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Block Design for Two-Level Factorial Experiments in Block Size Four

Anurag Rawat[1,2], Sukanta Dash[1], Rajender Parsad[1] and Kaushal Kumar Yadav[1,2]
[1]*ICAR-Indian Agricultural Statistics Research Institute, New Delhi*
[2]*The Graduate School, ICAR-Indian Agricultural Research Institute, New Delhi*

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SUMMARY

In experimental scenarios characterized by one source of heterogeneity within the experimental material, block designs offer significant value. Exploring the optimal replication(s) required for factorial experiments, conducted in blocks of size four has garnered significant attention among researchers. While experiments in blocks of size two have been extensively studied, there is growing recognition that experiments in blocks of size four might offer greater utility in practical applications. Particularly, when estimating main effects and specific two-factor interactions from two-level factorial experiments conducted within blocks, a considerable number of replicates may be necessary. This article delves into the exploration of designs that minimize the required number of replications for factorial experiments conducted in blocks of size four. The article presents methodologies aimed at obtaining such designs, which hold promise for enhancing the efficiency and effectiveness of experimental investigations.

Keywords: Block designs; 2^n Factorial experiments; Confounding; Efficiency.

1. INTRODUCTION

Factorial experiments conducted in blocks of size two has received significant attention in recent literature. Draper and Guttman (1997) initiated research in this area, suggesting that k replications are necessary to estimate all factorial effects of interest for a 2^k factorial experiment. Subsequent studies by Yang and Draper (2003), Wang (2004), Kerr (2006), and Wang and Cook (2012) explored various designs and methodologies aimed at estimating main effects and two-factor interactions while considering different assumptions and objectives. Dash *et al.* (2013, 2014), Godolphin (2019a, 2019b), and Yadav *et al.* (2023) have emphasized block setup designs for two-level factorial experiments with the minimum number of replications to estimate important factorial effects.

In practical scenarios, experiments with block sizes three or four can be more useful than those with block size two. For example, in agriculture experiments, particularly in post-harvest technology experiments, designs with blocks of size four are often encountered. Similarly, experiments involving four testing wafers allowed in a lot, four units in a tray, or four experimental runs in a day or production line may be of more interest. However, the research on block size four has received less attention compared to block size two.

Wang (2016) investigated minimum number of replicates consisting of factorial experiments with blocks of size four and proposed a method to generate the design using orthogonal arrays to obtain such designs at least up to 2^8 factorial experiments. But sometimes, either due to resource limitations or also due to unavailability of lots of plots, it is not feasible to estimate all two factor interactions. Also, in some cases it is not possible for experimenters to estimate all two factor interactions or experimenter may want to estimate all main effects and subset of two factor interactions. Therefore, it is also necessary to establish construction procedures to obtain designs for this situation with the lowest number of replications for the estimation of main effects and particular two factor interactions.

In this article, we have introduced a construction method tailored for two-level factorial experiments in

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Corresponding author: Sukanta Dash
E-mail address: sukanta.dash@icar.gov.in

block size four which focuses on estimating all main effects along with specific two-factor interactions of interest, allowing researchers to hone in on particular interactions deemed crucial to the study's objectives. Consider an experimental situation involving 3 or 4 factors where experimenter is not interested in all pair of two factor interaction and interested only 2 factor interactions between some of the factor of interest. This method offer flexibility in experimental design, catering to various research goals and analytical needs.

2. PRELIMINARIES

Consider a 2^n factorial experiment having factors A_i are with levels $x_i \in \{0,1\}$. If $x_i = 0$, then A_i assuming at low level and, if $x_i = 1$, A_i is assuming at high level.

The statistical model used is as follows:

$$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}$$

Here, y_{ij} is the response from i^{th} treatment effect in the j^{th} block. μ is general mean and τ_i, β_j are the effects of i^{th} treatment combination and j^{th} block respectively. ϵ_{ij} are random error with variance σ^2.

3. METHODOLOGY

The paper discusses the limitations in estimating all two-factor interactions in factorial experiments, especially when experimenters are primarily interested in main effects and specific interactions. As the number of factors increases, maintaining homogeneity within blocks becomes challenging. In these cases, fractional factorial experiments are preferred over full factorial designs. However, there's limited research on efficient block designs for factorial experiments with a block size of four. The paper addresses this gap by investigating methods to construct efficient block designs with a block size of four, with specific construction methods outlined in Section 3.1.

3.1 Designs for two-level factorial experiments in block size four

This section introduces a construction method specifically designed for generating block designs tailored to factorial experiments with block size four. The method focuses on scenarios where certain interaction effects between factors are not of interest to the experimenter. The proposed design enables the estimation of all main effects and select two-factor interactions deemed relevant, providing researchers with a streamlined approach to analyzing the key interactions aligned with their study objectives. The detailed construction method is described below:

Method of construction

Construction method

The paper proposes a construction method for block designs aimed at estimating all main effects and specific two-factor interactions, tailored to the experimenter's interests in factorial experiments. This method minimizes the number of replications needed, optimizing efficiency. Specifically, the block designs are constructed for $2^n \left(3 \le n \le 9\right)$ factorial experiments where the experimenter wishes to estimate all main effects and most two-factor interactions, excluding a select few that are deemed irrelevant. The detailed construction procedure is presented in two steps, allowing flexibility to focus on only those interactions that are of primary interest.

Step 1: To construct a block design of size 4 for a 2^n factorial experiment, represented as (2^n, 2^2), the block containing (1) where all factors are at their lowest levels is designated as the principal block. Once the treatment combination assigned to (1) in the principal block is selected, the remaining blocks can be systematically constructed by choosing other sets of treatment combinations within the block. The estimability properties of the blocked replicate are determined based on the $2^n - 1$estimable effects that remain unconfounded.

Step 2: To construct the design, select blocking types in $\begin{pmatrix} 2^n - 1 \\ 3 \end{pmatrix}$ different ways. Among these $\begin{pmatrix} 2^n - 1 \\ 3 \end{pmatrix}$ blocking types, choose those combinations that enable the estimation of the desired factorial effects (all main effects and two-factor interactions) with the minimum number of replications. Let ttt represent the number of blocking arrangements or replications in which a given factorial effect remains unconfounded. The efficiency factor of a factorial effect is then defined as t / r, where is r is the total number of replications. Therefore, it is crucial to identify the specific interactions to be assigned as key elements in individual replications to ensure the desired efficiency and estimability.

Say $r = 2$ i.e. out of n factors, $(n-2)$ factors interactions are not of interest then for obtaining principal block select 3 different factorial effects to allot with key element using the procedure describe below

Replication 1:

Out of the three factorial effects to be assigned as key elements in the principal block:

i. One factorial effect will involve all factors for which two-factor interactions are not of interest.

ii. The second factorial effect will include all factors for which two-factor interactions are of interest.

iii. The third factorial effect will correspond to the highest-order interaction.

Replication2:

Out of the three factorial effects to be assigned as key elements in the principal block:

i. The first factorial effect will include all factors for which two-factor interactions are not of interest, along with one additional factor from the remaining factors.

ii. The second factorial effect will involve all factors for which two-factor interactions are of interest, excluding those selected in the first interaction.

iii. The third factorial effect will correspond to the highest-order interaction.

Keep the selected interactions with the key element in the first block (principal block) for both replications and rest of the $2^{(n-2)} - 1$ blocks can be constructed from the principal block for both replications.

If $r = 1$ i.e. if out of n factors, $(n-1)$ factors interactions are not of interest then for obtaining principal block select 3 different factorial effects to allot with key element using the procedure describe below

- Out of 3 factorial effects to be assigned with key element in principal block, one will be the factorial effect containing all factors those two factor interactions not of interest, second factorial effect will be containing all those factors which two factor interaction is of interest and third factorial effect will be highest order of interaction.
- Based on the above procedure factorial effect can be obtained to be allotted with key element. Then after getting the principal block other $2^{(n-2)} - 1$ blocks can be generated easily.

Example 1: Consider four factors *A*, *B*, *C* and *D*. A block design of size four for a 2^4 factorial experiment can be constructed in two replications to estimate all main effects and all two-factor interactions, except for the *AB* interaction, which is of least importance to the experimenter. Following the steps outlined above, this design ensures the estimation of all main effects (*A*, *B*, *C* and *D*) and two-factor interactions of interest, while excluding the *AB* interaction from consideration.

Select three specific interactions for each replication to be allotted with key element in the principal block and rest of the $2^{4-2} - 1 = 3$ blocks for each replication can be constructed using principal block. Here, AB, CD and ABCD interaction factors are selected for first replication and ABC, C and ABCD for second replication.

Replication 1				**Replication 2**			
B 1	**B 2**	**B 3**	**B 4**	**B 5**	**B 6**	**B 7**	**B 8**
0000	1000	0010	1010	0000	1000	0100	1100
1100	0100	1110	0110	1110	0110	1010	0010
0011	1011	0001	1001	0001	1001	1001	1101
1111	0111	1101	0101	1111	0111	1011	1100

If in above example, experimenter is interested in all main effects (*A*, *B*, *C* and *D*) and all two factor interactions except AB, BC & AC which is least importance for experimenter then, the design can be constructed by following above steps in one replication.

Select three specific interactions to be allotted with key element in the principal block and rest of the $2^{4-2} - 1 = 3$ blocks can be constructed using principal block. Here, ABC, C and ABCD are allotted with key element for obtaining principal block**.**

B 1	**B2**	**B3**	**B4**
0000	1000	0100	1100
1110	0110	1010	0010
0001	1001	1001	1101
1111	0111	1011	1100

Table 1. Efficiency factor of estimable main effects and two-factor interactions in block designs for 2^n factorial experiments $(3 \le n \le 9)$

Factor	r	A	B	C	D	E	AB	AC	BC	AD	BD	CD	AE	BE	CE	DE
3	1	1.00	1.00	1.00			0.00	1.00	1.00							
4	2	1.00	1.00	1.00	1.00		0.00	0.50	0.50	0.50	0.50	1.00				
	1	1.00	1.00	1.00	1.00		0.00	0.00	0.00	1.00	1.00	1.00				
5	2	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.50	0.50	0.50	1.00	1.00	1.00	0.50
	1	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00
		A	B	C	D	E	F	AB	AC	BC	AD	BD	CD	AE	BE	CE
6	2	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.50	0.50
	1	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		DE	AF	BF	CF	DF	EF									
		0.50	1.00	1.00	1.00	1.00	0.50									
		0.00	1.00	1.00	1.00	1.00	1.00									
		A	B	C	D	E	F	G	AB	AC	BC	AD	BD	CD	AE	BE
7	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00
		CE	DE	AF	BF	CF	DF	EF	AG	BG	CG	DG	EG	FG		
		0.00	0.00	0.50	0.50	0.50	0.50	0.50	1.00	1.00	1.00	1.00	1.00	0.50		
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00		
		A	B	C	D	E	F	G	H	AB	AC	BC	AD	BD	CD	
8	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	
		AE	BE	CE	DE	AF	BF	CF	DF	EF	AG	BG	CG	DG	EG	
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.50	0.50	0.50	0.50	
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
		FG	AH	BH	CH	DH	EH	FH	GH							
		0.50	1.00	1.00	1.00	1.00	1.00	1.00	0.50							
		0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00							
		A	B	C	D	E	F	G	H	I	AB	AC	BC	AD	BD	CD
9	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
		AE	BE	CE	DE	AF	BF	CF	DF	EF	AG	BG	CG	DG	EG	FG
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		AH	BH	CH	DH	EH	FH	GH	AI	BI	CI	DI	EI	FI	GI	HI
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.50
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

4. CONCLUDING REMARKS

The article introduces a method for constructing block designs for factorial experiments with a block size of four, aimed at estimating all main effects and specific two-factor interactions. This approach builds on Wang (2016), who studied similar designs for estimating all interactions. However, this method addresses scenarios where experimenters are only interested in a subset of two-factor interactions, making it more adaptable. The proposed construction is simple to apply, accommodates any number of factors, and requires fewer replications, enhancing efficiency. The article provides a catalogue of designs for 2^n factorial experiments with up to nine factors (i.e., n<10). Table 1 presents the efficiency factor, indicating the proportion of replications in which each estimable effect appears relative to a complete design. This method thus offers a practical and resource-efficient approach for experimenters needing targeted interaction estimates in factorial designs with block size four.

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REFERENCES

Box, G.E.P., Hunter, W.G. and Hunter, J.S. (2005). *Statistics for experimenters*. Second edition New York, Wiley.

Cochran, W.G. and Cox, G.M. (1957). *Experimental Designs*. Second edition, Wiley, New York.

Dash, S., Parsad, R. and Gupta, V.K. (2013). Row–column designs for 2^n factorial 2-colour microarray experiments for estimation of main effects and two-factor interactions with orthogonal parameterization. *Agricultural Research*, **2(2)**, 172-182.

Dash, S., Parsad, R. and Gupta, V.K. (2014). Efficient row–column designs in two rows. *Journal of The Indian Society of Agricultural Statistics*, **68(3),** 377-390.

Draper, N.R. and Guttman, I. (1997). Two-level factorial and fractional factorial designs in blocks of size two. *Journal of Quality Technology***, 29,** 71-75.

Godolphin, J.D. (2019a). Two-level Factorial and Fractional Factorial Replicates in Blocks of Size Two. *Computational Statistics and Data Analysis*, **133**, 120-137.

Godolphin, J.D. (2019b). Construction of row-column factorial designs. *Journal of the Royal Statistical Society*, Series **B**, **81**(2), 335-360.

Kerr, K.F. (2006). Efficient 2^k factorial designs for blocks of size two with microarray applications. *Journal of Quality Technology*, **38(4)**, 309-318.

Wang, P.C. (2016). Design Two-level Factorial Experiments in Blocks of Size Four, *Communication in Statistic- Theory and Methods*, **46(1),** 9-16.

Yadav, K.K., Dash, S., Mandal, B.N. and Parsad, R. (2023). Row-column designs for two level factorial experiments. *Journal of the Indian Society of Agricultural Statistics*, **77(2),** 233-236.

Yang, Y.J. and Draper, N.R. (2003). Two-level factorial and fractional factorial designs in blocks of size two. *Journal of Quality Technology*, **35(3),** 294-305.