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# Mitigating Autocorrelation in Richards Growth Model Analysis using Incremental Growth Data with Application to Turkey Growth

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**SUMMARY**

The Richards curve is frequently used to model cumulative growth in science. Recent studies have shown that the resulting residuals in regression applications fitting the Richards curve are autocorrelated. Such autocorrelation has typically not been taken into account in the regression model, which calls into question the subsequent statistical inferences based on the model. This paper shows how fitting the derivative function of the Richards curve to the incremental weight gains, i.e. to the 'first differences', may mitigate autocorrelation inherent in growth data. The procedure is illustrated on turkey growth data from Syria. This new approach should apply to growth data in general, and to suitable poultry growth data in particular.

*Keywords*: Logistic growth model, Power-law logistic model, Durbin-Watson test, First differences.

## 1. INTRODUCTION

The Richards growth curve, also known as the power-law logistic curve (Banks 1994), is widely used to model the growth of natural populations (Seber and Wild 1989). The curve has recently found particularly widespread application for modeling growth curves of poultry, including not only turkey (Sengul and Kiraz 2005, Ersoy *et al*. 2006) and chicken (Norris *et al.* 2007), but also partridge (Cetin *et al*. 2007), duck (Knizetova *et al*. 1991), goose (Knizetova *et al*. 1994), emu (Goonewardene *et al*. 2003) and quail (Hyankova *et al*. 2001). It also has been used to model the growth in weight of species, such as cattle and elk (Goonewardene *et al*. 2003). The Richards curve was shown in these papers to fit data from these species apparently adequately, using the standard regression model assuming independent errors. However, many of the more recent papers (e.g. Sengul and Kiraz 2005, Cetin *et al*. 2007, Norris *et al*. 2007) recognize that the residuals from the regression analysis indicate that the error terms are not independent, as assumed, but rather are serially correlated. Even aside for any formal statistical testing, one would have to assume logically that consecutive cumulative weight data over time, a natural time series, would be autocorrelated.

When autocorrelation is present, in violation of the usual assumption of independence in a regression model, the estimated standard errors of the parameter estimates would be underestimated (Neter *et al*. 1996, Franses 2002, Lindsey 2004). Consequently, subsequent statistical inferences involving these standard error estimates would be subject to question. The commonly recommended solution to this problem is to append some assumed correlated error structure onto the model (see e.g. Berny 1989, Glasbey 1979, Seber and Wild 1989). However doing so adds complexity to the analysis, and choosing a suitable model for the error structure is no trivial task.

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The Richards curve is one of a number of commonly used growth curves for poultry, two others being the logistic and the Gompertz curve. These other curves would also be expected to exhibit autocorrelation when fitted to data. Matis *et al.* (2010) recently outlined a procedure for mitigating autocorrelation resulting from fitting the logistic model to cumulative count data. The procedure is based on transforming the data to first differences, and then fitting the derivative of the logistic model to these incremental changes. This paper applies this first difference methodology to the Richards curve.

Section 2 develops the Richards curve from a mechanistic basis, and reviews some of the pertinent properties of the curve. The procedure is illustrated with some turkey growth data from Mohammed *et al.* (2010). Section 3 develops the first difference procedure for the Richards curve, and illustrates this new methodology by applying it to the turkey growth data. Concluding remarks are given in Section 4.

## 2. REVIEW OF THE RICHARDS GROWTH MODEL

### 2.1 A Derivation and Some Properties

The Richards curve is typically given in the form

$$Y(t) = \frac{K}{[1 + Be^{(-Ct)}]^D} \tag{1}$$

with parameters $K$, $B$, $C$, $D > 0$. For subsequent purposes, we outline a mechanistic derivation of the model, given in Matis and Kiffe (2000, p. 50).

Let $Y(t)$ denote population size at time $t$, and $Y'(t)$ its derivative. Consider the mechanistic model

$$Y'(t) = aY(t) - bY(t)^{s+1} \tag{2}$$

with parameters $a$, $b$ and $s > 0$. Parameter $a$ in the linear term is called the 'intrinsic rate of growth', and parameter $b$ in the 'density-dependent' term is the coefficient for slowing the growth rate. The solution, often called the power-law logistic, is given in Banks (1994, p.108) in the form:

$$Y(t) = \frac{K}{[1 + Be^{(-ast)}]^{1/s}} \tag{3}$$

where

$$K = (a/b)^{1/s} \text{ and} \qquad B = (K/Y_0)^s - 1 \tag{4}$$

with $Y_0 = Y(0)$. Some investigators state the Richards curve in form (3). Clearly (3) is equivalent to (1) with $C = as$ and $D = 1/s$, hence the intrinsic rate $a$ in (2) is

$$a = C \cdot D\,. \tag{5}$$

That $K$ in (4) is the equilibrium population size, also called the 'carrying capacity', is easily seen by setting $Y'(t) = 0$ in (2). Many of the poultry papers focus on the point of inflection, denoted $t_{inf}$. One can show that

$$t_{inf} = [\log_e(BD)]/C. \tag{6}$$

The population size at this time, $Y_{inf}$, is

$$Y_{inf} = K/\,(1 + D^{-1})^D. \tag{7}$$

Note that for the (ordinary) logistic growth curve, with $D = 1$, the point of inflection occurs at population size $K/2$. This is a useful property for model discrimination, often even visually. As an example, it is visually apparent that the point of inflection for some growth curves of the size of certain muskrat populations in the Netherlands, given in Matis *et al.* (1997), exceeds the midpoint $K/2$. Therefore the Richards curve was used as an alternative to the logistic curve to describe the growth of these muskrat populations.

### 2.2 Statistical Regression Model

We fitted the Richards curve in (1) to data sets using standard nonlinear least squares (Neter *et al.* 1996), as implemented in SPSS (2007). Letting $y(t)$ denote the observed (mean) population weight at time $t$, it is standard practice to assume regression model

$$y(t) = Y(t) + \varepsilon \tag{8}$$

where $\varepsilon$ denotes an *independent* random error term with constant variance.

We note, however, that an observed population growth curve is a time series, and due to its cumulative nature, its observations are likely to be serially correlated. Serial correlation, also called autocorrelation, violates the assumption of independent observations in regression model (8). A consequence of such violation is that the estimated standard errors of the parameter estimates from standard nonlinear least squares would be too small, indicating greater apparent precision than warranted (Neter *et al.* 1996). Therefore, after fitting model (8) to data, we test for serial correlation using the standard Durbin-Watson $d$ statistic. The expected value of $d$ under the null hypothesis of no serial correlation is $d = 2$ (Neter *et al.* 1996).

The Richards curve is challenging to fit to data as it has four parameters, $K$, $B$, $C$, and $D$ (Seber and Wild

1989). Ghosh *et al.* (2011) reviews the general estimation problem for the Richards curve, and proposes a new method for parameter estimation when there are sufficient data. We have utilized a simplified fitting procedure in previous applications of the power-law logistic model by fixing $D$ at assumed integer values (see e.g. Matis *et al.* 1997). One could implement this practical iterative procedure with the present data as follows: 1. Fix $D = 1$ and fit the (ordinary) logistic curve, and 2. Continue fixing $D$ at successively larger integer values, always fitting the resulting conditional Richards curve with three parameters, until the mean squared residual (MSR) is reduced to an acceptable value.

### 2.3 Examples with Cumulative Turkey Growth Data

The Richards curve has been applied successfully to turkey growth data in the literature. Sengul and Kiraz (2005) fits the curve to cohorts of 144 male and of 144 female white turkeys from birth to 18 weeks of age, and estimates $D$ to be 10.0 and 11.1, respectively. Ersoy *et al.* (2006) fits the curve to cohorts of 41 male and 62 female American Bronze turkeys from 11 to 24 weeks of age, and estimates $D$ to be 18.83 and 18.24, respectively. These papers followed the generally accepted practice of assuming regression model (8).

Mohammed *et al.* (2010) gives data on the observed growth curves for a flock of 2000 male turkeys raised starting in May 2008 in Syria. Random samples of size 100 were taken from the flock weekly starting at age 7 to age 24 weeks. Data on the mean weights of the samples are given in Table 1 and illustrated in Fig. 1. This figure also illustrates the curve with $D = 1$ in (1) (i.e. the ordinary logistic curve) fitted to the growth data set assuming regression model (8). The curve fits the data set well visually, with MSR = 123288. The parameter estimates (with standard errors) are: for $K$, 26504 (827); for $B$, 23.0 (1.7); and for $C$, 0.200 (0.009). The fitted curve overestimates the weights at the initial times, which suggests that the logistic curve overestimates the point of inflection. Therefore we proceeded to implement the iterative process for $D$ = 5, 10, 50, 100, 500 and 1000. The mean squared residual (MSR) and parameter estimates for this sequence of Richards curves are given in Table 2. The MSR decreases monotonically with increasing $D$. Because the decrease in MSR is minimal after $D = 100$, and to be somewhat consistent with previous results in the literature, we deemed the model with $D = 100$ to give an adequate fitting for the present data.

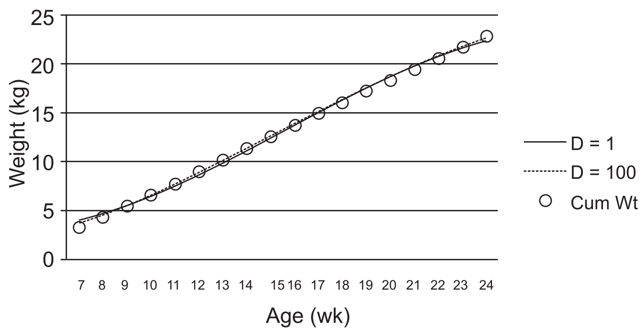


**Fig. 1**. Observed Weight (kg) vs Age (wk) with Fitted Richards Curves for $D = 1$ (solid line) and $D = 100$ (dashed line)

The fitted curve with $D = 100$ is also given in Fig. 1. Though the curve with $D = 100$ does not differ by much visually from that with $D = 1$, the Richards

**Table 1.** Mean Cumulative and Incremental Growth Weights

| Age (wk) | Cumulative Wt (gm) | Age (wk) | Cumulative Wt (gm) | Midpoint (wk) | Increment Wt (gm) | Midpoint (wk) | Increment Wt (gm) |
|---|---|---|---|---|---|---|---|
| 7 | 3195 | 16 | 13685 | 6.5 | 870 | 15.5 | 1195 |
| 8 | 4245 | 17 | 14900 | 7.5 | 1050 | 16.5 | 1215 |
| 9 | 5450 | 18 | 16000 | 8.5 | 1205 | 17.5 | 1100 |
| 10 | 6580 | 19 | 17170 | 9.5 | 1130 | 18.5 | 1170 |
| 11 | 7700 | 20 | 18270 | 10.5 | 1120 | 19.5 | 1075 |
| 12 | 8934 | 21 | 19425 | 11.5 | 1234 | 20.5 | 1155 |
| 13 | 10100 | 22 | 20500 | 12.5 | 1166 | 21.5 | 1075 |
| 14 | 11310 | 23 | 21630 | 13.5 | 1210 | 22.5 | 1130 |
| 15 | 12490 | 24 | 22755 | 14.5 | 1180 | 23.5 | 1125 |

**Table 2.** Mean Squared Residual, MSR, and Parameter Estimates for Fitted Richards Curves

| A. From Cumulative Growth Data | | | | | | |
|---|---|---|---|---|---|---|
| $D$ | MSR | $K$ | $B$ | $C$ | $t_{inf}$ | $Y(t_{inf})$ |
| 1 | 123288 | 26504 | 22.99 | 0.200 | 15.58 | 13126 |
| 5 | 46105 | 30968 | 1.255 | 0.122 | 15.05 | 12440 |
| 10 | 38248 | 31966 | 0.536 | 0.113 | 14.86 | 12327 |
| 50 | 32367 | 32897 | 0.094 | 0.105 | 14.82 | 12222 |
| 100 | 31659 | 33027 | 0.046 | 0.104 | 14.81 | 12214 |
| 500 | 31097 | 33126 | 0.009 | 0.103 | 14.79 | 12192 |
| 1000 | 31026 | 33139 | 0.005 | 0.103 | 14.79 | 12120 |

| B. From Incremental Growth Data | | | | |
|---|---|---|---|---|
| $D$ | MSR | $X_{max}$ | $t_{max}$ | $C$ |
| 1 | 4505 | 1194 | 15.63 | 0.089 |
| 5 | 4141 | 1198 | 15.23 | 0.070 |
| 10 | 4075 | 1198 | 15.16 | 0.068 |
| 50 | 4017 | 1199 | 15.10 | 0.065 |
| 100 | 4009 | 1199 | 15.09 | 0.065 |
| 500 | 4003 | 1199 | 15.08 | 0.065 |
| 1000 | 4002 | 1199 | 15.08 | 0.065 |

curve tends to have uniformly smaller residuals. These residuals yield MSR = 31659, which represents a large (over 74%) reduction in MSR as compared to the $D$ = 1 model. The estimated parameter values for the fitted curve with $D$ = 100 are: for $K$, 32023 (960); for $B$, 0.046 (0.001); and for $C$, 0.104 (0.004). Though the MSR is reduced substantially, the three pairwise correlations between the parameter estimates are larger in every case for the fitted model with $D$ = 100. This leads to mild multicollinearity, which tends to inflate the standard errors. Nevertheless the parameter estimates for the model with $D$ = 100 generally tend to have smaller comparable standard errors than for $D$ = 1. If the assumed regression model in (8) were correct, one could use these parameter estimates with their standard errors from the curve with $D$ = 100 for subsequent statistical inferences, e.g. for confidence intervals for the parameters.

The residuals for the fitted logistic growth curve, with $D$ = 1, are illustrated in Fig. 2. It is easy to recognize visually that autocorrelation is present, as there is a strong pattern with only 4 runs (i.e. changes of sign). The Durbin-Watson test statistic is $d$ = 0.34, which is highly significant (as $d < d_{L,0.05}$ = 1.05). The residuals from the Richards curve with $D$ = 100 are also given in figure. These residuals tend to be noticeably smaller in absolute value, resulting in the reduced MSR, however autocorrelation is again striking as this curve also has only 4 runs. The Durbin-Watson statistic is $d$ = 0.42, which is just a bit larger than before but still well below $d_{L,0.05}$ = 1.05, and hence highly significant statistically. It is clear therefore that the independent error assumption in model (8) is not tenable for the Richards curve, and hence that the statistical inferences based on these estimates are not valid. Other investigators (e.g. Norris *et al.* 2007) have pointed out the same problem of serial correlation without offering any solution.

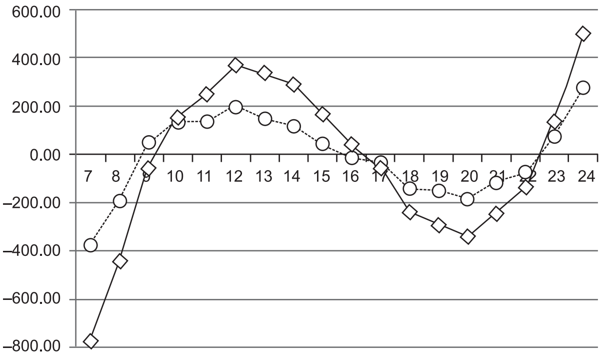


**Fig. 2.** Residuals (gm) vs Age (wk) from Fitted Richards Curves with $D$ = 1 (solid line) and $D$=100 (dashed line)

## 3. USE OF THE DERIVATIVE MODEL WITH FIRST DIFFERENCES

### 3.1 First Differences Procedure

A simple and well-known procedure for analyzing time series data in economics is to transform the data by taking "first differences" (Neter *et al.* 1996). In this procedure, one calculates the differences between consecutive values of the independent variable and also the differences between consecutive values of the dependent variable. Under certain conditions, most notably under a first-order autoregressive error assumption, the residuals from a regression model relating these first difference variables would no longer be serially correlated (Neter *et al.* 1996).

An adaptation of this standard procedure is proposed for the logistic growth model in Matis *et al.* (2010). With $y(t)$ denoting the observed cumulative weight, let $t_{mid}$ denote the average of consecutive times $t$ and $(t + 1)$, i.e $t_{mid} = (2t + 1)/2$. Let $x(t_{mid})$ denote the difference between the consecutive weights at their midpoint, i.e. $x(t_{mid}) = y(t + 1) - y(t)$. These first differences of the cumulative weights for *equally spaced data* are proportional to the estimated derivatives of the function, and hence the differences were fitted to the derivative of the logistic curve. The procedure is shown to mitigate the autocorrelation problem for the logistic model in Matis *et al*. (2010). We now investigate its application to the Richards curve. We derive the derivative model for the Richards curve, apply it to the first difference data, and then test whether the procedure reduces the autocorrelation previously observed in the cumulative data.

### 3.2 The Derivative Model

Let $X(t)$ denote the derivative of $Y(t)$. It follows from (1) that

$$X(t) = \frac{K \cdot B \cdot C \cdot D \cdot e^{-C \cdot t}}{(1 + B \cdot e^{-C \cdot t})^{D+1}} \tag{9}$$

Let $t_{max}$ denote the time at which $X(t)$ reaches its maximum. One can show that

$$t_{max} = \log (BD)/C, \tag{10}$$

which is equivalent to the point of inflection, $t_{inf}$, in (6) for the $Y(t)$ cumulative weight model. The value at $t_{max}$, denoted $X_{max}$, is

$$X_{max} = \frac{K \cdot C}{(1 + D^{-1})^{D+1}} \tag{11}$$

Model (8) can be reparameterized by substituting (10) and (11) into (9) to yield

$$X(t) = \frac{(1 + D^{-1})^{D+1} \cdot X_{max} \cdot e^{-C(t-t_{max})}}{(1 + e^{-C(t-t_{max})} / D)^{D+1}} \tag{12}$$

Though (12) is less elegant than (9), response function (12) has naturally interpretable parameters, $X_{max}$ and $t_{max}$. The other parameter, $C$, is a rate constant related to the intrinsic rate of growth in (5). This parameterization is useful for iterative nonlinear least squares procedures, as reasonable initial values for $X_{max}$ and $t_{max}$ are easy to obtain by inspection of the data. The parameters are also stable and hence readily estimable (Ross *et al*. 2010). Function (12) with $D = 1$ reduces to the logistic density function form of the model given in Matis *et al*. (2010).

Derivative function (12) can be fitted to the first differences of the previous data sets, assuming again standard regression model

$$x(t) = X(t) + \varepsilon \tag{13}$$

where as before $\varepsilon$ denotes an *independent* random error term with constant variance. The derivative model $X(t)$ can be implemented again using the iterative process on $D$, and the assumption of independent errors can again be tested with the Durbin-Watson $d$ statistic.

### 3.3 An Application to Incremental Turkey Growth Data

The incremental weight gains are plotted in Fig. 3A. These first differences constitute a much better diagnostic tool for model lack-of-fit. Note, for example, that the weight gains at $t = 6.5$ and $t = 8.5$ stand out in the figure and might merit further investigation. Such unusual observations, which may be of scientific interest, tend to be masked in the cumulative data model. For example, these unusual weight gains are hardly noticeable in the cumulative graphs in Fig. 1.

The model with $D = 1$, i.e. the logistic derivative model, fits the data set well, with MSR = 4505. The parameter estimates are: for $X_{max}$, 1194 (24.3); for $t_{max}$, 15.63 (0.75); and for $C$, 0.089 (0.014). However the Richards derivative models with $D > 1$ fit better, with the curve with $D = 100$ reducing the MSR to 4009. This value appears from Tables 2 to be very close to the global minimum, and hence the curve with $D = 100$ is regarded as adequate for the data. The parameter estimates for this fitted curves are: for $X_{max}$, 1198 (22.7); for $t_{max}$, 15.09 (0.68); and for $C$, 0.065 (0.009). Because the parameter estimates are stable, there is no problem with multicollinearity, as the largest of the three correlation coefficients between parameter estimates is only 0.711, as opposed to the case for the cumulative weights, for which all three correlation coefficients exceed 0.810.

The residuals for the fitted derivative curve with $D = 1$ are plotted in Fig. 3B. The Durbin-Watson statistic is $d = 1.67$, which indicates that there is no evidence of serial correlation (as $d_{U,\,0.05} = 1.53$). The residuals for the fitted curves with $D = 100$ are also plotted in Fig. 3B, and are a bit smaller on average than those for $D = 1$, leading to the reduced MSR. The Durbin-Watson statistic is $d = 1.78$ which also indicates the lack of serial correlation.

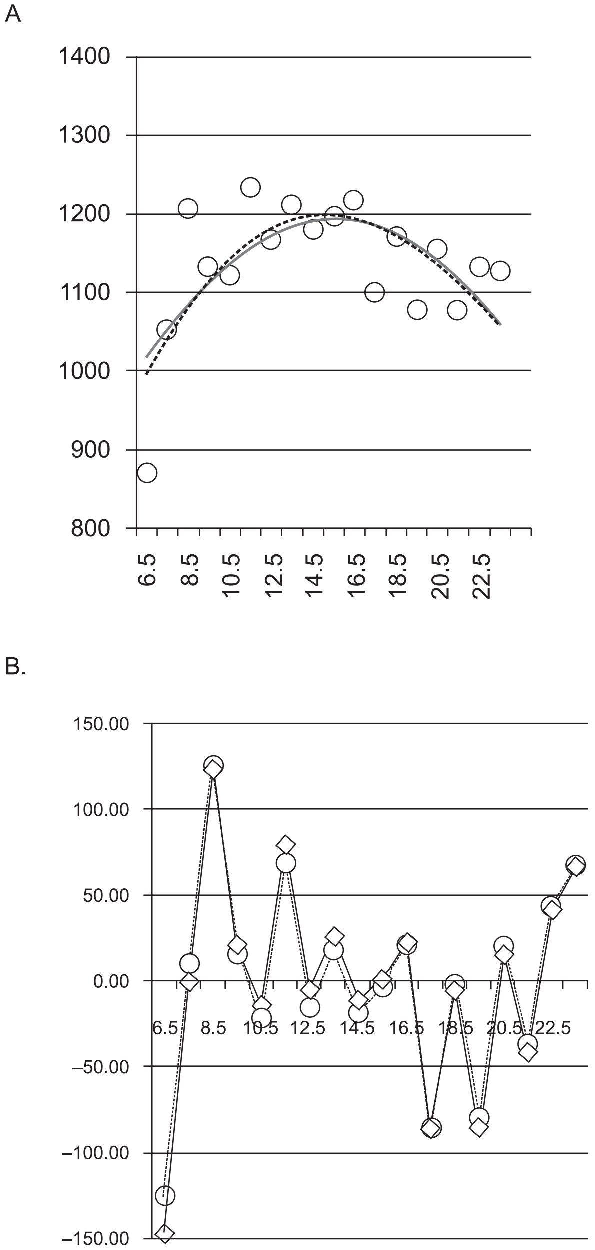


**Fig. 3.** A. Observed Incremental Weight Gains (gm) vs Age (wk) with Fitted Richards Derivative Functions for $D = 1$ (solid line) and $D = 100$ (dashed line). B. Residuals (gm) vs Age (wk) from Fitted Functions with $D = 1$ and $D = 100$.

## 4. DISCUSSION

### 4.1 General Comparisons of Cumulative and Incremental Weight Gain Approaches

It is clear in this example with turkey data that the use of the derivative models with the first differences has achieved the objective of reducing autocorrelation observed in the Richards model with the cumulative data. This result is immediately apparent visually by comparing the distinct pattern of residuals in Fig. 2 to the apparent lack of pattern in Fig. 3B. One could make statistically valid comparisons using this first differences approach, due to the absence of significant autocorrelation, e.g. to investigate whether there are gender differences in the underlying parameters, such as $X_{max}$.

We *do not* claim that every (cumulative) Richards growth curve application results in significant serial correlation. However we *do* suggest that sizeable "random errors" might occur logically at various times in the growth cycle due to a number of factors, and that serial correlation would result naturally as such perturbations are accumulated over time. This is apparent in our turkey data, and probably as well as in the other instances of the problem pointed out in the literature. Similarly, we *do not* claim that this procedure will always eliminate problematic serial correlation which might be present in the cumulative data. For example, we also investigated the growth data for two breeds of native chickens displayed in Norris *et al.* (2007), which were noted to have serial correlation. Based on our approximate readings from their graphs, the Durbin-Watson statistics for the residuals, from the Richards curve with $D = 100$ fitted to the cumulative data, for these breeds are $d = 0.54$ and $d = 0.47$, which clearly indicate serial correlation. The corresponding test statistics fitting model (12) to the first differences are $d = 2.76$ and $d = 0.58$. The value in the first case rejects serial correlation, as expected, but the value in the second case indicates that serial correlation is still present.

However we *do* propose fitting model (12) to the first differences as a general method for growth curve analysis. It is reasonable to assume that the $x(t)$ weight gains in model (13) could be independent. Under that assumption, however, the $y(t)$ cumulative sizes *could not* be independent, as assumed in (8), as each $y(t)$ is the sum of the past $x(t)$. Though we do not claim that the residuals of $x(t)$ will be uncorrelated, one can easily investigate *empirically* for any given data set whether the serial correlation has been substantially reduced under this transformation. The important point is that *when there is serial correlation in the analysis of the* $y(t)$ *cumulative sizes, the* $x(t)$ *incremental weight gains fitted to the new derivative model, which retains equivalent parameters, are likely to have far less serial correlation*.

### 4.2 Some General Considerations using the First Differences Approach

This paper considers the common case where the $y(t)$ observations are equally spaced over time. In cases where that is not so, one could use a slight variation in which the dependent variable is the estimated rate of change, say $r(t) = x(t) / \Delta t$, where $\Delta t$ is the size of the time interval. Derivative model (12) could be fitted directly to the $r(t)$.

Parameter $D$ is usually difficult to estimate as mentioned previously. Although it is estimable in theory, the estimation of $D$ is usually ill-conditioned in practice. As noted in Seber and Wild (1989, p 336), "Curves with quite different ($D$ values) look very similar, and we would anticipate difficulty in distinguishing between them with even small amounts of scatter". This is apparent in the present example, because as apparent in Table 2, large increases in $D$ result in only tiny decreases in MSR. With such a flat MSR surface, any estimate of $D$ would have enormous standard error. In these two problems, one might note that as $D$ approaches infinity, the point of inflection, $Y_{inf}$ in (7) approaches $K/e$.

### 4.3 Summary of Case for using First Differences

If experimenters were aware of both the cumulative and the incremental model approaches, they might prefer the cumulative model approach using the Richards curve in (1) because its graphs obviously tend to be smoother. We summarize the broader case for using the derivative model in (12) as an alternative to the cumulative model as follows:

1. *Model "lack-of-fit" is more apparent with the incremental data than with the cumulative data, making the former a better diagnostic tool for finding outlying or unusual observations which may have heuristic value and also suggest areas for future model refinements.*
2. *Model (12) for the incremental data is based on naturally interpretable parameters, and initial estimates of $X_{max}$ and $t_{max}$ are usually apparent in the data. These parameters are also stable, which yields relatively precise estimates.*
3. *When there is an indication of serial correlation in the residuals from the cumulative model, the residuals from the incremental model may have far less serial correlation. Statistical tests should be used to verify this assertion, and if it is correct, a data analysis based on the incremental model should be considered.*

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