffering from mastitis along with different measures of goodness of fit

Model	Parameter	Mean	S.E.	R^2(adj)	MAE	RMSE	AICc	BIC
AS	a	14.473	0.544	0.9929	0.1571	0.0586	-1720.1	-1701.6
	b	-14.627	0.965					
	c	3.683	0.458					
	d	-0.154	0.281					
	e	-0.353	0.036					
CL	a	12.121	0.034	0.9879	0.1842	0.0639	-1671.6	-1660.5
	b	0.029	0.000					
	c	0.167	0.003					
ME	a	12.228	0.030	0.9919	0.1607	0.0499	-1820.1	-1805.3
	b	0.792	0.014					
	c	0.124	0.003					
	d	0.029	0.000					
MG	a	2.073	0.003	0.9909	0.1689	0.1056	-1363.3	-1348.6
	b	0.411	0.003					
	c	0.075	0.003					
	d	2.424	0.065					
ML	a	3.762	0.088	**0.9930**	**0.1576**	**0.0397**	**-1962.0**	**-1950.9**
	b	-1.481	0.009					
	c	4.451	0.039					
ND	a	0.512	0.024	0.9487	0.4251	0.1140	-1318.5	-1307.3
	b	0.054	0.001					
	c	0.001	0.000					
WD	a	5.036	0.088	0.9839	0.2115	0.1284	-1246.0	-1234.9
	b	0.275	0.005					
	c	0.006	0.000					
WL	a	12.611	0.048	0.9799	0.2194	0.1683	-1081.0	-1069.9
	b	-7.250	0.155					
	c	-0.031	0.000					

Table 3. Descriptive Statistics of Predicted Milk Production by different Model of mastitic Vrindavani Cattle

Model	Predicted Milk Production	
	Peak Production	**Total Production**
AS	10.86	2300.86
CL	11.22	2298.44
ME	11.12	2300.84
MG	10.84	2299.54
ML	10.83	2300.81
ND	11.52	2308.79
WD	10.84	2300.97
WL	10.84	2300.89
Observed Milk Production	11.18	2300.84

Table 4. Test for the presence of autocorrelation and normality of residual in mastitic Vrindavani cattle by different lactation curve models

Model	Durbin-Watson	Kolmogorov-Smirnov[a]		Shapiro-Wilk	
	Statistics	**Statistics**	**Sig.**	**Statistics**	**Sig.**
AS	0.8140	0.028	0.200	0.986	0.004
CL	0.4031	0.089	<0.001	0.964	<0.001
ML	0.7931	0.034	0.200*	0.996	0.655
ME	0.6327	0.050	0.061	0.991	0.053
MG	0.6246	0.052	0.045	0.917	<0.001
ND	0.0923	0.096	<0.001	0.959	<0.001
WD	0.3252	0.075	<0.001	0.872	<0.001
WL	0.3067	0.115	<0.001	0.798	<0.001

In case of mastitic Vrindavani cattle milk production, the Kolmogorov-Smirnova test showed that the test statistics obtained from residual of AS and ML function are significant, but the Shapiro-Wilk test statistics obtained from residual of different function were non-significant *i.e.,* residuals were normally distributed. The DW statistics value range from 0.0923 to 0.8140 *i.e.,* the residuals were positively autocorrelated. The residuals obtained from different functions were plotted graphically (Fig. 2) and test statistics are mentioned in Table 4.

8. CONCLUSION

The dairy sector is contributing significantly to alleviating poverty by providing regular income and reducing malnutrition, particularly in rural areas. As animal breeders are interested in production during all stages of lactation, the lactation curve provides adequate information regarding daily, monthly, total milk production patterns. The present study shows

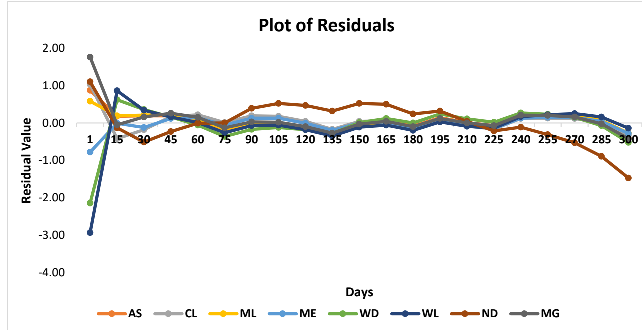

Fig. 2. Plot of residuals for mastitic Vrindavani cattle by different lactation curve models

the significant difference in milk production, peak production and lactation length of healthy to mastitic animals and the ML model was the best model to describe the production pattern of mastitic Vrindavani cattle. The mastitic Vrindavani cattle recorded 29.39% less milk production with respect to healthy animals. The peak production and lactation length of mastitic Vrindavani cattle were also decreased significantly.

ACKNOWLEDGEMENTS

Authors are highly thankful to the In-charge, Livestock Production and Management Section, ICAR-IVRI, Izatnagar, Bareilly, Uttar Pradesh for permission to use the data for the present study. The authors are also thankful to the Director ICAR-IVRI, Izatnagar (U.P.) for providing the necessary facilities for the execution of this study. The authors are highly thankful to the learned reviewer for his valuable comments on the original version of the paper.

REFERENCES

Adediran, S.A., Ratkowsky, D.A., Donaghy, D.J., Malau-Aduli, A.E. (2012). Comparative evaluation of a new lactation curve model for pasture-based Holstein-Friesian dairy cows. *Journal of Dairy Science*, **95**(9), 5344-5356.

Ali, T.E. and Schaeffer, L.R. (1987). Accounting for covariances among test day milk yields in dairy cows. *Canadian. Journal of Animal Science*, **67**, 637-644.

Brody, S., Ragsdale, A.C., Turner, C.W. (1923). The rate of decline of milk secretion with the advance of the period of lactation. *Journal Genetics Physiology,* **5**, 441-444.

Brody, S., Turner, C.W. and Ragsdale, A.C. (1924). The relation between the initial rise and subsequent decline of milk secretion following parturition. *The Journal of General Physiology*, **6**, 541-545.

Cobby, J.M. and Le Du, Y.L.P. (1978). On fitting curves to lactation data. *Animal Production*, **26**, 127-133.

Dohare, A.K., Yogesh, C.B., Sharma., V.B. and Verma., M.R. (2021). Modelling the effect of mastitis on milk yield in dairy cows using covariance structures fitted to repeated measures. *Indian Journal of Animal Research*, **55**(1), 11-14.

Dongre, V.B., Gandhi, R.S. and Singh, A. (2012). Comparison of different lactation curve models of Sahiwal cows. *Turkish Journal of Veterinary & Animal Sciences*, **36**(6), 723-726.

Green, P.J. and Silverman, B.W. (1993). Nonparametric regression and generalized linear models: a roughness penalty approach. CRC Press

Guo, Z. and Swalve, H.H. (1997). Comparison of different lactation curve sub-models in test day models. Interbull Bulletin 16 (Proceedings of the 1997 Interbull meeting. Vienna. Austria. 75-79.

Halasa, T., Huijps, K., Osteras, O. and Hogeveen, H. (2007). Economic Effects of Bovine Mastitis and Mastitis Management: A Review. *Veterinary Quarterly*, **29**(1), 18-31.

Kirkpatrick, M., Hill, W.G. and Thompson, R. (1994). Estimating the covariance structure of traits during growth and ageing, illustrated with lactation in dairy cattle. *Genetics Research*, **64**(1), 57-69.

Kshandakar, S., Verma, M.R., Singh, Y.P., and Kumar, S. (2017). Modeling the effect of metabolic diseases on lactation curves of Vrindavani cattle. *Int. Arch. App. Sci. Technol.*, **4**, 58-65.

Kshandakar, S., Verma, M.R., Singh, Y.P., and Kumar, S. (2018a). Effect of parity and calving season on modelling of lactation curves of Vrindavani cattle. *The Pharma Innovation Journal*, **7**(9), 185-193.

Kshandakar, S., Verma, M.R., Singh, Y.P., Kumar, S and Paul, A.K. (2018b) Effect of clinical mastitis on lactation curves of Murrah buffaloes. *Indian Journal of Animal Sciences*, **88** (5), 585–592

Kumar, P., Kumari, R.R., Kumar, A., Raman, R.K., Chandran, P.C. and Kumar, M. (2021). Status of subclinical mastitis in crossbred cattle of peri-urban unorganized herd of middle indo-gangetic plains. *Indian Journal of Animal Research,* **55**(12), 1468-1475.

Morant, S.V. and Gnanasakthy, A. (1989). A new approach to the mathematical formulation of lactation curves. *Animal Science*, **49**(2), 151-162

Murphy, M.D., O'Mahony, M.J., Shalloo, L., French, P. and Upton, J. (2014). Comparison of modelling techniques for milk production forecasting. *Journal of Dairy Science*, **97**, 3352-3363.

Nelder, J.A. (1966). Inverse polynomials, a useful group of multifactor response functions. *Biometrics*, **22**(1): 128-141.

Pandey, H.O. (2007). Studies on first lactation curve and preliminary sire evaluation in vrindavani cows and murrah buffaloes. Doctoral Dissertation submitted to ICAR-IVRI (Deemed University).

Rook, A.J., France, J. and Dhanoa, M.S. (1993). On the mathematical description of lactation curves. *Journal of Agricultural Science*, **121**, 97-102.

Ruelle, E., Delaby, L. Wallace, M. and Shalloo, L. (2016). Development and evaluation of the herd dynamic milk model with focus on the individual cow component. *Animal*, **10**(12): 1986-1997.

SAS Institute. (2011). SAS/STAT User's Guide (8.2). SAS Institute Inc., Cary, NC.

Sikka., L.C. (1950). A study of lactation as affected by heredity and environment. *Journal of Dairy Research,* **17**, 231-252.

Singh, P.J. and Singh. P.B. (1994). A study of economic losses due to mastitis in India. *Indian Journal of Dairy Science*, **47**, 265-272.

Singh, R.R., Dutt, T, Kumar, A. Tomar, A.K.S. and Singh, M. (2011) On-farm characterization of Vrindavani cattle in India. *Indian Journal of Animal Sciences*, **81**(3), 267-271.

Sinha, M.K., Thombare, N.N., and Mondal, B. (2014). Subclinical mastitis in dairy animals: incidence, economics, and predisposing factors. *The Scientific World Journal* Article ID 523984 | https://doi.org/10.1155/2014/523984.

Thirunavukkarasu, M.J. and Rajarathinam, A. (2018). Stochastic modelling for milk production. *Research Journal of Animal Husbandry and Dairy Science*, **9**(2), 26-35.

Wilmink, J.B.M. (1987). Comparison of different methods of predicting 305-day milk yield using means calculated from within-herd lactation curves. *Livestock Production Science*, **17**, 1-17.

Wood, P.D.P. (1967). Algebraic model of the lactation curve in cattle. *Nature,* **216**, 164-165.